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7686 DISCUSSION

A mathematical examination of urban run-off prediction

S. G. NEWTON & R. B. PAINTER

Dr Newton and Dr Painter

A mistake in the simple example given at the end of the Paper was noticed shortly before publication, but unfortunately not in time to rectify the error.

51. In § 26, line 4, and in § 41, lines 8 and 10, Q_L should read $Q_L{}^*$. The last sentence in § 41 should read, 'This discharge at outfall has a peak of approximately 30·5 l/s.' The dashed line in Fig. 11 is incorrect and should be ignored.

52. The mistake originated from an error in the calculation of storage in the example. To clarify the situation a revised version of Fig. 11 is shown here.

53. These corrections in no way affect the main argument or conclusions of the Paper.

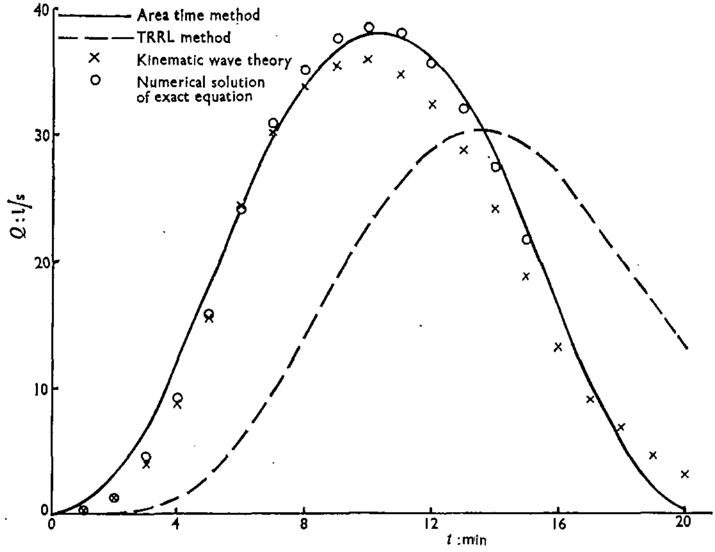

Fig. 11

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DISCUSSION

Mr R. Hepworth, M

The Authors take care in differentiating between the flow and wave velocities but then show that for given conditions the two are approximately equal. This presumably justifies the use of tabulated full bore velocities in the Lloyd-Davies calculation[4] and in area–time diagrams, which I agree are reasonable methods—until time of entry is considered.

55. I agree with the suggestion that the TRRL method falls down by neglect of consideration of surface storage (not that this is the only reason). The originator of the method appreciated that the area–time diagram gave too high a maximum flow but then proceeded to reduce the flow by an illogical method, for routing the flow through the sewer system after an area–time calculation is illogical.

56. I have no comment to make on the derivation of the exact formula quoted in the Paper, but I do not believe such a formula is necessary in practice. The Authors appear to be falling into the trap of considering the storm from the start of rainfall. It is more beneficial to consider the storm from the start of run-off, or better still from the time of maximum run-off.

57. At the time of initial run-off the water at the point in the pipe under consideration must necessarily have fallen on the surface which is nearest to the point on the pipe and scrutiny of rainfall/run-off records shows that this is as a result of high intensity rainfall in the middle of the storm. At the same time other points on the pipe are receiving water from adjacent areas and this water proceeds down the pipe under gravitational forces.

58. At the time of maximum flow, which the same records show to be at about the end of the storm, a maximum number of areas are contributing to the maximum flow. Generally the furthest area contributes with an intensity of rainfall similar to that which created the first run-off at the point under consideration and the nearest area contributes with a much lower intensity at the end of the storm. Intermediate areas contribute with intermediate intensities. The maximum run-off does not necessarily come from the whole area but more probably from part of the area, because it is unlikely that the length of the storm is critical for the area on which it falls.

59. Before the period of rainfall which causes maximum flow, sufficient rain falls to generate flow from the surface, and during the period of rainfall which causes maximum flow a two minute time of entry is applicable.

Table 1

Time, *h min*	Impermeable area, *acres*	Intensity, *in./h*	Run-off increment, *cusecs*
14 34	2·4	0·45	1·08
14 33	2·4	0·5	1·20
14 32	4·8	0·75	3·60
14 31	8·5	0·85	7·23
14 30	10·9	0·85	9·27
14 29	14·6	0·7	10·22
14 28	14·6	0·75	10·95
14 27	15·8	1·6	25·28
14 26	15·8	3·1	48·98
14 25	10·9	1·8	19·62
14 24	9·8	3·0	29·40
14 23	3·6	2·5	9·00
14 22	2·4	2·2	5·28
14 21	2·4	3·2	7·68
14 20	2·4	4·3	10·32

60. Using these principles rainfall and run-off records can be correlated. For the storm at Oxhey, area 11 on 5 June, 1954, considered by Watkins,[13] the time of peak run-off was 14 h 36 min. If a time of entry of two minutes is allowed a theoretical flow of 199·11 cusecs is calculated as shown in Table 1.

61. The maximum intensity of rainfall occurred after 14 h 20 min. The storm started effectively after 14 h 7 min, finished after 14 h 36 min and lasted 29 minutes. The actual measured flow for this storm was 200 cusecs and so the 199·11 cusecs compares favourably. Fifteen increments of time–area diagram, each multiplied by its own respective rainfall intensity, were required to give the maximum run-off.

62. This storm was critical for the whole of the catchment area of 121 impermeable acres and the length of storm was approximately twice the time of flow of the full area. The examination of records of 47 storms has shown the reliability of this correlation method.

63. Much of the Authors' time and effort could be reduced in this way. In particular the storm should be considered from the time of maximum flow; the size of the area producing the maximum flow is not necessarily the full area, but depends on the length of the storm.

64. The Authors suggest that surface water storage is of the same order of magnitude as pipe storage. Do they agree that the correlation method described approximates to this suggestion?

Dr R. K. Price and Mr R. W. Pethick, Hydraulics Research Station

As the Authors indicate in § 51, the TRRL method for the pipe model gives a peak discharge at the outfall of approximately 30·5 l/s. We also found that when we ran a computer program for the TRRL method the peak of the resulting hydrograph

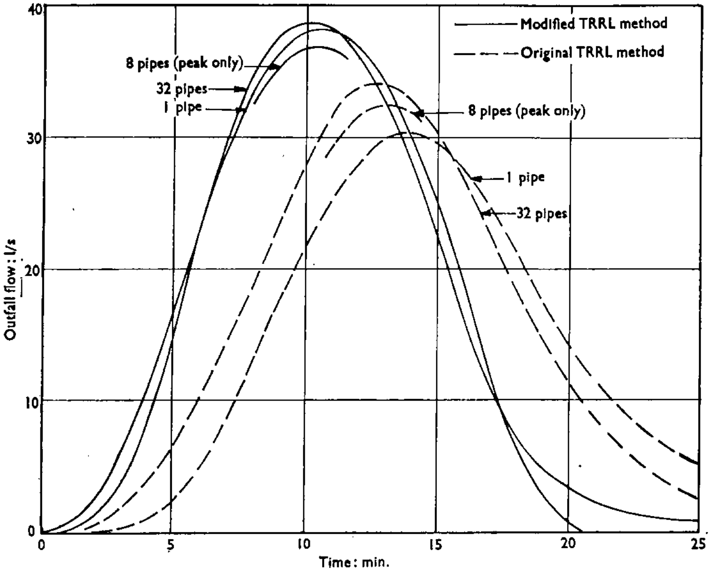

Fig. 12. Outflow hydrographs for the modified and original TRRL methods

had a lag of about three minutes compared with the peak flow of the other methods. Obviously too much storage is created by the TRRL method.

66. To remove this additional storage we made two amendments. The first-stage routing of the combined hydrograph formed by the results of the area–time method and the hydrograph from the pipe immediately upstream was removed. Then, after the program had routed the hydrograph from the previous pipe through the current pipe, we added the hydrograph produced by the contributing area. Obviously for the Authors' example of one pipe this modified TRRL method gives the area–time results, so to test the modified method and also to examine in more detail the storage routing procedure adopted by the TRRL method, we divided the original pipe into a number of sub-pipes, and apportioned the lateral inflow accordingly. Fig. 12 shows the results for 1, 8 and 32 pipes using the modified TRRL method and the corresponding results for the original method. In the modified TRRL method the storage routing produces a variation of about 4% in the peak discharge, although the corresponding variation for the original method is about 13%.

67. One can deduce from these results that the modified TRRL method as described here produces a more accurate description of the time of travel and peak discharge in steep pipes than the original TRRL method. However, in extensive pipe systems the original TRRL method may give a final peak discharge which is approximately correct, with a possible increase in the time of travel of the peak compared with the time from the modified method. This may go part of the way towards explaining why the original TRRL method can give reasonable results.[7–9] With the modified TRRL method it would be necessary to increase the time of entry to each pipe in order to reproduce similar results for these catchments. This supports the Authors' suggestion that there is considerably more storage in the surface run-off process than has previously been acknowledged. They recommend that the urban run-off problem should be regarded as two processes—surface run-off and pipe flow—and defining the entry time as the time of concentration for the surface flow they give an improved expression for Q_L in equation (48). As they point out, the expression differs markedly from equation (46) when the rainfall curve is sharply peaked. However, for a long period storm and a pipe system for which the time of concentration for the pipes alone is considerably greater than the time of concentration for the surface flow, the difference between the two expressions is marginal. It can be deduced therefore that a precise definition of the time of entry may only become important for smaller catchments; for larger catchments improved predictions may be obtained by adopting a time of entry greater than previously advocated and by using the modified TRRL method. The accuracy of the method in this latter case will also benefit when the engineer has a reliable method of calculating, rather than estimating, the (global) time of entry.

68. Further research on the surface run-off process should isolate the relevant parameters, although it is to be hoped that the values of such parameters in particular cases will be easy to calculate. This raises the question of whether or not an approach based on deterministic hydraulic principles will produce such parameters, particularly in view of the fact that the run-off process is much more complex than the flow in a pipe system. It is probable that a statistical approach to run-off from urban areas will be better in isolating the parameters which are useful to the drainage engineer.

69. We would also draw attention to the fact that the analysis carried out by the Authors should be restricted to catchments and pipe systems which are reasonably steep, i.e. with gradients of the order of 1 in 100. This is because the kinematic wave approximation is usually adequate for describing the flow in such systems. However, in flatter catchments and pipe systems dispersive effects can become important and should accordingly be allowed for. The range of conditions under which kinematic wave theory or the area–time method are sufficient to describe pipe flow are therefore limited.

70. A considerable amount of work is still needed before it is possible to produce accurate routing methods for flow in pipes when dispersive effects are important and when surcharging and backwater effects dominate the flow.

Dr T. M. Prus-Chacinski, C. H. Dobbie and Partners

Why do the Authors say in § 45 that the TRRL method makes double allowance for storage?

72. Did they consider the surcharge?

73. Do they consider that the example selected by them is fair to the TRRL method? Sometimes a given situation suits some methods of estimation better than others.

Dr M. J. Hall, Department of Civil Engineering, Imperial College of Science and Technology

Before the introduction of the so-called rational method at the beginning of the twentieth century, drainage systems in the UK were designed on the basis of flat-rate rainfall intensities. British literature credits Lloyd-Davies[4] with the introduction of the rational method, whereas American literature refers to an earlier paper by Kuichling.[14] However, Dooge[15] has pointed out that the principles of the rational method were clearly expounded by Mulvaney[16] in 1851 and his account compares favourably with that provided a century later by Williams.[17]

75. One of the most critical steps in the application of the rational method involves the choice of a suitable run-off coefficient. This coefficient is strictly defined as the ratio of the peak rate of run-off to the average rate of rainfall during the time of concentration but, using the assumptions of constant transmission velocity and an average rainfall intensity which is both temporally and spatially uniform, the same coefficient may also be shown to equal the ratio between the total volume of run-off and the total volume of rainfall.[17] In defining p as the proportion of rainfall which runs off (§ 9) the Authors appear to have taken the identity for granted. Nevertheless, because transmission velocities are not constant in nature and rainfall rates are never obligingly uniform in both time and space, the problem of determining values of the run-off coefficient for design purposes remains. Many designers appear to use the proportion of impervious area within the catchment as an approximation. An attempt to verify this assumption was made by Meek,[18] but the results obtained were only fully explained after further work by Appleby.[19] Watkins[5] presented further evidence on this but much more work will be required both in the field and in the laboratory before a better understanding of the contribution made by the impervious area within a catchment is obtained.

76. The inability of the rational method as presented in equation (14) to deal with catchment areas in which the rate of increase in contributing area is variable led to the introduction of design methods based on the use of the time–area diagram. The various tangent methods[20–23] are biased in their estimation of peak run-off rates and have now largely been abandoned. The approach treated by the Authors in §§ 12–24 and typified by equation (32) is perhaps more generally referred to as the typical storm method. Apparently first introduced by Ross[24] in 1921, the many versions of this method differ only in the variation of rainfall intensity with time, which is assumed for a given frequency of occurrence. Arbitrary storm profiles constructed from the rainfall intensity–duration–frequency relationship for a given recurrence interval were proposed by several authors[24–27] and few appear to have used the more logical alternative of a recorded storm profile.[28] Whether or not arbitrary times of entry are added to times of flow to form the times of concentration used in constructing the time–area diagram, the peak rate of run-off which results from applying equation (32) depends intimately on the form of the storm profile, as the Authors point out in paragraph (34). The peak run-off rate is obtained when the

contribution from the highest R_{i-j} falling on the largest $\delta\alpha_{j+1}$ reaches the outfall. The Authors show in § 32 and Figs 6–8 that the use of the time of entry can result in different sequences of increments in contributing area $\delta\alpha$, depending on the manner in which the total area is subdivided. However, if the original[20] basis of subdivision is applied, i.e. sub-catchments for which contributing area could reasonably be assumed directly proportional to time up to the time of concentration are used, the peak rate of run-off is perhaps less sensitive to this source of error than that which accrues from the use of an arbitrary storm profile.

77. One of the most interesting aspects of the Paper is the proof that the rational method and typical storm method can be described by special solutions of the kinematic wave approximation to the St Venant equations. Nash[29] showed that the same design methods were particular cases of the unit hydrograph method. Nash's results follow directly from the equation which gives the ordinate of the surface run-off hydrograph at time t, $q(t)$, as the integral of products of the volume of effective rainfall during a time increment $\delta\tau$, $R(\tau)$, and the ordinate of the instantaneous unit hydrograph at time $t-\tau$, $U(t-\tau)$

$$q(t) = \int_0^t U(t-\tau)R(\tau)\mathrm{d}\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

78. For the rational method, comparing equation (14) with equation (61) shows that for $0 \leqslant t \leqslant t_c$

$$U(t-\tau) = Bp/t_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

i.e. the rational method is equivalent to the use of the unit hydrograph method with an instantaneous unit hydrograph having a constant ordinate from $\tau=0$ to $\tau=t_c$ and a zero ordinate outside this range.

79. For the typical storm method, comparing equation (33) with equation (61) yields

$$U(t-\tau) = p\frac{\mathrm{d}}{\mathrm{d}t}\left[\alpha(t-\tau)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

i.e. the instantaneous unit hydrograph is equivalent to the derivative of the time–area diagram.

80. While conceding that the kinematic wave approximation may provide the more rigorous description of the run-off process, I am inclined to believe that the analogy with a linear time-invariant system, which is implied by equation (61), may be more useful for design purposes. To date, techniques of linear systems synthesis which use simple conceptual models of the linear reservoir type have found little favour in urban hydrology. The use of two linear reservoirs in series, with the first representing the overland flow storage and the second the pipe storage, by Sarginson and Bourne[30] has provided a notable exception to this trend. Provided that the parameters of such models can be related to site or storm characteristics or both, the construction of a design hydrograph is reduced to a simple iterative procedure. Swinnerton *et al.*[31] have shown the use of this approach for the drainage of motorway carriageways running in cutting, using an alternate parameter linear reservoir. The general implication of the Authors' approach tends to suggest that they foresee the development of new design procedures based on the kinematic wave approximation or even the full shallow water equations. If so, would they enlarge on their proposals and state whether such techniques would have the simplicity and economy in application for the accuracy of forecasting of which conceptual model design methods have been shown to be capable?

Mr L. H. Watkins, Mr C. P. Young and Mr D. Fiddes, Transport and Road Research Laboratory

One of the main conclusions of the Paper is that the TRRL hydrograph method 'would in general cause overestimation of the peak discharge'. It has brought many enquiries to the TRRL from practising engineers thinking that the method may be leading to overdesign. The invalidity of this conclusion is evident from the results of about 280 comparisons of recorded and predicted flood hydrographs published in reference 13.

82. We wish to assure practising engineers that the method does not in general cause overestimation of peak discharges and to indicate where the weaknesses lie in the evidence brought forward by the Authors to support their conclusion.

83. Recently the TRRL method has been the subject of two independent enquiries,[32, 33] both of which comment favourably on the method. We are conducting an investigation in East Africa to establish what modifications are required to the standard method to allow for the significant run-off from unpaved areas often observed in the tropics towards the end of the rainy seasons. Our results confirm that at the beginning of the rains, when conditions are similar to those assumed for design purposes in the UK, the method works well.[34]

84. The Authors give no experimental evidence to support their conclusions, and so, because all available experimental evidence supports the supposition that the method gives a close approximation to reality under a wide range of conditions, it must be concluded that some of the Authors' assumptions are invalid.

85. It is a pity that in deciding that the storage within the sewer system was comparable to surface storage the Authors looked only at the two smallest catchments studied by the TRRL. If they had looked at larger catchments (e.g. Oxhey main area) they would have found this not to be so—such areas contain much larger sewers.

86. The only support for the Authors' conclusion appears to be a hypothetical worked example using a very long non-tapering pipe. We know by experience

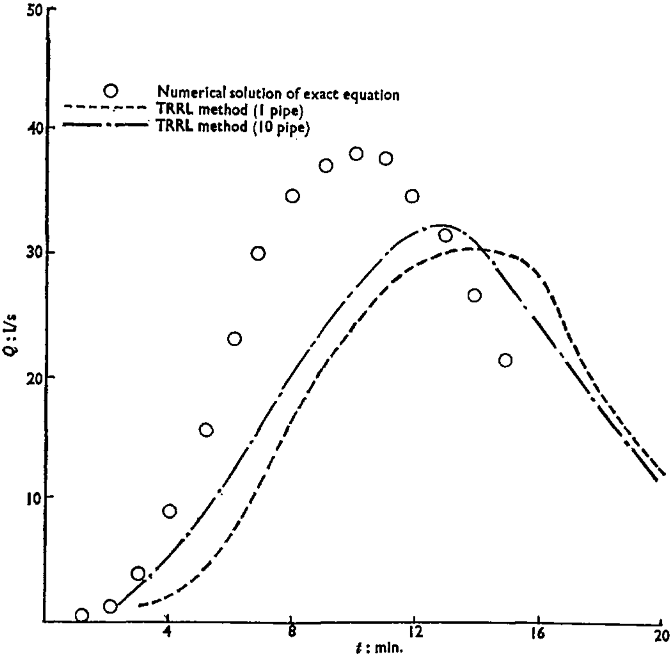

Fig. 13

under these circumstances that the hydrograph method tends to underestimate, not overestimate, the peak flow. Splitting the pipe into reaches for the computation gives a closer fit to recorded flows. Using the standard hydrograph method as used by most local authorities in the UK and the input data quoted in the Paper, the calculation produces the hydrographs shown in Fig. 13. The line for one pipe corresponds to the Authors' example; the line for ten pipes is more in accord with recommended practice. These lines confirm our expectation in what is an artificial example. This slight underestimation does not matter for design purposes; for small areas the rational and TRRL methods produce the same sewer systems.

87. Those engineers who use the method can continue to have confidence in it. If a new method is ever produced that field data show to be superior, we shall be the first to acknowledge it.

Dr Newton and Dr Painter

The correlation method described by **Mr Hepworth** appears to be equivalent to lagging the rainfall by the entry time (two minutes) and then performing an area–time calculation. It follows that the peak run-off is obtained by applying this calculation in the period equal to t_c preceding the time of peak flow. However, it is necessary first to determine the time at which the peak flow occurs. Under this interpretation the correlation method does not account for surface water storage as well as pipe storage.

89. The modified TRRL method prepared by **Dr Price** and **Mr Pethick** gives a very similar result to the area–time calculation, which is not surprising as the storage–discharge relation for circular pipes is approximately linear. Clearly it is dangerous to extrapolate from a single simple example, but because the dimensionless relationship between proportional discharge and proportional storage is virtually the same for all circular pipes, it is reasonable to expect close agreement between the area–time result and that of the modified TRRL method.

90. We agree with the point made in § 69 and would emphasize that the domain within which kinematic wave theory is valid needs to be established.

91. In reply to **Dr Prus-Chacinski** the TRRL method makes double allowance for pipe storage by using the area–time method and then routing the result through the pipe reservoir. Surcharge conditions were not considered in the Paper. The simple example given in the Paper demonstrates the significant difference between the TRRL solution and that obtained by numerical solution of the shallow water equations.

92. We would query the statement made by **Dr Hall** that 'The peak run-off rate is obtained when the contribution from the highest R_{i-j} falling on the largest $\delta\alpha_{j+1}$ reaches the outfall'. Consider the trivial example given in Table 2. Q_i obtained from formula (32) has a peak value of 33 at $i=5$, whereas the highest R_{i-j} falling on the largest $\delta\alpha_{j+1}$ reaches the outfall at $i=3$ (the product of R_2 and $\delta\alpha_2$).

93. Concerning § 80 we would agree that linear reservoir models, because of their simplicity, should be tested on urban systems. However, it is probable that a linear

Table 2

i	R_i	$\delta\alpha_i$	Q_i
1	2	2	4
2	4	4	16
3	3	3	28
4	2	2	32
5	3	1	33
6	0	0	28

model will be inadequate to represent the surface run-off phase of the urban regime, because of the non-linearity of the storage flow relation for overland flow.

94. There are several points we should like to clarify in response to the contribution by **Messrs Watkins, Young and Fiddes**. In § 81 they preface a quote from the Paper with their own subject and imply a statement which we never made. They are presumably referring to § 48 where the following statement was made: 'However, the method by which the TRRL method constructs the area–time diagram using the entry time seems unreliable and can result in artificially steep slopes in the curve which would in general cause overestimation of the peak discharge.' It should be noted that counteracting this possible overestimation in the result of the area–time diagram, is the reduction in peak caused by routing through pipe storage. In the simple example given, the latter effect dominates (the former having no effect in this case of a straight line area–time diagram) and the TRRL method underestimates the peak flow by approximately 21%.

95. Aitken,[33] in comparing the TRRL method with the rational formula applied to cities in Australia with similar hydrological conditions to those in the UK concluded, 'The application of the RRL Hydrograph Model . . . is considered equally satisfactory to that of the application of the Rational Formula.'

96. The reason why Oxhey Road and Kidbrooke were used in comparing pipe storage to surface storage was not a subjective choice; these were the only two areas for which values of the total volume of the pipes were available. It is not surprising to learn that areas such as Oxhey main area 'contain much larger sewers'—because it has an area of 611 acres compared with 1·12 acres for Oxhey Road—the important figure is the ratio of the total pipe volume to the impermeable area.

97. The underestimation of 21% in the peak flow by the TRRL method may be 'slight' but this should be considered against the fact that the much simpler rational method gives a peak which is within one per cent of the numerical solution of the exact shallow water equation.

98. Concerning the validation of the TRRL method using results from 286 storms,[13] at that time Manning's formula (or occasionally Crimp and Bruges' formula) was used to calculate flow velocities and to obtain the area–time diagram and retention characteristics for a system. We have been unable to ascertain why a value of 0·009 was often adopted for Manning's n as this is roughly the value appropriate to glass or polished brass pipes.

References

13. WATKINS L. H. *The design of urban sewer systems.* HMSO, 1962, Road Research Technical Paper 55.
14. KUICHLING E. The relation between the rainfall and the discharge of sewers in populous districts. *Trans. Am. Soc. Civ. Engrs*, 1889, **20**, 1–60.
15. DOOGE J. C. I. The rational method for estimating flood peaks—Irish contributions to the technique. *Engng, Lond.*, 1957, **184**, 311–313, 374–377.
16. MULVANEY T. J. On the use of self-registering rain and flood gauges in making observations on the relation of rainfall and of flood discharge in a given catchment. *Trans. Instn Civ. Engrs Ire.*, 1851, **4**, No. 2, 18–31.
17. WILLIAMS G. R. Hydrology. *In Engineering hydraulics.* ROUSE H. (ed.), John Wiley, New York, 1950, ch. IV, 229–320.
18. MEEK J. B. L. Sewerage, with special reference to runoff. *Engineering Conference Report.* Institution of Civil Engineers, London, 1928. Discussion, 162–174.
19. APPLEBY F. V. Impervious factors. *Proc. Instn Munic. Cty Engrs*, 1937, **63**, 1077–1100.
20. REID J. The estimation of storm-water discharge. *Proc. Instn Munic. Cty Engrs*, 1927, **53**, 997–1027.

21. Riley D. W. Notes on calculating the flow of surface water in sewers. *Proc. Instn Munic. Cty Engrs*, 1932, **58**, 1483–1494.
22. Norris W. H. Estimation of runoff from impervious surfaces. *Proc. Instn Munic. Cty Engrs*, 1946, **72**, 425–438.
23. Escritt L. B. *Sewerage and sewage disposal.* C. R. Books, London, 1965, 3rd edn.
24. Ross C. N. The calculation of flood discharges by the use of a time contour plan. *Trans. Instn Engrs Austr.*, 1921, **2**, 85–92.
25. Judson C. C. Runoff calculations, a new method. *Proc. Instn Munic. Cty Engrs*, 1933, **59**, 861–867.
26. Ormsby M. T. M. Rainfall and runoff calculations. *Proc. Instn Munic. Cty Engrs*, 1933, **59**, 889–894.
27. Hart C. A. Correspondence on rainfall and runoff. *Proc. Instn Munic. Cty Engrs*, 1933, **59**, 978–980.
28. Coleman G. S. and Johnson A. Rainfall runoff. *Proc. Instn Munic. Cty Engrs*, 1932, **58**, 1403–1415.
29. Nash J. E. Determining run-off from rainfall. *Proc. Instn Civ. Engrs*, 1958, **10**, 163–184.
30. Sarginson E. J. and Bourne D. E. The analysis of urban rainfall run-off and discharge. *J. Instn Munic. Engrs*, 1969, **96**, 81–85.
31. Swinnerton C. J. *et al.* Conceptual model design for motorway stormwater drainage. *Civ. Engng Publ. Wks Rev.*, 1973, **68**, 124–132.
32. Stall J. B. and Terstriep M. L. *Storm sewer design—an evaluation of the RRL method.* US Environmental Protection Agency, 1972, Report EPA-R2-72 068.
33. Aitken A. P. *Hydrological investigations and design in urban areas—a review.* Australian Water Resources Council, Canberra, 1973, Technical Paper 5.
34. Forsgate J. A. and Temiyabutra S. *Rainfall and runoff from an industrial area in Nairobi, Kenya.* Road Research Laboratory, Crowthorne, 1971, Report LR 408.