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7778 DISCUSSION

Non-uniform shear wall–frame systems with elastic foundations

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The Author's solution of the shear wall–frame interaction problem in the form of a single second order differential equation, derived from the consideration of complementary energy, is commendable. The total energy of the system using the Author's approach is always upper bound and so, unlike displacement approaches, the solution will converge down to the exact values.

32. A similar analysis of the same problem has been presented by Heidebrecht and Stafford Smith[10] who used a displacement approach for the derivation of the governing second order differential equation. Their solution is presented in both analytical and graphical form which allows the consideration of shear wall–frame structures with a wide range of heights and/or relative stiffnesses. A transfer matrix treatment is presented for non-uniform shear wall–frame structures.

33. We have some reservation about equation (22). For a fixed base the conditions are

$$K_{\theta w} = K_{\theta t} = \infty$$

Therefore equation (22) becomes

$$\frac{M'_{Hn}}{S_{wn}} = \frac{\overline{T}-M'_{Hn}}{S_{tn}} \qquad (23)$$

34. Equation (23) means that for a particular iteration, the shear force at the base would be shared by the shear wall and the frame according to their relative stiffnesses which, we think, negates the purpose of the Author's analysis.

35. One of the disadvantages of the continuum approach (both force and displacement approaches) is that it cannot predict the individual share of shear and axial forces in the frame members, namely columns and beams. A further analysis using one of the conventional methods of frame analysis would therefore be necessary before the results can be used in the design of members.

Professor I. A. MacLeod, Paisley College of Technology

I feel it would take me some time to work out how to set up and solve the equations in the Paper as the method of solution is not described.

37. I believe that nowadays to be worthwhile hand solution must require a minimum of calculation because it is inconceivable that an engineer involved in the design of a tall building cannot readily procure a simple computer frame solution, i.e. what the Author describes as a discrete frame analysis.

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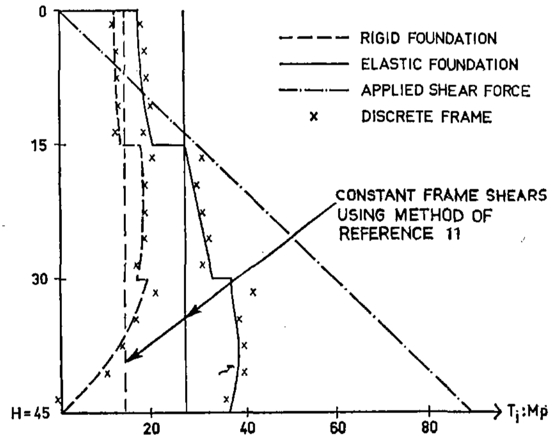

Fig. 6. Shear force distribution in the frame

38. Reference 11 (see also references 12 and 13) describes a method which I believe is worthwhile for hand analysis of shear wall–frame interaction. The basic assumption made in this method is that the frame takes constant shear. It is thus less accurate than that described by the Author although all the important variables are taken into account. From the data given and using the method described in reference 1, and with very little arithmetic, I was able to make estimates of the load distribution for the example given in the Paper. Fig. 6 shows that the constant frame shear assumed in the simplified method gives a good first approximation and Table 2 shows that the predicted maximum wall moments are sufficiently accurate for design. In design I would prefer such a method for preliminary work and would otherwise use computer frame solutions.

Mr Arvidsson

In my opinion the main aim of the paper by Heidebrecht and Stafford Smith[10] is to simplify the treatment of uniform shear wall–frame systems. Their method, contrary to mine, is founded on the basis that the shear wall has an infinite shear stiffness, that the frame has an infinite bending stiffness, and that frame and shear wall are rigidly founded. In this case it is possible to draw graphs, which very much facilitates the calculations.[4] For the general case, where all deformations are taken into account as well as at non-uniform systems, it is impracticable to make solutions in the form of graphs as the number of graphs needed would be too great.

Table 2. Maximum wall moments

	Author*	Reference 1	Percentage difference
Rigid foundation	1420	1380	−2·4
Elastic foundation	755	797	+5·6

* Scaled from Fig. 4.

40. The solution of the non-uniform system presented by Heidebrecht and Stafford Smith is based on transfer matrix treatment, which I have classified as a simplified discrete frame analysis and not as a hand method.

41. **Dr Khan**'s and **Dr Choudhury**'s reservation about equation (22) is surprising. One of the basic assumptions of the continuous approach on shear wall–frame systems is that the link beams between the shear wall and the frame are infinitely closely spaced, which means that the deflexion and the angular deformation respectively are identical in the frame and shear wall. If the frame and the shear wall are rigidly founded, the shear force distribution will be proportional to the shear stiffnesses of the shear wall and the frame in accordance with equation (23). In most cases $S_{fn}/S_{wn} \simeq 0$, which gives $\overline{T}_H = M'_{Hn}$, i.e. the shear force in the frame will be zero. which can normally directly be taken as a boundary condition for rigidly founded systems.

42. In reply to **Professor MacLeod**, the solution of the linear equation system is done by a process of adaptive approximation as pointed out in the Paper. To begin with equations (17) and (18) are set up for every element. By choosing $T_1(0) = T_0$ at the top of the frame, and with regard to the fact that $M_1(0) = 0$, two boundary conditions will be obtained at the top of the system, which will directly give constants C_1 and D_1. When C_1 and D_1 are known, M_1 and T_1 are obtained in the second part of the first element by using equations (17) and (18). Equations (20) and (21) will then give the upper boundary conditions for M_2 and T_2 of element 2. With M_2 and T_2 known, C_2 and D_2 can be worked out. Analogously, C_i and D_i can be obtained, and finally C_n and D_n. When C_n and D_n are known, the angular deformation of frame and shear

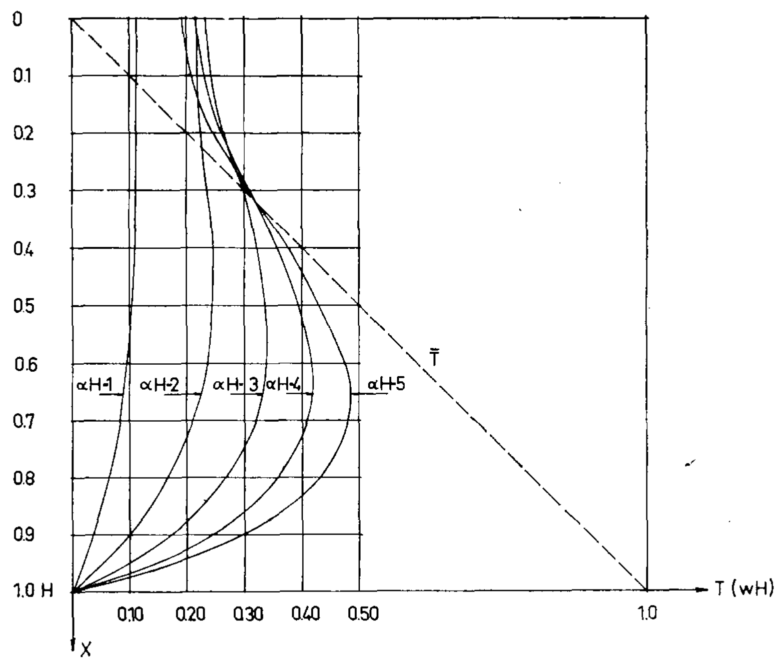

Fig. 7. Shear force distribution in the frame, uniform loading, rigid foundation, $K_f = \infty$

wall can be calculated (equation (22)). If the angular deformation of the frame is larger than that of the shear wall, the chosen value of $T_1(0)$ is too high and at the next iteration a smaller value should be chosen. In the contrary case of the angular deformation a larger value should be chosen for $T_1(0)$ for the following attempt. The iteration is then repeated to the desired accuracy.

43. The method corresponds well with the continuous methods for coupled shear walls, which are well established, but an iterative operation is added as the last step of the solution of the problem. I think the additional amount of work in the last stage is moderate and acceptable in practice.

44. I agree that a designer of a tall building nowadays readily procures a simple discrete frame analysis. However, the solution presented in the Paper is also easy to compute and requires less machine time and input work than a common discrete analysis. The method is thus in line with the modern approach to the design of tall buildings.

45. The approximate solution with a constant frame shear mentioned by Professor MacLeod gives good values of the moment of the shear wall, which has also been pointed out by Rosman[14,15] who also uses a constant frame shear assumption. However, the shear force in the frame is roughly estimated. In cases with relatively stiff frames $\alpha H > 3$ the shear force distribution in the frame is far from constant (Fig. 7).[4]

46. As a first approximation the graphs presented in reference 4 and by Heidebrecht and Stafford Smith[10] give a convenient and reasonably accurate solution of both bending moment in the wall and shear force distribution in the frame at an arbitrary value of αH, thereby being preferable to Professor MacLeod's method.

References

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12. MacLeod I. A. Simplified analysis of shear wall-frame interaction. *Bldg Sci.*, 1972, **7**, 121–125.
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14. Rosman R. Laterally loaded systems consisting of walls and frames. *In Tall buildings*. Pergamon, London, 1967, 273–289.
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Corrigenda

Equation (10) should read

$$M''_t - \alpha_t^2 M_t = -\beta_t \overline{M} + \kappa_t \overline{T}'$$

The horizontal axis in Fig. 5 should be labelled T_t:Mp.