

Hydrodynamic levelling of an offshore tide gauge

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We found the Paper of considerable value in connexion with a recent levelling operation in Southampton Water.

40. In the maintenance of approach channels to ports situated in estuaries a high standard of accuracy of echo sounding is demanded, but in the case of fairly wide estuaries the reference tide staffs for establishing datum may well be at the shallow water margin of the estuary some distance from the actual sounding area and in a different current regime. For economic reasons it may be desirable to conduct survey operations during the run of the tide as well as at slack water and in these circumstances it is most important to establish the true relationship between the water level in the survey area of the main channel and the tide staffs at the margin.

41. Such was the object of an experiment conducted in collaboration with Captain M. J. Ridge, Hydrographic Surveyor of the British Transport Docks Board, Southampton, in which the transverse variation in level on the western side of the estuary at Hythe Pier was examined. Stations were set up at the pier, and at a point about 250 m off-shore of the pier head at the western edge of the shipping channel, together with intermediate positionings.

42. The experiment was conducted during the ebb of a spring tide, and, in order to use the hydrodynamic equations, it was necessary to measure current velocity (magnitude and direction), salinity and tidal level at a number of points. Wind (direction and magnitude) and barometric pressure were recorded at a nearby meteorological station.

43. Calculations showed that the maximum difference in water level between the pier head and the edge of the shipping channel was only 0.01m, of which the density difference accounted for the greater part. The transverse gradient was therefore extremely small and the resulting discrepancy well within the limits of accuracy of any normal echo sounding operation. It also lends confidence to the idea that the difference in level would still be negligible even if the distance were much greater than 250 m.

44. It was hoped to confirm the computations by means of bubbler-type pneumatic tide recorders, with the recording unit at the pier head linked by small bore tubing to a pressure box placed just above the bed at the appropriate stations. Unfortunately, one of the instruments malfunctioned on the occasion so that this part of the experiment would have to be repeated. It is coincidence that the bubbler

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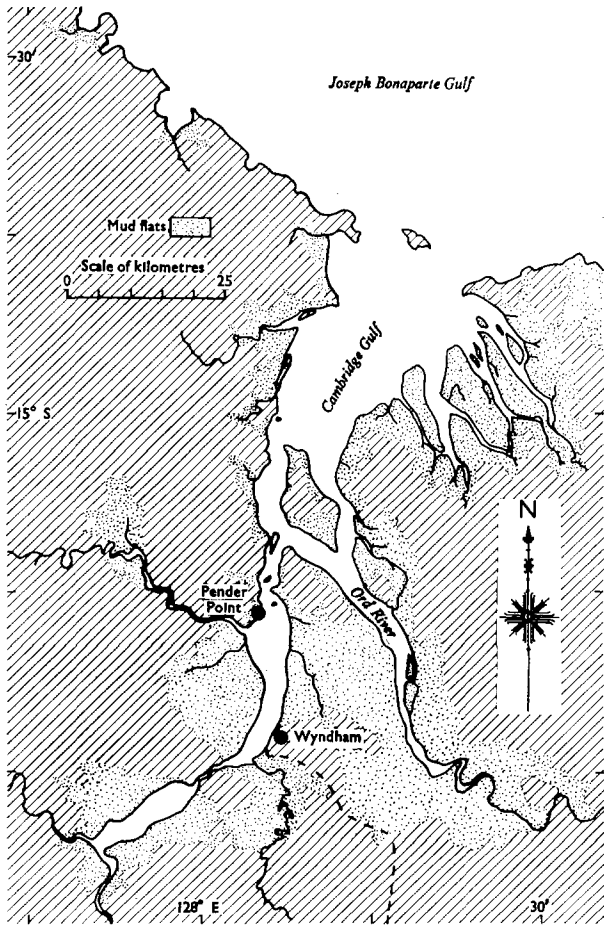


Fig. 7

Table 4

Accepted mean sea level at Wyndham Correction from tidal differences (Fig. 8)	4.36 m 0.23 m
Height above zero of Pender Point tide gauge corresponding to mean sea level at Wyndham	4.13 m

units used were those that had been used in the Wash feasibility study to which the Paper refers.

45. The technique of hydrodynamic levelling is a powerful one, and has a number of useful applications, but there is quite an amount of organizational and field work involved in mounting an operation to measure the various parameters that constitute the basic equations.

Mr D. F. Wallace, Senior Hydrographic Surveyor, Public Works Department, Perth, Western Australia

The solution given by the Authors to the problem of connecting an off-shore tide gauge to a land levelling system can similarly be applied to the interconnexion of near-by coastal tide gauges where the terrain presents uneconomic surveying by spirit levelling, e.g. mud flats, swamps and rugged mountains.

47. In Western Australia I have developed a technique of transferring level by the correlation of high and low water turning points throughout the spring-neap cycle. The method entails plotting the differences of observed turning point heights at the known and unknown station on corresponding tides (not necessarily simultaneous) against the observed turning point height at the known station. A best fit curve is drawn or calculated through these points. The point on this curve corresponding to the mean sea level at the known station gives the correction to datum at the new station.

48. The method in effect gives a hydrostatic solution to hydrodynamically influenced observations by taking into consideration the trend towards zero tide at neaps. In the region where this technique has been applied very small range neap tides occur. An indication of the accuracy of the correlation can be obtained from the scatter of

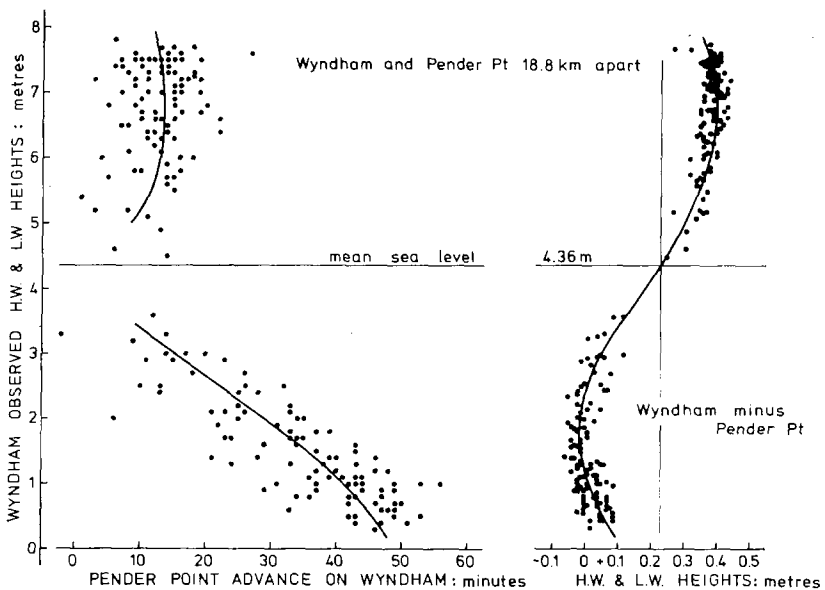


Fig. 8. Cambridge Gulf tidal differences, 10 August, 1973, to 10 October, 1973

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the plotted points. This, and the precision of the tide recorders, give a measure of the probable error. The records required for this treatment need not be continuous and the result can be enhanced by obtaining extra neap observations.

49. The transference of level by this technique has been carried out in Cambridge Gulf between the permanent tide recording station at Wyndham and a temporary recorder at Pender Point about 18.8 km downstream (Fig. 7). Wyndham is 450 km south-west of Darwin. Conventional surveying methods would have involved the running of 125 km of level traverse over rough terrain and four river crossings: a formidable task in high temperatures.

50. Tides at Wyndham were recorded on a continuous chart and at Pender Point a digital recorder sampled the height every 15 min. The estimated accuracy of the tide recorders was ± 0.02 m. At Pender Point the spring tide range was 0.4 m less than at Wyndham. Surface waves at both sites seldom exceeded 0.3 m. Ocean swells do not penetrate that far into the Gulf and the meteorological effects of wind and pressure gradients are slight and variable.

51. Figure 8 shows the correlation between Wyndham and Pender Point. This correlation is applied as shown in Table 4. The time differences shown in Fig. 8 show the distorted nature of the tide. The time and height differences may also be used to predict the Pender Point tides from the standard port predictions for Wyndham.

Mr Alcock and Dr Pugh

The application, by Mr Webber and Mr Blain, of the hydrodynamic levelling technique to determine instantaneous differences of water level in the approach to Southampton Harbour illustrates the general applicability of our method. By calculating theoretical differences along a transverse section their v components of current are presumably small, as the flow tends to parallel the coastal boundaries. As a result the acceleration term $\partial v/\partial t$ and the bottom friction term, which is proportional to $v\sqrt{(u^2 + v^2)}$, will also be small. In the case of an estuary or basin open at only one end, tidal levels have the characteristics of a standing wave, so that the acceleration term is a maximum at high water, while the bottom friction term is a maximum around mid-tide when the currents are greatest. Our results for the Wash (Fig. 2) show the acceleration gradients at high water to be approximately 0.012 m/km and 0.006 m/km at spring and neap tides respectively, along the north-east, south-west axis. Clearly, over a distance of 250 m, level differences of the order of 0.1 m (often the required accuracy of a hydrographic survey) are improbable.

53. To produce a level difference of 0.1 m over a distance of 1 km, due to the Coriolis effect, would require at the latitude of the Wash a u component of 8.6 m/s. To produce the same difference due to density variations alone, in a water depth of 8 m would require a density difference of 25 kg/m³: the difference between fresh and average sea water. Although in a harbour or estuary where fresh water discharge occurs there will be lateral density gradients, their effect on water levels may be neglected for normal hydrographic surveying. The term most likely to introduce a significant error could be that involving the velocities at the ends of the line. If the current at the tide pole were zero, a velocity of 1.41 m/s at the measuring site would produce a level difference of 0.1 m.

54. The pneumatic gauges used for the Wash survey would not have been capable of resolving the level differences calculated for the experiment at Southampton (0.01 m) as their individual accuracy is 0.02 m.⁸ However, the pneumatic system itself is more accurate than the recording instrument; we suggest that the pneumatic tubes be connected directly to opposite sides of a sea water manometer for future experiments, as this gives a direct measure of differential levels to much greater accuracy.

55. We agree that a considerable programme of field observations is necessary

before application of the hydrodynamic levelling technique in shallow water. Hence our conclusion is that it is usually better to correct the mean levels by applying a simple model, requiring few measurements, and then to estimate the accuracy of the level transfer from the discrepancy between the observed and theoretical instantaneous differences.

56. The method developed by Mr Wallace for transfer of level by interpolation between high and low water level differences to a mean sea level is interesting. We have applied our formalism to his technique to determine whether or not the intuitive notion that tidal effects are removed, leaving only residual effects, is valid. To do this we assumed that the currents and elevations could be expressed as

$$\begin{aligned}\zeta_w &= w_0 + w_1 \cos(\sigma t + \psi_{1w}) + \beta_w w_1^2 \cos(2\sigma t + \psi_{2w}) \\ \zeta_p &= D + p_0 + p_1 \cos(\sigma t + \psi_{1p}) + \beta_p p_1^2 \cos(2\sigma t + \psi_{2p}) \\ u &= 0 \text{ everywhere} \\ v &= v_0 + v_1 \cos(\sigma t + \phi_1) + \alpha v_1^2 \cos(2\sigma t + \phi_2)\end{aligned}$$

where ζ_w is the instantaneous level at Wyndham, w_0 is the mean level at Wyndham relative to the survey datum, w_1 is the amplitude of the semi-diurnal tidal constituent of angular speed σ , and β_w is a coefficient which relates the amplitude of the fourth-diurnal, shallow water term to that of the semi-diurnal constituent. ψ_{1w} and ψ_{2w} are phase angles, relative to a selected time origin. The same notation applies for Pender Point levels, ζ_p , except that these are related to a level D m below the survey datum. The co-ordinate axes are chosen, as for the Wash, with the positive v current component directed from the station already connected to the survey datum, towards the station to be levelled. We assume the orthogonal u component to be zero for our discussion. Assuming that v_1 tends to zero as w_1 and p_1 tend to zero, during the spring-neap cycle modulation, we may investigate the implicit assumption of Mr Wallace's method, that $w_0 = p_0$ when w_1 and p_1 are zero, by considering the terms of equation (5) individually.

57. The group of terms due to conditions along the line disappears. Differentiating v with respect to time leaves only terms with coefficients in v_1 which disappear as v_1 tends to zero; the residual current v_0 has no effect. As we have set u to zero, both the advective and Coriolis terms are zero.

58. The terms relating to conditions at the ends of the line do not disappear, but they are likely to be small, as for the Wash exercise. The difference in the squares of the v velocities at Pender Point and Wyndham now becomes the difference in the squares of the residual velocities. To produce an error of 0.01 m, if the velocity at Wyndham is zero, the residual velocity at Pender Point would have to be 0.45 m/s. The differences in atmospheric pressure are not related to tidal variations, but will certainly be negligible over 18.8 km in a region where winds which are slight and variable. The density term becomes $h \ln(\rho_p/\rho_w)/2$ where the densities are the mean values. Assuming h to be 10 m, for an error of 0.01 m the difference in the mean densities would have to be approximately 2 kg/m³.

59. The bottom and surface stress terms are a little more complicated. Writing for the bottom stress term $G_b = K\rho v|v|$, assuming h is constant along the section, approximating to ζ_w for the average tidal level along the line, and omitting the shallow water terms which go to zero with v_0^2 gives

$$\frac{KL}{g} \left[\frac{v_0^2 + 2v_0v_1 \cos(\sigma t + \phi_1) + v_1^2 \cos^2(\sigma t + \phi_2)}{h + w_0 + w_1 \cos(\sigma t + \psi_{1w})} \right]$$

Here we have dropped the modulus sign temporarily, for convenience. The third term in the numerator goes to zero with v_1^2 so may be eliminated. The second term may be expanded into

$$\frac{KL}{g} \frac{2v_0v_1}{h + w_0} \cos(\sigma t + \phi_1) \left[1 - \frac{w_1 \cos(\sigma t + \psi_{1w})}{h + w_0} + 0(w_1^2) \right]$$

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where $w_1/(h+w_0) < 1$, which again vanishes as v_0 tends to zero. In the limit as w_1 tends to zero, the first term is simply the level difference due to bottom friction opposing residual currents

$$\frac{KL}{g} \frac{v_0 |v_0|}{h+w_0}$$

where the modulus sign is now restored. Substituting in values for these parameters shows that a level difference of 0.01 m due to this term would require a residual v_0 of 0.15 m/s.

60. Writing for the wind stress $V = \text{constant}$, $U = 0$ and arguing as for the bottom friction, the level difference becomes

$$K_a \frac{\rho_a}{\rho} \frac{LV|V|}{g} \frac{1}{(h+w_0)} \left[1 - \frac{w_1 \cos(\sigma t + \psi_{1w})}{h+w_0} + O(w_1^2) \right]$$

which, as w_1 tends to zero, becomes

$$K_a \frac{\rho_a}{\rho} \frac{LV|V|}{g(h+w_0)}$$

Substituting in values for these parameters shows that a level difference of 0.01 m due to a constant wind stress term would require a mean wind speed of approximately 4.1 m/s.

61. From these arguments it appears that Mr Wallace's method contains only errors due to the residual currents. Its application at Wyndham is favoured because neap tides have such a small amplitude, but it could be usefully applied elsewhere. It would be of interest to apply the method between two sites where the datum levels had already been established by conventional techniques.

Reference

8. PUGH D. T. *Sea level measurements using the Neyrpic bubbler pressure gauge*. Institute of Coastal Oceanography and Tides, 1971, Internal report 22.