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8110 DISCUSSION

Plasticity, composite action and collapse design of unreinforced shear wall panels in frames

R. H. WOOD

Dr R. J. Mainstone, Building Research Establishment

As an early critic of the use of linear elastic analyses to predict the ultimate strengths of infilled frames, I welcome any attempt at a more realistic analysis. A truly valid analysis would certainly provide a sounder basis than yet exists for the wider application in design of the available experimental data, but the observed behaviour in the tests at the Building Research Establishment (BRE) was more complex than the opening paragraphs of the Paper suggest. In the face of that complexity I had to admit defeat and I feel that the Author, in returning to the problem ten years later, can claim success only by ignoring the same complexity to an inadmissible extent.

85. In terms of overall behaviour, the most important observation was probably that all infills had limited ductility and, in nearly all cases of practical relevance, had passed their peak contributions to the composite strengths well before plastic hinges had developed in the frames. This is shown in Fig. 22, although the curves for additional loads taken by the composite panels include additional loads taken by the frames by virtue of displacements of the hinges from the corners, and therefore underestimate the decreases in load taken directly by the infills after the peak had been reached.

86. The mode of failure of the infill was always essentially diagonal compression. This occurred under highly non-uniform stress and strain conditions, but over a greater or lesser width according to the relative stiffness of the beams and columns of the frame (adequate strength to resist the hydrostatic wall pressure also being necessary without being the major determinant). The more homogeneous infills of micro-concrete failed in a clearly defined manner, usually after cracking once diagonally, and after failure lost strength rapidly. The brickwork infills cracked initially to a greater extent with the cracks running rather irregularly, partly through the weaker bricks and partly through the joints. Failure was then less clearly defined and more progressive, being partly the result of the disruption of the panel into multiple struts bounded by the irregular cracks.

87. In the later stages of collapse, characterized by the presence of developed plastic hinges in the frame, I would still describe the mode as diagonal compression in almost every case, although of a degenerate form as far as the infill panel was concerned. The mode identified by the Author as corner crushing was merely the form taken by the DC mode when the effective diagonal strut was narrow and the band of crushing was thus close to one of the loaded corners. When the effective strut was wider the crushing occurred farther from the corners. An approximation to the Author's shear mode was

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enforced in a few tests devised for the purpose with very stiff beams and columns and relatively weaker joints, but the strain distributions in the infill panel were still far from uniform, both when the panel reached its peak strength and subsequently. I therefore think that this mode is best regarded as an ideal limit rather than as a real natural mode. Approximations to the Author's shear rotation mode were also observed in a few tests; Figs 2 and 3 show two examples. A predisposing factor in the full-scale test (Fig. 2) was the weak connection of the beams to the columns. In all the model tests the mode was more apparent than real, with less fully developed hinges also in the columns. (Observe particularly the right-hand column in Fig. 3 and both columns in the instance in reference 16.)

88. Quantitatively it is predicted that mode SR should occur roughly in the range of $0{\cdot}2 < m < 1$ and mode S if $m > 1$. Correlation with the experimental observations is difficult on account of the invalid assumption, implicit throughout, that these final modes are always associated with the peak composite strengths. However, if it is accepted for the moment that the values of γ_p derived on this basis form the BRE measured peak composite strengths, the corresponding ranges in terms of the nominal values of m for full crushing strengths become, for brickwork, $0{\cdot}075 < m < 0{\cdot}23$ and $m > 0{\cdot}023$, and, for concrete, $0{\cdot}07 < m < 0{\cdot}2$ and $m > 0{\cdot}2$. Thus all BRE models with the heavier frame designated C in reference 5 should have collapsed finally in mode S, whereas they all clearly did so in mode DC. And all BRE models with brickwork infills and the lighter normal frames designated B should have collapsed finally in mode SR, whereas only the two referred to at the end of the previous paragraph did so even to a first approximation. All 16 others clearly followed mode DC again, with well developed column as well as beam hinges.

89. Even on this rather artificial basis it is clear that the empirical γ_p factor that reconciles calculated and measured strengths is being called on to do duty as much more than a stress reduction factor. This wider duty becomes doubly clear when it is remembered that the measured strengths were not those of these final modes but those of earlier modes in which the frames were mostly still deforming elastically. And, because the duty is undefined, there is no reason to believe that the values derived in Figs 20 and 21 have any validity beyond the range of specimen types and parameters covered by the tests.

90. Where there is no valid quantitative theory to guide the application of experimental data I prefer to seek meaningful relationships directly from the data in terms of clearly defined parameters suggested by a more intuitive appreciation of all the relevant phenomena. Here the relevant parameters appear to be

(*a*) the relative stiffness and/or strength of frame and infill panel
(*b*) the effective width of the infill panel regarded as a uniformly stressed diagonal strut acting in the characteristic panel mode DC
(*c*) the relevant dimensions and individual strengths of frame and infill panel, the strength of the latter being expressed as a unit crushing strength.

91. Ideally (*a*) should probably be a composite parameter based on both stiffnesses and strengths. However, in practice relative strengths are closely related to relative stiffnesses so that it makes little difference which is chosen and there are various reasons[17] for preferring a simple relative stiffness parameter. Unit crushing strengths for the infill panels should be those appropriate to the loading condition, calling strictly for another (pure) stress reduction factor. This factor must be linked with (*b*), and can conveniently—and without overall loss of generality—be absorbed into it, calling merely for different effective widths for brickwork and concrete.

92. In my own analysis on these lines of the BRE test results[17] I derived simple relationships between effective panel widths for peak strengths w'_{ec}, cracking strengths w'_{et}, pre-cracking stiffnesses w'_{eK} and the stiffnesses of the panels relative to those of the frame members K. It was convenient to express the effective panel widths non-dimen-

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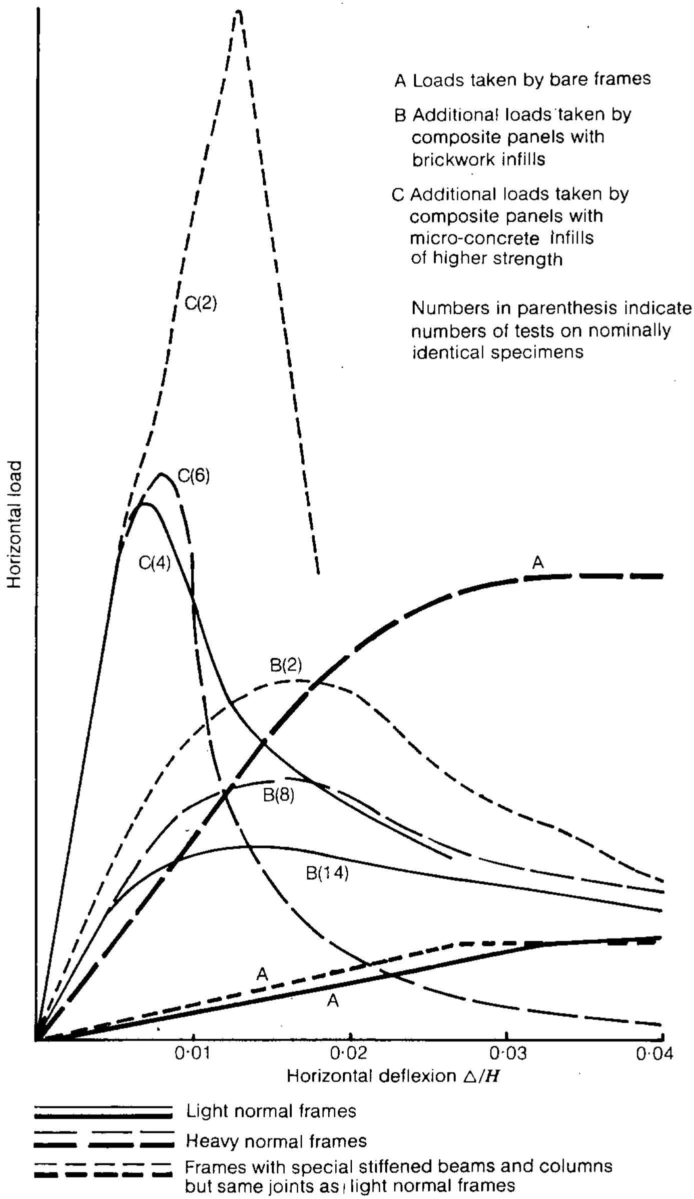

Fig. 22. Mean load–deflexion curves for BRE tests of model steel frames infilled with panels of model brickwork and micro-concrete (the micro-concrete has a nominal crushing strength about twice that of the brickwork)

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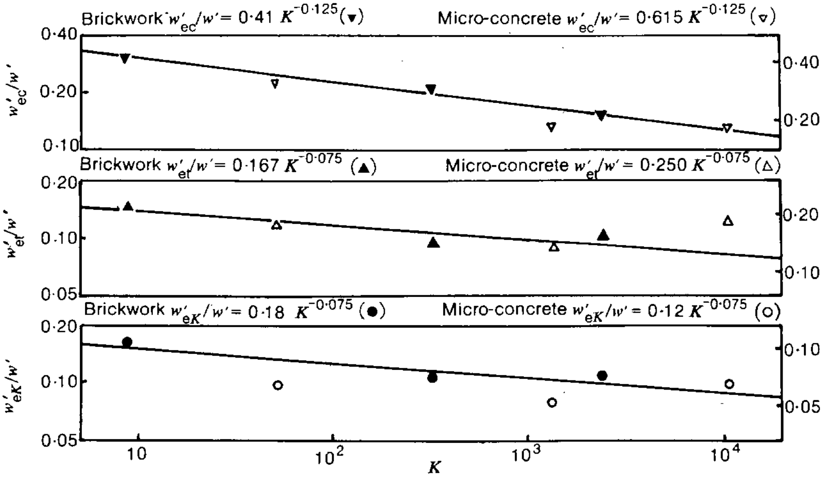

Fig. 23. Relationships between panel crushing, cracking strengths and pre-cracking stiffnesses (expressed in terms of parameters w'_{ec}/w', w'_{et}/w' and w'_{eK}/w' respectively) and the ratios of panel stiffness to stiffness of frame members K

sionally by dividing by the total widths w' normal to the loaded diagonal. The basic plots corresponding to the load–deflexion curves in Fig. 22 are shown in Fig. 23, slightly amended from the original plots to correct an arithmetical error in the conversion to SI units of the stiffness of the columns and beams of the stiffest frames. As in Figs 20–22, mean values are plotted for each group of nominally identical specimens. As there were more tests on brickwork panels, the relationships were derived first from these tests. Similar relationships were then assumed for concrete panels with, in effect, different stress reduction factors incorporated in the multipliers of K^{-B}. (In reference 5 all individual test results were finally plotted against the related parameter $\lambda_h h (= K/4)^{1/4}$.)

93. Although I find these relationships more physically meaningful and more convincing than those between γ_p and m in Figs 20 and 21, they give incomplete guidance for design. For estimates of composite cracking strengths and pre-cracking stiffnesses it is both conservative and reasonable to ignore the contributions of the frames, but the question of what frame contributions should be added to obtain composite ultimate strengths is left open. However, this approach has the merit of posing that question directly instead of first assuming an invalid (completely plastic) answer and then introducing a hidden correction in an ambiguous global γ_p factor. The load–deflexion curves provide guidance and suggest that, for code of practice purposes, the simple assumption of fairly low values of w'_{ec}/w', independent of K, to derive panel strengths which would then be added to the full plastic bare frame strengths would achieve an acceptably uniform level of safety.[5,17] It would compensate for the reduced ductilities of panels surrounded by stiff frames, which are ignored in the Author's approach, and it would deal automatically with the case of panels on both sides of a beam or column—although this presents no further difficulty whichever procedure is adopted.

94. The Author's solutions should be regarded, for practical purposes, as conceivable upper bounds but well above anything likely to be achieved. More realistic predictions of ultimate strengths would have to give full recognition to the limited strain capacities of concrete and brickwork at peak compressive stresses, to the weakening effect of tensile cracking in brickwork in particular, and to the actual departures from the assumed normality of relative displacements to idealized crack directions.

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Mr D. G. E. Smith, Scott Wilson Kirkpatrick and Partners

The Author's method for the design of infilled frames involves an abrupt departure from Stafford Smith's concept[10] in which the infill provides the frame with an elastic foundation, with contact chiefly at the loaded corners.

96. The two methods give similar strengths for infilled panels with weak frames, the new method being the more accurate. Serious discrepancies between the two methods appear with stronger frames, for which the Author postulates alternative failure modes, giving enhanced strengths for single bay frames. It is impossible to judge conclusively between the two methods because insufficient test data for specimens with strong frames are available, although for single bays with weak frames tests tend to favour the Author's method.

97. The Author's method gives the better results largely because of parameters which vary with m and it is probable that similar results could be achieved using Stafford Smith's method if a parameter varying with his relative stiffness factor λ were introduced.

98. Irreconcilable discrepancies between the two methods arise with multi-bay infilled frames with weak panels or strong frames. The Author attributes most of the strength to the columns and Stafford Smith attributes it to the infill. In multi-bay frames the ratio of infill panels to columns is much higher than in single bay frames and consequently Stafford Smith's method gives the greater strengths. No suitable tests have been conducted to resolve this issue. The Author quotes a test with a reinforced concrete frame, but there are special considerations which apply to such frames and they are not really suitable for deciding such an important issue.

99. Another factor tends to reduce the design strengths given by the Author's method and which relates to the emphasis on the strength of the frame; this is the reduction in the panel strength due to vertical loading, as illustrated in the example.

100. Should the Author's method prove to be the more precise it may yet be shown that the underestimate of the frame strength in Stafford Smith's method is compensated for by the two unsafe factors just considered. In many cases Stafford Smith's panel cracking criterion will govern design, irrespective of which method is used to assess panel strengths. I have had difficulty in analysing the two-bay frames of Kadir and Hendry[11] and Stafford Smith[10,18] which they tested as deep frames, subject to central point loads. The central column is clearly undeflected and can therefore contribute no strength.

101. It is to be expected that in future design infills will replace diagonal members in many braced structures. These structures usually have pinned connections and infilled frames with pinned connections therefore constitute the most important design situation. The proposed design method gives very low strengths for these frames. Riddington[19] indicates that strengths given by an extension of Stafford Smith's approach are only slightly less than those of similar infilled frames with rigid corners. Polyakov[20] conducted many tests on suitable frames but only records that the ultimate failure load was appreciably above that at cracking. Isolated tests by Hendry[3] and Mainstone[5] are insufficient in number and in range of frame stiffness to be of assistance; they suggest no obvious conclusions.

102. An interesting feature of the design method is the very low value of the penalty factor γ_p (0·23–0·45 for brickwork and 0·20–0·35 for micro-concrete). If one assumes a reduction on the cube strength of 1·5 to give the strength in the test specimen and a similar value for brickwork (which is probably not justified) γ_p becomes 0·34–0·67 for brickwork and 0·30–0·52 for micro-concrete. A possible interpretation of the fact that these values are still so far from unity is that the pressure exerted on the frame by the infill is far from hydrostatic and may be of quite a different nature.

103. Another revolutionary proposal in the Paper is the new safety factor γ_{comp}. Is this factor just for this and future design methods for infilled frames or does it apply to composite beams and columns? If the latter, then it would be a sufficient penalty factor to discourage the use of composite construction in buildings.

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Mr P. A. C. Sims, Building Research Establishment

I have applied the Author's approach to reinforced shear wall panels. Theoretically, mode SR is redundant for square panels comprising equal beams and columns because conditions of symmetry will enforce plastic hinges in the columns and must therefore give the same result as mode DC. This can be checked by considering equations (19)–(26), and noting that the best lower bound solution occurs when M_{max}/M_p in the columns is unity. This can be interpreted as being a plastic hinge and so mode DC equally applies; thus all entries in the second column of Table 3 could have been annotated as having been derived from mode DC. Any departure from symmetry would cause mode SR to appear.

105. A point which needs clarification arises in the design examples (§§ 75 and 78), regarding the values which are purported to correspond to the contributions from the frame and the wall. From the definition of f (equation (6)), it would appear quite natural to consider the first term of the denominator as being due to the frame and the second due to the wall. For mode S, this is certainly true, but for the other modes, the true contributions from the frame and wall are masked by this definition of f. The way to obtain the correct individual contributions is by reworking the solutions keeping these contributions separate. For mode CC square panels, doing this results in the frame and wall each contributing

$$F = \frac{\sqrt{m}}{1+m}\left(\frac{4M_p}{H} + \frac{1}{2}\gamma_p\sigma_c tB\right)$$

Inserting the values from § 78 yields

$$F(\text{frame}) = F(\text{wall}) = 112{\cdot}5 \text{ kN}$$

Comparing this with the split shown in § 78, it can be seen that the frame contributes more, and the wall correspondingly less. Thus by considering the true individual contributions, any impression that the wall is being called on to contribute, by plastic theories, a disproportionate amount relative to the frame, for low m values, can be alleviated.

106. There is evidence, some of which is shown in Figs 1 and 3, to support the justification for a penalty factor. The introduction of γ_p indeed causes plotted test results, based on nominal values, to be shifted towards the weak wall mode S (§ 63), and it is quite likely that this operation will result in different modes applying to the nominal point and to the modified point. This is best illustrated by considering the panel shown in Fig. 3. For this panel $B=0{\cdot}61$ m, $H=0{\cdot}406$ m, $t=0{\cdot}01875$ m, $\sigma_c=8{\cdot}6$ MN/m^2, $M_p=0{\cdot}00044$ MN m and $F=0{\cdot}01266$ MN.

107. The nominal m value, from equation (7), is 0·06 with the nominal f value being 0·23. The range of the design curve applicable to each mode is given in § 62; the above values lie within the range applicable to mode DC. However, Fig. 3 shows that this is not the correct mode. Applying the procedure for determining γ_p results in $\gamma_p=0{\cdot}216$, the modified values of m and f being 0·272 and 0·820 respectively. This point now lies in the range covered by mode SR which is the mode shown in Fig. 3. For the full-scale panel in Fig. 1, the nominal values indicated mode SR, with mode S being indicated after modification by the relevant γ_p value. This is confirmed by Fig. 1. The use of γ_p not only affects the values of m and f, but also ensures that an altered mode is applicable to the panel, which is verified realistically by the tests giving considerable support to the theory. However, I feel that the role of γ_p is a little more complex than just being a penalty for the use of idealized plasticity theory. In deriving it from test results, it must also contain effects from other parameters not considered in the basic theory, e.g. the effect of elastic deformations and the use of an idealized yield criterion. γ_p already includes effects due to insufficient ductility in compression (§ 62), but in a qualitative manner rather than quantitatively. It is likely that this limited ductility could be another parameter to be considered.

108. I have evaluated the design examples using the methods proposed by Stafford Smith[18] and Mainstone,[5] although for purposes of comparison only their expressions

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for the compressive failure load have been considered. The additional information required for this exercise has been assumed as Young's modulus (frame) = 200 GN/m², Young's modulus (infill) = $900\sigma_c/\gamma_{mm}$.[21] Using Stafford Smith's method the collapse load F was 496 kN for example 1 and 393 kN for example 2; Mainstone's method gave $F = 446$ kN for example 1 and $F = 261$ kN for example 2.

109. These values are higher than those obtained in the Paper. Furthermore, the method of Stafford Smith assumes that the frame makes no contribution and hence the values quoted above are to be taken by the wall alone. Mainstone's method also yields higher wall values but makes some allowance for the contribution of the frame. Both these approaches, although based on elasticity parameters, predict stronger panels than the Author's plasticity approach.

Dr Wood

Rigid-plastic theory is only an approximation for real behaviour, but photographs indicate that it must be the most powerful contributor. Years ago Prager,[22] referring to the Building Research Station tests, suggested that cracking could almost be regarded as an ideal plastic state, with constant zero yield stress, and near normality of displacement for extensive cracking, provided cracks died out somewhere and were held longitudinally. Only now are near exact solutions offered, closing upper and lower bounds. Had they been available throughout the test programme, they would have altered the instrumentation and interpretation.

111. Designers and critics must not fall into the trap of thinking that plasticity must necessarily overstate wall resistance. It would do so if σ_c were used, but the penalty factor γ_p causes smaller wall resistance to be derived than is given by elastic theory, if both theories are calibrated against the same test results. Plastic theories enforce greater frame strength, so that less is claimed for the wall. This surprising result is illustrated with respect to the second worked example.

112. Although acceptable stress fields and mechanisms have been found alternative plastic solutions can exist, but these cannot give noticeably different collapse loads. Parallel diagonal bands are not necessary, and curvilinear bulging bands might be more realistic. In early tests it was believed that extensive cracking at the centre led to a

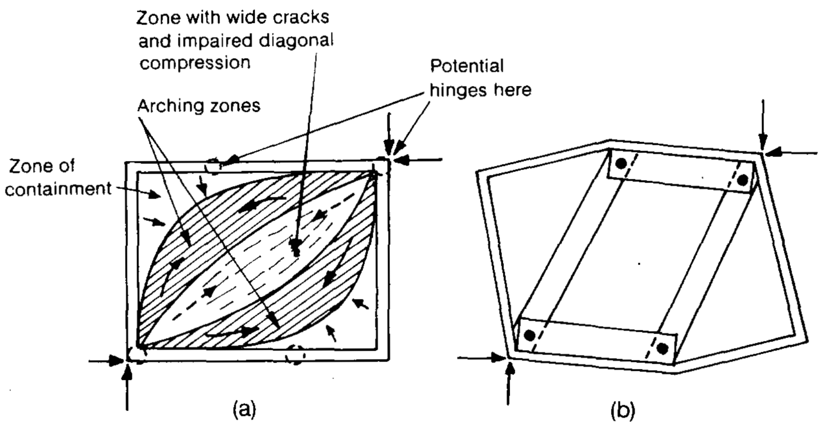

Fig. 24. Attempts at explaining observable panel behaviour; (a) impressions of arching action reported during early full-scale tests, (b) Mainstone's simplified pinned linkage mechanism to demonstrate the SR mode

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partial abandonment of diagonal thrust in favour of dual arching between corners (Fig. 24(a)).

113. A distressing feature about test results is the wide scatter from nominally identical specimens. Although Dr Mainstone took exceptional care with specimens, some specimens intended to be identical show as much as a 2:1 ratio of collapse loads. If it were possible to identify powerful elastic and plastic parameters, against which to plot results, it would take an enormous number of specimens merely to identify statistic trends. Theories then become of greater importance, and differ from mere dimensionless plots—as Dr Mainstone presented. Thus correcting factors like γ_p become weighted by predetermined theoretical trends.

114. The collapse load does not then rely solely on the plot. The ideal form of presentation would be to plot the test results divided by the best available theoretical result against the ratios of predictions of other theories (e.g. the Merchant–Rankine plot). Apart from existing relatively weak but significant elastic theory,[10] the next important advance must come from finite element or similar analysis, with restricted plastic strain. Outstanding research now lies in determining an extra theory to deal with plastic strain limitation—better than just γ_p—as Mr Sims suggests.

115. It is now clear that Dr Mainstone has misinterpreted my intentions. With such a scatter of results and with no composite elasto-plastic theory at the time to help with observations, Dr Mainstone cannot say with any certainty that he has observed forms of behaviour—still less stress fields. Thus his frame contribution has been considerably underestimated for the DC mode, in value and in redistribution, so that his remaining wall contribution is exaggerated. It was never claimed that mechanisms form exactly at the point of maximum load. It is impossible to say, by eye, just what proportion of a full plastic hinge does form, because at the moment of formation there is no hinge as such. However, within this tendency γ_p operates in a strongly weighted fashion to account largely for the missing plastic strain limitation, with slight interference from elasticity.

116. The biggest discrepancy in argument seems to be Dr Mainstone's insistence that tests are best described with reference to diagonal struts. This is not what he wrote when commenting on Holmes's diagonal strut approach:[16] 'the wall stiffened the frame, first by carrying most of the load itself, then, after cracking commenced, by changing the mode of deformation of the frame.... The composite panel ... more slowly developed strength of the frame in a manner which Dr Holmes's equivalent structure ... failed to represent ... The effective "ductility" of the infilling wall was obviously of great importance when designing either for strength or for energy absorption.... The changed mode of deformation of the frame ... could be simulated by replacing the wall by a linkage' (see Fig. 24(b) and fig. 7 of reference 16).

117. Misunderstanding arises from failure to realize that modes S, SR, DC and CC all have varying extents of diagonal crushing, but that the important observation is what happens to the frame. It is not, as Dr Mainstone argues, that frames nominally strong enough for mode S should have failed DC-wise; the progression must be the other

Table 5

Author	Total load, kN	Frame, kN	Wall, kN
Stafford Smith[18]	393	0	393
Mainstone[5]	261	58	203
Wood	235*	107	128

* Using Δf (Fig. 19).

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way round as Mr Sims shows. Any strong wall, nominally predicting a DC mode, could be weakened by uneven crushing to revert to SR or even S modes. Indeed, this has happened. Note should be taken of Mr Sims's point that with square panels there is no difference in collapse load between DC and SR modes, so it is not surprising that Dr Mainstone found evidence of plasticity in columns.

118. Plastic theory easily allows lower bound stress fields to be devised for continuous panels, unequal beams and columns, pinned joints and so on. The suggested increase in γ_{comp} (§ 72) allows for Dr Mainstone's justified fears of limited ductility. This is an instance of how Mr Smith should interpret γ_{comp}. This could be increased for complicated composite structures where bare frame analysis is never justified, and reduced for composite beams where it is justified.

119. There is a difference between an M_p strength control and an EI stiffness control, as the second worked example shows. Braced columns can approach the squash load with drastic reduction of M_p but not of EI. Mr Sims rightly points out that, to find out the sharing of load by wall and frame, one should go back to the theory, and not use the nominal contribution to f. In the second test example, allowing $4M_p/H$ to be in Dr Mainstone's treatment attributed to the frame, the split-up occurs as shown in Table 5. This makes it clear why designers should not be afraid of plastic theories, in that they do not overload the wall, especially as Mr Smith has hinted that γ_p need not have been so low if some reduction had already been made for variable strength of brickwork throughout the wall.

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