

Shear in web-reinforced concrete T beams

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In Paper 7632 the Authors assume that 'the major shear crack opens at the mid-depth at the middle of the shear span'. However, quite different behaviour is much more common. Often there are many equally significant cracks, and in other cases the critical crack is located differently, e.g. see Fig. 7 of Paper 7656.

D2. Shear stresses are apparently assumed to be uniformly distributed across the breadth of the flange. Physically this cannot be possible because within the web breadth there will be shear stresses at the soffit of the flange, and in the remaining $b - b_f$ such stresses are impossible. Further, so long as the main reinforcement is elastic, the neutral axis depths for beams with equal areas of main steel will decrease with increasing breadth b . However, the decrease will not be proportional to $1/b$, so the predicted shear strength must increase significantly with increasing flange breadth. This is not in fact the case as can be seen from Fig. D1.

D3. The resistance V_o is taken to be proportional to the depth $k_s d$. Thus it must be predicted that, if all other factors are constant, the shear resistance should not be influenced by the thickness of the flange. Again this is not the case as can be seen from Fig. D2 (results from reference D1).

D4. The derivation of $k_s d$ is not explained in detail, but the depth of the compressive zone is said to be 'related to the depth of the compression block at flexural failure, thus satisfying compatibility conditions.' Unless the beam is over-reinforced the two statements are contradictory as the ultimate neutral axis depth of an under-reinforced beam is determined only by strength and not deformation characteristics. It would appear that the Authors would predict variations of shear resistance dependent on the grade of the main steel, even if there were no yielding of it in any case.

D5. I have great reservations about any method dealing with only a single assumed location of failure. In reality it is necessary that all parts of a member should possess adequate strength, and methods such as that proposed here can tell the designer nothing about the distribution of force in the main steel (for curtailment) or the compression in the web concrete (web crushing failure) and so on.

D6. In Paper 7656 the Authors comment briefly on crack widths but seem to ignore two important points: first, the stage at which crack widths matter is the serviceability and not the ultimate limit state and, second, their use of very small beams may give a misleading impression of crack widths, which can be expected to scale up in direct proportion to the size of the member if the stirrup spacing is related to the effective depth.

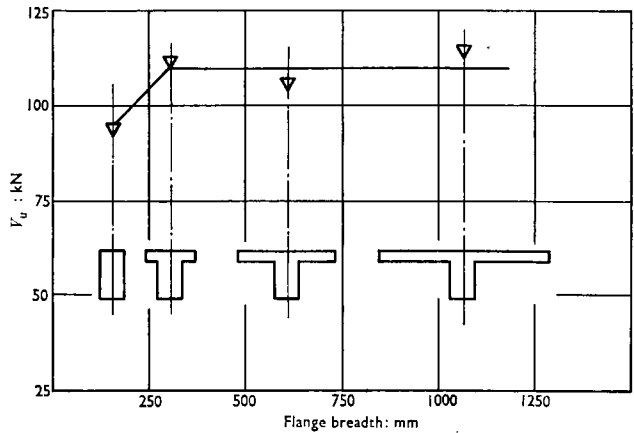
D7. The observation that the amount of shear reinforcement required to give truly ductile behaviour may be greater than that needed to develop the flexural capacity of a beam, is significant. However, neither Paper gives a real definition or

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solution of the problem. A calculation of shear capacity as a function of the ratio M/V_d is clearly not sufficient.

D8. It is widely thought that minimal shear reinforcement should be sufficient to prevent a failure occurring immediately upon shear cracking, i.e. that the stirrups should not fracture directly when inclined cracks form. The Authors' requirement that the ultimate shear resistance should be increased, even in cases where the ultimate capacity of a beam without stirrups would exceed the cracking load, is clearly more conservative. However, their work still does not seem to indicate any inadequacy in CP 110 as the largest minimum value of r_{fyw} mentioned by them is 0.30–0.48 N/mm² (44–70 lbf/sq. in.) and CP 110 requires percentages of stirrups corresponding to a value of 0.50 N/mm².

Fig. D1



All beams failed in shear
 $A_{sv} f_{yv} / b_w s_v = 0.58 \text{ N/mm}^2$
 $a_v / d \approx 5$

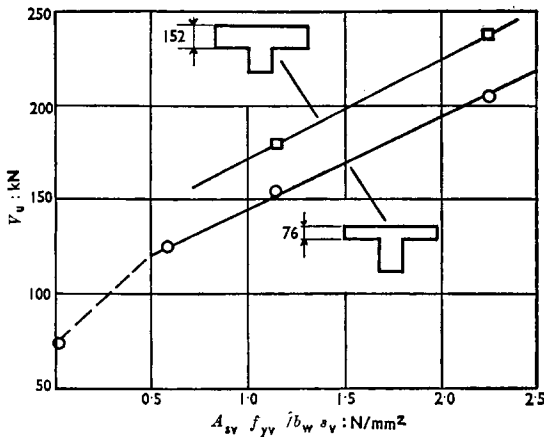


Fig. D2

D9. CP 110 calls for shear reinforcement to be designed to resist the difference between the applied shear force and the concrete's contribution to shear resistance, and limits the maximum spacing of stirrups to $0.75d$. There therefore seems to be no problem about the end zones near supports. The code certainly does not propose details such as those used in some of the reported tests, where lengths equal to $2d$ were left without web reinforcement.

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Dr Regan is correct about the fact that in a beam failing in bending and shear there may be more than one significant crack within the shear span, and that often the critical crack is located outside the mid-shear span. Indeed, failure need not occur either along the first diagonal crack to form. In the beams we tested and reported in Paper 7656 and elsewhere^{D2, D3} this is indeed so, and it has also been emphasized that the location of the failure plane can never be pre-determined satisfactorily. However, it would clearly be undesirable to produce different theories for each location of the failure plane under a particular mode of failure.

D11. Even though the critical shear span may be located outside the mid-shear span, we have shown in § 30 of Paper 7656 that there is a limit to the shifting of the critical diagonal crack towards the support and the loading points. Thus within the shear span there is a critical region within which the failure plane is most likely to occur. Having recognized this, we have made the simplifying assumption about the location of the failure plane to formulate a generalized single theory which would include all possible failure planes within the shear span. There is no reason why this assumption should invalidate the theoretical approach; indeed the fact that the theory correlates well with the results of tests of all beams with different locations of failure planes is evidence of the validity of the assumption.

D12. The uniform distribution of the shear stress at failure across the breadth of the flange is again an assumption. Obviously, the precise distribution of the shear stress in the compression zone is likely never to be known, because whatever method of computation is used will have to be based on certain assumptions.^{D4, D5} It is also true that the major part of the shear in the compression zone is resisted by the flange over and adjacent to the web^{D6} and we have also shown^{D2} that as the width of the flange increases the effectiveness of the added flange area to the resistance of the beam decreases; this is just as true in bending as in bending and shear. The assumption of a uniform shear stress distribution simplifies much of the calculation in the same way as in flexure where it is clearly understood that the actual shape of the stress distribution depends not only on the concrete strength and amount of tension steel but also on the strain gradient over the depth of the beam and along the length of the beam. In any case, as k_s is related to the width of the flange, the effects of assuming a uniform shear stress are corrected for in evaluating k_s .

D13. The ultimate shear capacity of a reinforced concrete T beam depends on the ratio of the flange to web width, the amount of tension steel and the moment-shear ratio. For a given area of tension steel and moment-shear ratio of 3.5, the ultimate shear strength increases with the b/b_r ratio up to about 3 and then remains sensibly constant.^{D2} In Fig. D1 the amount of tension steel is not given, and the results show that the ultimate shear resistance increases up to a b/b_r ratio of 2 for a moment-shear ratio of about 5 and then remains almost constant.

D14. However, our tests show^{D2} that for a given moment-shear ratio, the limiting b/b_r ratio up to which the shear capacity will increase depends on the amount of tension and web reinforcement. Fig. D1 can at best be regarded only as a trend and not the final answer to the effect of the flange width.

D15. Our theory will in fact predict this increase in ultimate shear strength related not only to the b/b_r ratio, but also to the amount of tension and web reinforcement, and moment-shear ratio, and it can be shown that the theory presented in Paper 7632 can satisfactorily predict test results with these variables. Dr Regan's argument of

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significant increases in shear resistance is based on the assumption that the main reinforcement remains elastic, but in fact this is not so. In the tests reported in the Paper 7656 and in reference D2 extensive steel strain measurements were taken throughout the span of the beam and, far from being elastic, the tension steel in the shear span can even yield locally, particularly in the vicinity where the critical diagonal tension crack meets the longitudinal steel.

D16. There is the further point that rectangular and T beams having the same web width and amount of tension reinforcement cannot be directly compared. The addition of the compression flange will clearly increase the shear-compression strength of the beam, but the longitudinal steel ratio based on web width then becomes unmeaningful because the effective compression zone determines the amount of tension steel required for balanced failure conditions. A comparison of rectangular and T beams would be more realistic from a design point of view if beams of equal p/p_b ratio are considered; if this is done it can be shown^{D2} that flange widths up to about seven times the web width can contribute effectively to the shear resistance of the beam. Most tests on T beams are carried out without any special attention to the composite action of the flange with the web, and tests show that in wide-flange beams the instability of the flange and splitting at the junction between the web and flange are major causes of failure. These prevent the effective mobilization of the flange from carrying a greater part of the shear. The implication is that although theoretically no shear stresses can exist at the soffit of the flange outside the web, the flange is nevertheless a part of the web, and in practice a part of the action of this integral beam-slab construction is lost. We consider that a more efficient structural performance could be obtained if the slab part of the beam can be more effectively tied to the web to achieve this shear transfer.

D17. Contrary to Dr Regan's assertion, our theory will also predict the influence of the flange thickness. The tests of series SR (group 6) (see Fig. 4 of Paper 7632) with varying flange thickness (25–50 mm) and effective depth were carried out to check the general applicability of our theory, and Table 2 of Paper 7632 shows that the theory also can predict these test results satisfactorily. We have shown that in beams without and with web reinforcement the thickness of the flange has an influence on the shear capacity. However, it appears that in Fig. D2 the thickness of the flange is out of proportion in relation to the total depth.

D18. We cannot see the apparent contradiction reported by Dr Regan in relating $k_s d$, the depth of the compression zone at shear failure section, to the depth of the compression block in the flexural zone. It is true that the neutral axis depth at failure in the pure bending region is influenced by the amount of steel in relation to the concrete strength, but that in the region of shear failure is influenced by the shear deformation characteristics as well. As the depth of the compression block in the flexural region is more clearly defined, both theoretically and experimentally, and because any computation of the neutral axis depth in the shear failure region would necessarily depend on the assumptions made, we have felt it more appropriate to relate $k_s d$ to the neutral axis depth in the flexural region. We do not see why our predictions should depend on the grade of the main steel; if Dr Regan implied the stress in the steel, this is so because it is the stress in the steel, whatever be its grade, at the failure section that influences the variation in the depth of the compression zone and hence the contribution of V_c .

D19. We appreciate that Dr Regan has some reservations about the assumption of a single location of the failure plane, but there is no need for this. When all the available test results on shear, including those of Dr Regan and ours, are analysed, it is clear that under combined bending and shear failure can occur in a variety of ways. However, most shear failures occur in a combination in the web, in the tension zone and in the compression area. Sometimes shear failure occurs along one plane; at the other times failure occurs along more than one shear crack. Sometimes the first shear crack that appears is not necessarily the crack along which ultimate failure occurs; sometimes the failure plane cuts across all the numerous inclined shear cracks

and the beam fails along an entirely new failure plane. It is thus extremely difficult to predict precisely the location of the failure plane under bending and shear. In any case the location of the failure plane *per se* is not the critical factor; the resultant stress distribution and deformations determine the shear capacity of the beams. The assumption of a single failure plane has the merit of simplifying the calculations and we would have thought it undesirable to produce a new analysis for each critical failure plane. This is not necessary because whenever the failure plane is assumed to occur it is related to the stresses and deformations, and the assumption would then be valid for all other locations of the failure plane. Indeed, we have shown that a single generalized theory can predict satisfactorily the results of all beam tests, irrespective of where the failure plane occurs.

D20. Neither we nor any designer would dispute the need for all parts of a structural member to possess adequate strength to resist whatever stress states and deformations they are subjected to, but this does not affect the assumption of the location of the failure plane for ultimate strength calculations. These are two different issues. Even within the shear span, not all sections are equally sensitive and critical to the shear resistance of the beam. We have shown that even when only part of the shear span is reinforced with stirrups, there is a limit to the shifting of the failure plane (§ 30, Paper 7656). Indeed there is a reasonably well defined region in the shear span within which the failure plane must occur, irrespective of whether there are one or more significant diagonal tension cracks in the shear span.

D21. We doubt if any theory can predict satisfactorily the distribution of force in the main steel in the shear region. Reference D1 does not make any such prediction either; most theories make an assumption on the state of the steel stress. The problem of curtailment is influenced by other factors as well, and there is no reason why every theory should predict all associated phenomena; however, the theory does predict the influence of a wide range of variables. The problem of web crushing does not generally occur with the normal proportions of flange and web used in practice in T beams, although the existence of this mode of failure is recognized in other cross-sections.

D22. Our comments on diagonal crack width were to emphasize the influence of web reinforcement and spacing of stirrups on crack width from the stage of initial shear cracking to failure and to show that the efficiency of the web reinforcement in controlling crack width does not vary linearly. This does not imply that the width of the crack is not important under service load conditions, the significance of which in relation to durability is well known. Unlike flexural cracks the width of the shear crack is difficult to measure precisely because of the unstable nature of the crack; wide variations have been shown to occur in measuring crack widths.^{D7} We have reservations about Dr Regan's equation for shear crack widths^{D1} because it includes the shear cracking load which itself is not easily defined or quantified, and cracking under bending and shear is influenced by other factors as well.^{D7} Our tests, like those of Dr Regan, are with laboratory size specimens because of necessity, but it is doubtful if the crack widths would scale up in direct proportion to the size of the member although the crack widths would be higher and different. In addition to other factors the ratio of aggregate size to size of member is a parameter which influences cracking, but our beams, unlike those reported by Dr Regan, are based on an extensive study of strength, cracking and deformation similitude of T beams under bending and shear.^{D7}

D23. One significant result of our investigation is that to ensure that flexural capacity is achieved with adequate ductility and to preserve the integrity of the beam, more effective web reinforcement is required than that calculated for the flexural load capacity. This requirement was reported in 1969.^{D8} The extent of ductility is not easily quantifiable but, we hope, is clearly understood by engineers as in the case of flexure. We have not yet quantified this because of our limited study. Nevertheless, the solution should be obvious to any designer, bearing in mind, in particular, that

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the efficiency of the web reinforcement does not increase in proportion to the quantity provided. The amount of web reinforcement obtained from shear considerations, as for example from CP 110, therefore needs to be substantially increased to obtain this ductility.

D24. Dr Regan's comment that it is widely thought that minimal shear reinforcement is adequate is difficult to accept. It is not enough that all parts of a member have adequate strength (§ D5). If a beam is designed for a given flexural capacity it is then necessary to ensure that this is achieved not only with adequate ductility but also without any distress due to shear that would impair the integrity of the beam. The function of web reinforcement is not only to ensure the full flexural capacity but also to control the diagonal crack width and to provide resistance against opening of the cracks at all stages of loading. Further, this should be achieved with ductile characteristics, and at no load stage should there be the danger of a diagonal crack opening up and precipitating a premature brittle type of failure. The fact that such a danger always exists even when flexural capacity is achieved is shown by our tests and the tests of Kani.^{D9} The provision of minimal shear reinforcement is then clearly not adequate for major structural members.

D25. There appears to be a fallacy here. Is the minimal reinforcement provided to prevent stirrups from fracturing? In the hundreds of tests we and our co-workers have carried out the fracture of a stirrup is an exception even in beams grossly under-reinforced in the web. Therefore this is not a valid criterion in deciding minimal shear reinforcement. It is not simply a question of whether a particular requirement of minimum shear reinforcement is conservative or not; rather the question is whether minimum shear reinforcement provided would enable a structural member not merely to reach its flexural capacity but enable it to achieve this with full ductility. This is a necessary engineering criterion of design, and this requirement has great practical significance as events have recently shown. Whereas it is often possible to improve the flexural capacity of a beam in an existing structure, inadequate shear reinforcement can never be satisfactorily remedied.

D26. The value of rf_{sw} of 44–70 lbf/sq. in. we reported is the minimum required to make the web reinforcement effective and not that which is required to produce either flexural failure or flexural failure with ductility. The requirements of CP 110 are clearly inadequate in this respect. Further, our tests clearly show that the location of the end stirrups is critical to the shear capacity and the integrity of the beam; we would prefer to see these specified unambiguously.

D27. The fact that we have carried out tests without web reinforcement for lengths of $2d$ from the support does not mean that beams should be designed that way or that CP 110 should specify such details. Likewise the fact that tests on beams without web reinforcement are carried out does not mean that beams should be designed without stirrups. We hope that this is not the type of inference derived by Dr Regan. The investigation of the influence of various parameters in a research study is necessary for the development of adequate and relevant design criteria, and in shear there are still many factors the effects of which are not fully established.

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