Proc. Instn Civ. Engrs, Part 2, 1977, **63**, Sept., 715–720

7956 DISCUSSION

Flow through quasi-rough circular pipes

A. KUMAR

Professor D. I. H. Barr, Department of Civil Engineering, University of Strathclyde

Although the Author has arrived at his position independently of the work of Schröder and Knauf,[17] his work takes its place in the general sequence of progress as a consolidation and extension of the findings of Schröder and Knauf. Schröder and Knauf evaluated α as

$$\alpha = 1 - \exp\left[-\frac{1}{b}\left(\frac{Re}{d/k} - a\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

giving values of 40 and 396 for a and b respectively, in order to simulate the Nikuradse sand roughness test data. This transition control function is of the same general nature as that proposed by the Author and gives equally satisfactory results for the cases treated. Equation (37) compares with the Author's equation (25). However, the Author can be regarded as having consolidated the previous work because the more rigorous Re_w has been adopted in equation (25) and throughout the new work in place of the explicit approximation given by the use of $Re/(d/k)$. The extension is represented by the much greater coverage and the amplification of the system of control functions provided by the Author. Schröder and Knauf restricted their evaluations to the Nikuradse results, and to new test data from concrete pipes, which showed the same general shape of transition route. In both cases the comparisons between test data and the evaluation have been presented in terms of α plotted against Reynolds number. The Author's diagrams have the roughness Reynolds number Re_w as the abscissa, and are therefore coalesced, whereas Schröder and Knauf adopted the basic Reynolds number Re, and their curves separate for different d/k values.

64. Thus first Schröder and Knauf and then the Author elected to proceed without applying a basic control to cause variation in rate of transition curvature. I believe that the adoption of a transition tightness control should be the first stage in the generalization of the Colebrook–White function, for the following reasons.

65. The basic Colebrook–White function can be regarded as the special case of

$$\frac{1}{\sqrt{\lambda}} = -2C \log\left[\left(\frac{2{\cdot}51}{Re\sqrt{\lambda}}\right)^{1/C} + \left(\frac{1}{3{\cdot}7d/k}\right)^{1/C}\right] \quad . \quad . \quad . \quad . \quad (38)$$

where $C=1$. Many of the sets of data which plot into continually turning curves but of different tightness from that given by $C=1$ can then be fitted well by adopting other values for C. The data of Halsey and Owen[13] are an excellent example, where $C=2$ is found suitable, and the mere adoption of this single value gives an almost perfect fit with the data. In making this assessment, it is necessary to depart from the opinion of the original investigators that rough turbulent conditions are reached at the highest values of Reynolds number. Often when investigators reach the highest Reynolds numbers attainable with their apparatus, they tend to undertake a number of tests at

Paper published: *Proc. Instn Civ. Engrs*, Part 2, 1976, **61**, Dec., 737–756.

about this Reynolds number; these tests plot as a cluster because of experimental scatter between individual data points, giving an incorrect impression of rough turbulent conditions having been reached. The whole number value of $C=2$ which gives a good fit of the data of Halsey and Owen has no more, or less, significance than the whole number value of $C=1$ of the basic evaluation. Some of the data of Harris[8] are well fitted by very small values of C and there are many other examples where good fits are obtained with $C\neq 1$.

66. On more complex transition routes, such as the classic Nikuradse[2] data, I found that the application of transition tightness control avoids what appeared to be a fundamental dilemma. Perhaps the best diagram for conveying the logic of processes of transition route control is Colebrook's[4] original coalesced mode plot (his fig. 1) of the Colebrook–White function. For any chosen tightness, the application of different values of α represents the sliding of the transition route up or down the smooth law. Values of $\alpha>1$ cause the transition curve to run out horizontally above the basic rough turbulent line on the coalesced plot, and values of $\alpha<1$ cause the transition curve to run out horizontally below this line. The effect in the non-coalesced mode of application of the α factor is to make the effective d/k value proportional to $1/\alpha$. Thus, to fit the Nikuradse curve (also shown in Colebrook's fig. 1), while maintaining $C=1$, α has to be reduced to zero, with decreasing value of friction number, while still representing conditions in the smooth turbulent region. Thus the effective d/k is allowed to increase to ∞. Then a smooth pipe surface value of roughness is being imposed at lower values of Reynolds number for the cases of the Nikuradse transition routes obtained with larger values of d/k.

67. The elegance and physical compliance of the Colebrook–White function surely arises from the correspondence of the physical submergence of the roughness within a quasi-laminar layer, with reduction in Reynolds number, with the numerical submergence of the rough turbulent component of the function when the function is evaluated at lower values of $Re\sqrt{\lambda}$. This gives a value of λ corresponding to almost that for smooth turbulent conditions despite the fact that the roughness number of the pipe remains appropriate to the actual surface. Thus, I think great care should be taken in generalizing the Colebrook–White function that physical compliance is not lost. The imposition of a smooth surface value of d/k within the function while dealing with a rough pipe appears to move in this direction.

68. However, if the tightness of transition is controlled to fit the early part of the Nikuradse curve, the limiting lower value of α, and the corresponding upper limit of the effective value of d/k for a given ultimate value of d/k, are not physically incompatible with the actual surface in question. The basic Colebrook–White function can be visualized as corresponding to a particular roughness size and density distribution, and the Nikuradse uniform sand roughness of the same ultimate value of friction factor acts, during the earlier stages of departure from smooth turbulent conditions, as a Colebrook–White roughness of lesser ultimate value of friction factor, and of greater density of peaks.

69. An additional benefit from the adoption of a value of $C<1$ in simulation of the Nikuradse data occurs when the curves approach the rough turbulent value of friction factor. Instead of overshooting and for a short range of Reynolds number reaching values of λ greater than that for rough turbulent flow, by virtue of adhering to a basic transition route of insufficient tightness because $C=1$, the curves can be caused to follow the quite sharp turn of the actual data, without the imposition of further correction factors. The same applies in the case of the Colebrook–White surfaces. The converse difficulty to that of the effective d/k value having to be increased to ∞ can occur when controlled variation in $\alpha>1$ is adopted to fit a line running over a single tightness transition curve field in order to simulate isolated roughness test data. Much of such data is incomplete, and there is a danger that evaluations are adopted which fit the known data, but which run out to high values of α and hence unreal values of d/k while still being within turbulent flow values of Reynolds number. Again, the use of transi-

tion tightness control readily leads to simulation of these sets of data which are sufficiently complete to show the physically essential reverse curvature trend once the stage of rapid increase in friction factor with decrease in Reynolds number has been passed.

Mr Kumar

I agree that the interpretation of roughness effects, by the formulation given in equation (19), is dependent on the accuracy and the range of experimental evidence. The derivation of the correct equivalent roughness projection length of a pipe surface is clearly important; the accuracy of the independent variable, the wall roughness Reynolds number and of the dependent variable, the scale factor, depends on the accuracy of the roughness projection length. Most of the frictional data used in the Paper extended well into the rough zone and it was possible to determine the roughness projection lengths of the surfaces from the nearly uniform values obtained from the data in the rough zone and the friction factor equation (8).

71. Velocity profiles in the transitional and rough regimes of flow, such as those reported by Nikuradse,[2] would have provided additional control on the description of the transitional characteristics of the pipes listed in Tables 2–5. Head loss observations in these pipes at lower Reynolds number of flow would also have been useful.

72. The proposed model has been developed from other studies (e.g. Garrett[18]) concerning the effect of roughness elements in the transitional regime of flow. It was found that the Colebrook–White equation (13) may be used as an alternative method for separating viscous effects and that the roughness projection length, obtained from the experimental data and the equation, provided a measure of the effect of roughness elements at a stage in the transition.

73. Roughness 5 of Colebrook and White (Table 2) was one of the earliest investigated. The ratio of the transitional stage roughness to the terminal roughness, later termed the scale factor, was found to be greater than unity in the earlier stages of the transition and curve fitting, using the method of least squares, showed that

$$\alpha = 2{\cdot}2988 - 0{\cdot}1279\,(Re_w) + 0{\cdot}003219\,(Re_w)^2 \quad . \quad . \quad . \quad (39)$$

74. The use of wall roughness Reynolds number in the description of the transitional characteristics of pipe surfaces is based on dimensional analysis and on the use of the parameter by Nikuradse[2] and others.[3,4,6] I did not investigate alternative parameters because, in view of the relationships given in equations (9)–(12), the scale factor was expected to be adequately described as a function of wall roughness Reynolds number.

75. Transitional zones in pipes may be defined and compared with the transitional regime, of the pipes with uniform grain roughness, $3 < Re_w < 67$. Results given in Tables 2–5 show that the transition is completed by approximately $Re_w = 70$ for pipes with commercial roughness but that the transitional regime of pipes with surface and spot roughness extends beyond $Re_w = 70$.

76. The curve fitting approach, which was used in the derivation of the polynomial equation (39), was not actively pursued. The accuracy of mathematical interpretation should be compatible with the accuracy of experimental evidence. The expressions for the scale factors given in Tables 1–5 are, in spite of some rounding off of numerical constants, in close agreement with the experimental evidence.

77. Also, the mathematical expression for the scale factor should reflect the nature and the scale of fluid dynamic processes. A decay type exponential function with wall roughness Reynolds number as the independent variable appeared to be, and was later shown to be, a logical choice for the representation of the diminishing effect of viscosity with increasing flow in the transitional regime. The viscous effects concern the damping of eddies shed by closely spaced roughness elements in pipes with uniform grain roughness, and the viscous drag on widely spaced individual roughness elements in pipes with non-uniform roughness.

78. The exponential functions also satisfactorily describe the gradual decay of viscous effects in the later stages of the transition.

79. The expressions in Table 1 which describe the transitional characteristics of pipes with uniform grain roughness, conform with the physical explanation proposed by Nikuradse[2] which was based on the interaction between the viscous sub-layer and the roughness projections. The beginning of the transitional zone in pipes with uniform grain roughness, when the eddies shed by the grains are not completely damped by viscosity and begin to propagate, is gradual and is also well represented by the expressions.

80. However, the transitional characteristics of pipes with non-uniform roughness remain undefined at very low wall roughness Reynolds number. The scale factor for these pipes is greater than unity in the transitional regime. It increases with decreasing wall roughness Reynolds numbers and in some cases the scale factor in the early stages of transition is significantly greater than unity (Figs 3 and 4). The scale factor is not expected then to decrease systematically with decreasing Re_w in the earliest stages of transition, i.e. at very low values of Re_w. Systematic experimental data at lower than currently available Reynolds number of flow in the turbulent regime and in the laminar–turbulent transitional regime is needed to clarify the effect of roughness elements in the earliest stages of quasi-rough regime in pipes with non-uniform roughness.

81. Professor Barr has put forward an interesting proposal (equation (38)) to describe the resistance characteristics of quasi-rough pipes. The approach appears to be based on curve fitting requiring the determination of the magnitude of C from a set of experimental data. The magnitude of C has been shown to vary from pipe to pipe. Also, it is supposed that the exponent l/C is equal to $f(Re_w)$ generally, although constant exponents may satisfy the experimental data from only a few pipes.

82. Like the Colebrook–White equation (13), the proposed formula (38) has the following merits

(*a*) at low Reynolds number of flow in pipes with very large relative roughness, $d/k \rightarrow \infty$, the formula transforms into equation (7) for hydraulically smooth pipes

(*b*) for very large Reynolds number of flow, i.e. $Re \rightarrow \infty$, in pipes with small d/k the formula transforms into equation (8) for hydraulically rough pipes.

83. However, for pipes of engineering interest in which $0 \ll d/k \ll \infty$, neither formula (38) nor the Colebrook–White equation (13) merges into the frictional equation (7) for smooth pipes at low Reynolds numbers of flow.

84. As equation (38) is based on the work of Prandtl, von Karman and Nikuradse (§ 3), it may be compared with the resistance equation (6). Hence

$$2C = \frac{\ln 10}{\sqrt{8}} \frac{1}{\kappa} \sim \frac{0{\cdot}8}{\kappa} \qquad (40)$$

Values of C that are very different from unity imply a significant variation in the magnitude of mixing length coefficient κ. For example, $C=2$, which was used by Professor Barr to fit the data reported by Halsey and Owen,[13] corresponds to $\kappa=0{\cdot}2$. However, considerable experimental data indicate that κ is equal or very nearly equal to 0·4. The proposed formula (38) would then appear to provide an inadequate representation of the fluid dynamic processes under consideration. Nevertheless, in view of its successful use by Professor Barr, the applicability of the equation merits further investigation.

85. Nikuradse[2] used a plot of function F where

$$F = \frac{1}{\sqrt{\lambda}} - 2 \log \frac{R}{K} \qquad (41)$$

against Re_w to show the transitional characteristics of pipes with uniform grain roughness. He showed that a single curve may be used to describe the transitional characteristics of all the pipes with various relative roughnesses and derived F–Re_w relations, similar to the A–Re_w relations of equations (10)–(12), for the three sub-zones of the transitional regime in pipes with uniform grain roughness.

Table 7. Scale factor, functions F and G for pipes with non-uniform roughness, scale factor of unity and uniform roughness

Category of pipe	Re_w	3	5	10	15	20	30	40	50	60	70	80
With non-uniform roughness	α	1·595	1·504	1·332	1·219	1·144	1·063	1·027	1·012	1·005	1·002	1·000
	F	0·878	1·069	1·298	1·423	1·505	1·601	1·649	1·675	1·689	1·698	1·704
	G	2·237	2·965	4·037	4·689	5·171	5·910	6·489	6·975	7·388	7·747	8·062
Colebrook–White transition $\alpha = 1$	F	1·095	1·299	1·492	1·567	1·607	1·649	1·671	1·684	1·693	1·699	1·704
	G	1·614	2·303	3·480	4·277	4·880	5·772	6·428	6·947	7·376	7·742	8·062
With uniform roughness	α	0·068	0·140	0·298	0·426	0·531	0·687	0·791	0·860	0·906	0·937	0·958
	F	1·604	1·933	2·144	2·119	2·054	1·937	1·857	1·806	1·773	1·753	1·740
	G	0·149	0·480	1·606	2·689	3·594	4·945	5·892	6·595	7·143	7·588	7·959

86. It may be shown from equations (41) and (23) that

$$F = 1{\cdot}74 - 2\log\left(\frac{3{\cdot}3}{Re_w} + \alpha\right) \quad . \; . \; . \; . \; . \; . \; . \quad (42)$$

in the transitional regime.

87. I used equation (42), in conjunction with the F–Re_w relationships deduced by Nikuradse for the sub-zones, to derive the following relationship for the whole of the transitional regime in the pipes investigated by Nikuradse

$$\alpha = 1 - \exp\left(\frac{2{\cdot}1 - Re_w}{24{\cdot}4}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (43)$$

The equation was contained in the draft of the Paper and is practically identical with equation (25) derived from the A–Re_w relationships of equations (10)–(12).

88. The transitional characteristics of pipes with non-uniform roughness may also be shown by plotting F against Re_w. Colebrook and White[3] and Colebrook[4] used the function F_1, where $F_1 = 1{\cdot}74 - F$ and showed that pipes with natural and commercial roughnesses undergo gradual transitions.

89. However, function G (equation 22) provides greater scope for the description of transitional characteristics of pipes. It may be used to study and interpret the resistance as well as the velocity profiles in quasi-rough pipes (equations (21) and 20)).

90. Values of the functions F and G at various stages of transition in pipes with and without uniform roughness are given in Table 7. Equations (41) and (22) for F and G respectively and the scale factor equations (33) and (28) for pipes with non-uniform and uniform roughness respectively were used in the preparation of the Table. Table 7 may be used in the interpretation of experimental observations and in the design of pipes in the transitional range.

References

17. Schröder C. M. and Knauf T. H. Über das hydraulische Widerstandsverhalten von Beton—und Stahlbetonrohren im Übergangsbereich. *SchrReihe GWF*, 1972, **11**, 536–541.

18. Garrett R. T. An investigation of turbulent flow in a pipe of rectangular section. Department of Civil Engineering, Portsmouth Polytechnic, Project 1969. *Unpublished.*

Erratum

The left-hand side of equation (23) should read $1/\sqrt{\lambda}$, i.e. the equation should read

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2{\cdot}51}{Re\sqrt{\lambda}} + \frac{1}{3{\cdot}71}\frac{k\alpha}{d}\right) \quad . \; . \; . \; . \; . \; . \quad (23)$$