

On the influence curves for effective widths in non-prismatic composite beams

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Some aspects of the phenomenon studied by the Author, namely the influence of partial interaction on shear lag action in composite beams, are elucidated by the results shown in Fig. 10. These results were obtained from finite difference analyses¹⁰ of two composite T beams, one having an infinitely rigid shear connexion and the other having a weak shear connexion. In the presence of interface slip there is a decrease in the magnitude of the average strain and in the influence of shear lag action on the strain distribution. The decrease in the average strain would be mainly due to the reduction in interface shear, and the decrease in the influence of shear lag action would be mainly due to the reduction in the interface shear gradient.

33. As noted by the Author, the widely accepted definition of effective width based on extensional stresses cannot adequately reflect this phenomenon; the Author is to be complimented on defining a rational effective width for use in the design of composite beams having any degree of interaction. However, it has been shown that the effect of partial interaction can be neglected in the design of composite beams with practical shear connexions.¹⁰⁻¹² Consequently, it appears that it will not be necessary to adopt for general use the new and rather complex definition proposed by the Author.

34. Although the Author's influence curves for effective widths of two-span composite beams (Figs 5 and 6) would obviously prove useful in the design of a beam having such a support condition, it would be difficult to apply them to beams with other support conditions. In this respect, investigations¹³ have shown that the effective widths of simply supported steel box beams can be conveniently used in estimating those of beams with any other support condition; the technique involves treating each portion of the beam between adjacent points of zero moment as an equivalent simply supported span.¹³⁻¹⁵ It is likely that a similar technique could be used for composite beams with partial interaction.

35. For a statically indeterminate beam, the distribution of the longitudinal bending moments associated with a particular load system is, in general, affected by a change in the moment of inertia along some portion of the beam. This well-known behaviour could perhaps be used to explain the obscure results noted in § 30; the introduction of the small amount of shear connexion could have modified the distribution of longitudinal bending moments to such an extent that, notwithstanding the increase in effective section modulus, the stresses under consideration showed a slight increase in magnitude. One would not expect to observe this apparent anomaly in the case of statically determinate beams.

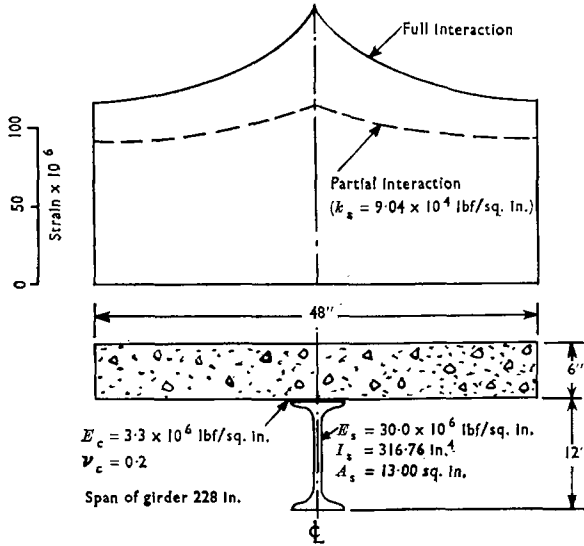


Fig. 10. Distribution of longitudinal extensional strain in slab at mid-span of simply supported composite beam under central point load of 3.36×10^4 lb

36. The Author noted that the utilization of composite action between a concrete slab and a steel beam can lead to a maximum reduction of 14% in steel bottom flange stress over that for the steel beam sustaining the loads alone. Although it is not clear from the Paper, this value of 14% must be only strictly applicable to the examples considered by the Author. The actual reduction in stress that can be achieved depends, of course, on the relative sizes of the concrete slab and the steel beam and can be estimated from the engineering theory of bending, provided that due allowance is made for shear lag.¹³

37. The Author is to be complimented on developing a series solution, for the analysis of composite T beams with partial interaction, that is more efficient than numerical methods such as the finite difference or finite element methods. However, by virtue of its generality, the finite element method can be readily used for studying the effect of partial interaction in much more complex structures. For example, we have developed a general purpose finite element computer program¹⁶ which is suitable for the study of such effects as slip and separation in multi-beam bridge decks as well as in composite plates and box girders.^{11, 17}

38. We are currently using this program to study the effect of shear connector distribution on the behaviour of composite box girder bridges. Details of some box girders being used in this study are given in Fig. 11; the girders are intended to be representative of those occurring in practice. The parameters of interest being varied are the shear connector distribution coefficient α (defined as the ratio of the stiffness of the shear connectors placed directly over the webs to the total stiffness of the connectors) and the breadth to span ratio b/a . The girders were analysed for a uniform load over the length of each web and for a point load over each web at the mid-span.

39. The maximum longitudinal extensional stresses in the concrete slab and the top steel plate at the mid-span of the girders are given in Table 3. In the case where

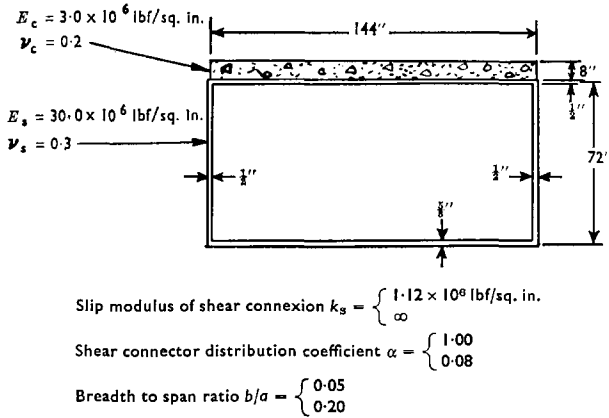


Fig. 11. Details of some simply supported composite box girders used in partial interaction parametric study

Table 3. Influence of shear connector distribution on maximum longitudinal extensional stresses in concrete slab and top steel plate at mid-span of composite box girders (the subscripts p and f signify partial and full interaction, respectively)

| | $\frac{b}{a}$ | σ_p/σ_f | | | |
|--------------------------------------|---------------|---------------------|-----------------|-----------------|-----------------|
| | | Concrete slab | | Steel plate | |
| | | $\alpha = 1.00$ | $\alpha = 0.08$ | $\alpha = 1.00$ | $\alpha = 0.08$ |
| Uniform load over length of each web | 0.05 | 1.00 | 0.98 | 1.01 | 1.04 |
| | 0.20 | 0.96 | 0.87 | 1.06 | 1.25 |
| Point load over each web at mid-span | 0.05 | 0.98 | 0.93 | 1.07 | 1.13 |
| | 0.20 | 0.93 | 0.80 | 1.12 | 1.38 |

Table 4. Influence of shear connector distribution on top flange effective widths at mid-span of composite box girders (ψ is the effective width ratio with respect to extensional stresses)

| | $\frac{b}{a}$ | ψ | | | |
|--------------------------------------|---------------|-----------------|-----------------|-----------------|-----------------|
| | | Concrete slab | | Steel plate | |
| | | $\alpha = 1.00$ | $\alpha = 0.08$ | $\alpha = 1.00$ | $\alpha = 0.08$ |
| Uniform load over length of each web | 0.05 | 0.98 | 0.99 | 0.97 | 0.96 |
| | 0.20 | 0.81 | 0.90 | 0.79 | 0.67 |
| Point load over each web at mid-span | 0.05 | 0.85 | 0.89 | 0.82 | 0.80 |
| | 0.20 | 0.53 | 0.59 | 0.51 | 0.45 |

the connectors are placed directly over the webs ($\alpha=1.00$), the stresses are not significantly affected by the connector flexibility. (A similar comment can be made about the stresses in T beams with practical shear connexions.¹¹) However, in the case where most of the connectors are uniformly distributed over the flange ($\alpha=0.08$), the stresses are significantly affected by the connector flexibility, especially for the girders with the larger b/a ratio. It would appear from these results that the connectors designed to transmit the longitudinal shear forces in a box girder should be confined to those regions of the flange immediately adjacent to the webs and that the width of these regions should reflect the b/a value of the flange.

40. The effective widths of the concrete slab and the top steel plate of the girders are given in Table 4. In the case where the connectors are placed directly over the webs ($\alpha=1.00$), the values of effective width are almost identical to those given in references 13 and 15 for equivalent steel box girders and, presumably, would also agree closely with those given by the Author's solution for equivalent T beams. However, in the case where most of the connectors are distributed over the flange ($\alpha=0.08$), the effective widths of the concrete slab increase whereas those of the steel plate decrease. These trends would be due to horizontal shear being transmitted from the steel plate to the concrete slab through the connectors distributed over the flange.

Professor Adekola

I should like to apologise for the wrong dimension of the overall depth of the steel section given in Fig. 3. This depth should be 625 mm, not 62 mm.

42. I should like to thank Dr Moffatt and Dr Lim for their discussion, especially the aspects dealing with my definition of effective width, the relevance to practice of the influence curves for effective widths at partial interaction, and the technique which they have developed of using simply supported steel box beams to simulate beams, presumably of any profile, with any other support conditions for the purpose of determining their effective widths.

43. I have used the finite difference approach for some years to study not only effective widths but also interface slip and uplift.^{6,18} I am also familiar with the work of Chapman and Teraszkiewicz.¹² It seems that recourse is often made to the finite differences or finite element approach when the problem at hand is too complex to deal with by a classical approach and/or when the research engineer is unable to find a suitable mathematical model. Problems that lend themselves to exact mathematical representation are better studied in depth in a generalized form by this approach. One can then easily test the validity of the results by obtaining known special cases through a limiting process. Such solutions also lead to much shorter computations than would be called for in the finite differences or finite element approach.

44. It is reassuring that the limitation of the effective width based on extensional stresses is appreciated by the contributors and one would have liked a simpler definition of a more rational effective width than the one given in the Paper. In as much as in day to day practice transformed section theory does not frighten the average practitioner, the approach adopted in determining the effective width as defined in the Paper, based as it is on this theory, would seem straightforward and simple. I must admit, however, that the definition in the Paper could have been less complex.

45. The reduction of 14% in steel bottom flange stress reported is only in respect of the section examined and cannot be said to be a universal value. Obviously when the effective width of slab is known for a given section, estimation of stresses when full interaction is assumed is simple, but the real point of interest is the determination of effective widths for various conditions of support and loading of a non-prismatic composite beam. I agree that one would almost always assume perfect connexion

between steel beam and concrete in design practice, so that effective widths at partial interaction may at first appear to have little relevance. In a practical section that is non-monolithic, is there really perfect connexion, and is it known at what stage of interface connexion this condition is approached? Through its incorporation of partial interaction the Paper has attempted to answer these questions.

46. Although the biharmonic equations which are the governing equations for deflexions and extensional stresses in a plate can be solved using finite differences or finite elements, one obtains a more exact solution and continuous deflexion and displacement surfaces using the exact mathematical solutions wherever they can be conveniently applied. The work on multi-beam bridge decks, composite plates and box girders Dr Moffatt and Dr Lim referred to is of great interest. In a non-prismatic continuous beam profile interacting with a concrete slab where there is imperfect shear connexion one does not know *a priori* the points of zero moment.

47. The effect of distribution of shear connexion on interaction is currently being studied at the University of Lagos. Composite action is so complex that even the common and apparently obvious assumption of constant shear connexion modulus for symmetrically located prismatic simply supported beams interacting with a concrete slab does not quite reflect the true picture.

References

10. TERASZKIEWICZ J. S. *Static and fatigue behaviour of simply supported and continuous composite beams of steel and concrete*. PhD thesis, University of London, 1967.
11. MOFFATT K. R. and LIM P. T. K. Finite element analysis of composite bridges having complete and incomplete interaction. In preparation.
12. CHAPMAN J. C. and TERASZKIEWICZ J. S. Research on composite construction at Imperial College. *Proceedings of the conference on steel bridges*. British Constructional Steelwork Association, London, 1969, 49–58.
13. MOFFATT K. R. and DOWLING P. J. *Parametric study on the shear lag phenomenon in steel box girder bridges*. Engineering Structures Laboratories, Imperial College of Science and Technology, London, 1972, CESLIC report BG 17.
14. FLINT A. R. and HORNE M. R. Conclusions of research programme and summary of parametric studies. *Steel box girder bridges*. Institution of Civil Engineers, London, 1973, 173–191.
15. COMMITTEE OF INQUIRY INTO THE BASIS OF DESIGN AND METHOD OF ERECTION OF STEEL BOX GIRDER BRIDGES. *Interim design rules*. Department of the Environment, London, 1973.
16. LIM P. T. K. and MOFFATT K. R. General purpose finite element program. *Proceedings of the symposium on bridge program review*. PTRC, London, 1971.
17. MOFFATT K. R. and LIM P. T. K. Discussion on steel–concrete composite plates with flexible shear connectors. *Proc. Instn Civ. Engrs*, Part 2, 1973, 55, 513–515.
18. ADEKOLA A. O. Analytical and relaxation solutions of shear lag problem in composite beams. *Civ. Engng Publ. Wks Rev.*, 1965, Nov., 1627–1634.