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DISCUSSION

Semi-analytic solution of skew plates in bending

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In the analysis the Authors adopt, for the strip displacement functions, a fifth order polynomial in conjunction with various harmonic functions satisfying different end support conditions. To accommodate the two extra arbitrary constants associated with the fifth order function (as compared to a cubic polynomial), nodal line curvature amplitudes (γ_{im} and γ_{jm}) are chosen as the additional unknown parameters. This procedure imposes curvature continuities at strip boundaries and as a result may not be effective in cases[7] where nodal line moment continuities are not feasible.

18. In an attempt to overcome this difficulty and to improve the cost-efficiency of this fifth order procedure, the Authors adopt a condensation procedure to eliminate the unknown curvature amplitudes from the stiffness equations of individual strips, i.e. in contrast to the conventional elimination of all unknowns from the overall matrix equation.

19. Although the condensation procedure has been applied successfully to eliminate (from individual strips) the unknown parameters associated with auxiliary (intermediate) nodal lines[8] in a higher order finite strip analysis of box girder bridges, we question the validity of the procedure used by the Authors.

20. The total potential energy U_T in a discretized system may be obtained by summing the contributions of individual strips[8]

$$U_T = \sum_{I=1}^{N} U_I \qquad (17)$$

where U_I is the total potential energy of strip I and N is the number of strips in the system. The minimum potential energy theorem requires that

$$\frac{\partial U_T}{\partial \{D_T\}} = \sum_{I=1}^{N} \frac{\partial U_I}{\partial \{D_T\}} = \{0\} \qquad (18)$$

in which $\{D_T\}$ contains the displacement parameters w_{im}, θ_{im}, γ_{im} and so on at all nodal lines. In view of the fact that U_I is written in terms only of nodal line displacement parameters at strip I, i.e. $\{D_I\}$, equation (18) becomes

$$\sum_{I=1}^{N} \frac{\partial U_I}{\partial \{D_I\}} = \{0\} \qquad (19)$$

or

$$\sum_{I=1}^{N} [S_I]\{D_I\} = \{F_I\} \qquad (20)$$

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in which $[S_I]$ and $\{F_I\}$ are the stiffness and force matrices respectively for strip I. Equation (20) represents the overall matrix equation and the summation sign indicates the assembling procedure.

21. It may be seen in § 20 that, except for those displacement parameters associated with auxiliary nodal lines[8] (which are not physically connected with nodal lines from other strips), no unknown may be eliminated before equation (20) is assembled if the minimum potential energy criterion is to be satisfied and the true solution obtained for the discretized model.

22. In the Authors' case the minimization with respect to the nodal line curvatures is not complete. The errors thus involved may not be appreciable in the gently varying moment fields for which the Authors quote results. The discrepancy between this incomplete solution and the true solution would be apparent for moments in the vicinity of highly concentrated wheel loads and in the regions of skew corners near supports.

23. Unlike the conventional analysis of simply-supported right and circularly curved structures, the proposed finite strip procedure for skew plates requires the solution of a large set of simultaneous equations (see equation (12)). In combination with the numerical procedure adopted for evaluating the individual stiffness matrices (equation (10)), this will lead to computational efforts considerably greater than a conventional strip procedure. As a result, the competitiveness of the proposed approach diminishes in the presence of the more versatile finite element procedure. This is particularly true when the analysis involves wheel loadings which require the use of greater numbers of strips and harmonics.

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Dr Loo and Professor Cusens express doubt but give no evidence that the condensation procedure applied in the Paper may lead to error in the moments in skew slab bridges in the 'vicinity of highly concentrated wheel loads and in the regions of skew corners near supports'. The Paper includes examples which show accurate results obtained by the skew strip solution when the load is uniformly distributed. This type of loading represents the major part of the load on actual bridges. Testing of the method for concentrated loading is done as a part of a study of the convergence of the skew finite strip solution. Skew finite strip analyses are applied to the plate shown in Fig. 1(a) subjected to a concentrated load P on one of its free edges. From the results in Table 6 it is apparent that the number of terms in the basic series has a more

Table 6. Convergence study; skew plate* $\alpha = 45°$, subjected to a concentrated load P (Fig. 1(a)); $a/l = 1{\cdot}0$, $\nu = 0{\cdot}2$

Number of strips	Number of terms	w_G	w_H	w_I	M_y point G	M_x point H	M_y point I
4	3	231	90·8	41·6	2290	857	282
8	3	231	90·8	41·6	2294	857	281
16	3	231	90·8	41·6	2294	857	281
8	3	231	90·8	41·6	2294	857	281
8	5	234	91·6	37·8	1652	909	350
8	7	242	91·7	35·7	1740	893	261
8	9	244	91·8	35·0	1488	912	313
8	11	245	91·8	34·3	2221	881	217
Multipliers		$(Pa^2/N) \times 10^{-4}$			$P \times 10^{-4}$		

* Edges AD and BC simply supported, AB and CD free.

significant effect on the convergence of the solution than the number of strips. No results obtained by other methods for this plate are available for comparison.

25. The skew bending strip has also been used in an extension of this research[9] for the analysis of skew box girder bridges. Accuracy of results has been reassured by applying the method to several bridges subjected to distributed loads or concentrated wheel loads, and comparing the results with values obtained by the conventional finite element method and by experiments on models.[10]

26. Dr Loo and Professor Cusens are concerned about the computational effort for the numerical evaluation of the integral in equation (10). Such integration requires a negligible computing time particularly when the structure can be divided into strips of one or two typical widths.

27. The finite strip analysis of right-angle or skew plates in bending requires the solution of a number of simultaneous equations ($2 \times$ number of nodal lines $\times$ number of terms in the series). In the case of a rectangular plate simply-supported, the equations uncouple and a solution for each term of the series can be found separately with saving in computing time. For this to be possible, the matrix $[S^*]_{mn}$ in equation (10) must vanish when m is not equal to n; this is not the case for skew strips or for rectangular strips with any of the two ends other than simply supported. In spite of this, inexpensive solutions are still possible.

28. The finite strip method offers the following advantages in the analysis of skew plates and skew box girder bridges. The results obtained (with smaller or comparable computing time) are generally as accurate as other methods of analysis, including the finite element method, yet—and more important—the finite strip method offers clear advantages in simplicity of use and preparation of data. Further, the finite strip method gives complete continuity of stresses along the length of each strip.

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