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DISCUSSION

Calculation of material pressures for the design of silos

R. T. JENKYN

Dr M. Herzog, Consulting Engineer

Unfortunately, the Author does not support his new proposal by experimental evidence. A recent report[14] on the proposal by Vivancos[11] shows by hind-calculations of failures and measurements that this reasonable approach is valid for many types of silo, e.g.

(*a*) octagonal wheat silo[4] ($D=5{\cdot}02$ m, $H=22$ m, $\gamma=800$ kg/m^3)
(*b*) rectangular coal bunker[15] ($b=2{\cdot}90$ m, $H=7$ m, $\gamma=850$ kg/m^3)
(*c*) circular cement silo[16] ($D=16{\cdot}00$ m, $H=21{\cdot}6$ m, $\gamma=1700$ kg/m^3)
(*d*) square barley bin[17] ($a=3{\cdot}92$ m, $H=65$ m, $\gamma=730$ kg/m^3)
(*e*) circular gravel silo[18] ($D=15{\cdot}12$ m, $H=20$ m, $\gamma=1590$ kg/m^3).

Mr J. Faber, Oscar Faber & Partners

In the light of present knowledge, the design of any sizeable silo remains the subject of an inexact art.

48. The fundamental requirement of economical silo design must be the assured success of the completed work. The cost of repairs and remedial measures following structural failure can far outweigh any hoped-for savings from working to skinned down design pressures; the formwork, scaffolding and/or sliding equipment, and the concreting process—all with attendant overheads—have to be provided in any case, so that in reality it is only a proportion of the reinforcement cost that comes into contention at the time of original design. This is true of the walls although it may not be so true of the bottoms. Any statement of wall design pressures can in itself be meaningless if it is not considered closely in relation to the stresses to be allowed in the construction materials, and the quality and types of those materials, particularly their elastic properties. For example, external walls, which may be perfectly alright in terms of strength, may develop tension cracks to the detriment of the stored material and to the life of the silo itself, and so be unsatisfactory in service.

49. The trouble in silo design with all mathematical formulae based on laboratory coefficients is that granular materials change their behaviour characteristics when effectively confined. Furthermore, friction values vary considerably depending on flow configurations which in practical terms are outside an engineer's prediction or his control. In no material is this more true than the cement and raw-meal referred to by the Author: their bulk densities vary on filling, on settling down, on being surcharged, on aeration and at emptying, and so do their angles of internal friction. Indeed when aerated fully so as to be truly fluidized, K_0 becomes unity and the bulk densities may be reduced to about three quarters.

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50. It is interesting to note the close agreement between the curve for P_h in Fig. 8 and tests conducted in a full-size silo 9·1 m in diameter.[19,20] Tests of this type normally fail to pick up the effects of surge on filling, switch-pressure on emptying, breakdown after funnelling or hang-ups, and landslide ruptures. Practical design must take these phenomena into account.

51. The most difficult thing to decide in silo design is not the wall pressures, but the bottom pressures. This is where any error in determining the factor K_0 can have sinister consequences.[21] In the design of really large silos one probably has to use Rankine's classical theory.

Mr G. C. Dunn, CMGM (Pty) Ltd

In his two examples the Author has, in my opinion, underestimated the wall pressures when using the ACI method because he has used the internal friction angle (approximate angle of repose) as his factor for μ. All the methods mentioned in the Paper are based on Janssen's formula,[3] which was originally derived by considering the resistance against motion of the stored material by friction on the wall. Figs 1 and 2 make recognition of this. I would therefore recommend that a much lower μ factor, say 20°, be used for the ACI method[2] in the examples in the Paper.

53. Using the Author's examples the following indicates the difference.

(*a*) *Example 1*

$D = 33{\cdot}5$ m
$\theta = 30°$
$H = 60{\cdot}0$ m
$\mu = 20°$
$\gamma = 1{\cdot}6\ \text{t/m}^3$
$H/D = 1{\cdot}8$

Using the ACI method with overpressure factors of 1·35 for vertical pressures and 1·65 for horizontal pressures

Maximum vertical pressure = $87\ \text{t/m}^2$
Maximum horizontal pressure = $35\ \text{t/m}^2$

The comparative figures using $\theta = \mu = 30°$ are $70\ \text{t/m}^3$ and $28\ \text{t/m}^2$.

(*b*) *Example 2*

$D = 27{\cdot}5$ m
$\theta = 30°$
$H = 48{\cdot}5$ m
$\mu = 20°$
$\gamma = 1{\cdot}6\ \text{t/m}^3$
$H/D = 1{\cdot}8$

Maximum vertical pressure = $70\ \text{t/m}^2$
Maximum horizontal pressure = $29\ \text{t/m}^2$

The comparative figures when $\mu = \theta = 30°$ are $57\ \text{t/m}^2$ and $23\ \text{t/m}^2$.

54. In the light of this, does the Author recommend the value of internal friction or wall friction as his value of $\mu°$ for Tables 2–5?

55. The cut-off position for the DIN 1055 method[1] in Figs 4 and 8 seems to be rather low down in the silo, e.g. Fig. 8 shows the cut-off at 26 m whereas $1{\cdot}2D = 33$ m and $0{\cdot}75h = 36$ m. Is this because there is a cone within the height of 48·5 m? This could also be the reason why the height to zero pressure only scales about 42·5 m.

56. The Paper considers only horizontal pressures. Although DIN 1055 gives higher wall and foundation loads (horizontal and vertical friction when emptying), the ACI method gives substantially higher forces on the floor, and should therefore be adopted for the design of suspended floor slabs.

57. The recommendations and methods in the Paper should only be considered for tall silos, i.e. ones in which the angle of rupture cuts the plane of the wall, and not the free upper surface of the contents. Coulomb's expression for the angle of rupture is

$$\beta = \frac{90+\theta}{2}$$

measured from the horizontal, where θ is the angle of repose. Therefore the limiting height h is equal to $D \tan \beta$. If h is more than the actual height of the material then Rankine pressures should be adopted. Vertical pressure equals the density of the material times the height and horizontal pressure equals K_0 times the vertical pressure where

$$K_0 = \frac{1-\sin\theta}{1+\sin\theta}$$

58. Other methods of determining whether the silo is deep or shallow are

(*a*) by Dishinger: if $h > 1{\cdot}5\sqrt{A}$, where A is the cross-section area of the silo, then the silo is deep

(*b*) by the Russian code for reinforced concrete: if $H > 1{\cdot}5$ times the diameter for circular silos, $H > 1{\cdot}5a$ for rectangular silos, where a is the width of the side.

Mr Jenkyn

In reply to Dr Herzog, the proposals made in the Paper were in part based on experimental measurements.

60. I have been making measurements of material pressures in a large silo for the storage of cement clinker. Pressure measuring instruments were installed at eight points on the wall and at two points on the floor of this silo during construction in 1975. Since then the material pressures have been recorded during three complete fill/empty cycles, with consistent results. It is hoped to complete this experimental programme during 1979.

61. With reference to the proposal made by Vivancos,[11] if the lateral pressure coefficient is taken as K (passive) (i.e. very large with respect to either K (active) or K_0), then the maximum horizontal pressure towards which calculated P_h tends is approximated at a comparatively shallow depth below the material surface. Thus it might be found that this method is more conservative in calculation of pressure in a silo having a height to diameter ratio close to or less than unity, e.g. examples (*c*) and (*e*) in § 46.

62. In examples (*a*), (*b*) and (*d*) in § 46—silos of small diameter or hydraulic radius and with $H/D > 2$—this advantage would be lost except possibly for smaller values of the angle of friction. In fact, the difference between the horizontal pressure calculated by the above method and my proposal is not great.

63. However, Vivancos[11] also suggests that in the emptying case there is an increase in the proportion of the material load carried on the wall by friction, and a reduction in bottom pressure, compared with the filling case. This is contrary to observations I have made; a substantial increase in bottom pressure was measured during discharge in the silo mentioned in § 60.

64. I am in full agreement with Mr Faber's comments regarding the necessity for avoiding over-optimistic design pressures and taking account of various causes of overpressure development in the structural design of silos.

65. It would be good practice to calculate design pressures by as many methods as seem appropriate in any example, and then to use for design either that which is most conservative overall, or an envelope containing the maximum pressures over the full height.

66. In reply to Mr Dunn, the coefficient of internal friction of the material should always be used in preference to the coefficient of wall friction, except in particular cases in which both the following criteria can be met.

67. If the material is to slide against the wall, then

(*a*) the coefficient of wall friction must be smaller than the coefficient of internal friction
(*b*) true mass flow must occur such that all the material in any cross-section moves at the same time.

68. With regard to the first criterion, in laboratory tests conducted on the material used in the examples shown in Figs 4 and 8 (cement clinker from two different sources) the angle of internal friction (initial) was measured at 30°, and the angle of wall friction (with slipformed concrete) was measured at 36°. Therefore shear failure will occur through the material before it will occur at the wall. There is no justification for assuming that the coefficient of wall friction is smaller than the coefficient of internal friction (kinetic) unless tests on the material indicate that to be so.

69. With regard to the second criterion, true mass flow is not something which occurs readily. For it to occur

(*a*) the outlet geometry and the silo proportions must be designed specifically to promote mass flow, using techniques such as those proposed by Jenike[22]
(*b*) there must not be any imperfections in the wall, such as weld seams in steel plate, cold joints in concrete and deviation from plumb, which are large enough to allow a stable boundary layer to be trapped against the wall.

70. With regard to the maximum horizontal pressure according to DIN 1055[1] (Figs 4 and 8), the cut-off values are

(*a*) at the bottom of the silo, P_h for the filling case
(*b*) in the middle height of the silo, the greatest of P_h (filling) at the bottom, P_h (emptying) at height $1{\cdot}2D$ or P_h (emptying) at height $0{\cdot}75H$.

71. In both the examples used, the maximum horizontal pressure of these three is P_h (filling) at the bottom, and thus a vertical line drawn at this co-ordinate on the pressure diagram will intersect the emptying curve at a height lower than either $1{\cdot}2D$ or $0{\cdot}75H$.

72. With regard to the vertical pressures calculated by the ACI method[2] and DIN 1055,[1] it is true that the ACI method will provide the higher value for vertical bottom pressure in most cases when the relationship

$$P_v = P_h/K_0 \qquad (16)$$

is used. Substantially higher pressures than these should be applied to the design of a silo floor (§ 30).

73. Considering the classification of silos into either deep or shallow using the plane of rupture criterion, several criticisms can be made of the practice of using Rankine's method for shallow silos and other methods for deep silos. There is no evidence—theoretical or experimental—to suggest that a fundamental difference in behaviour exists between shallow and deep silos so defined. Far more significant would be such factors as outlet geometry, bottom or hopper slope and wall roughness.

74. Rankine pressure $K_0\gamma h$ is linear in distribution from top to bottom of the wall. This is not likely to be more conservative than any pressure distribution based on Janssen,[3] which requires higher pressures at the middle part of the wall height.

75. One of the main points in the Paper is that material flow in a silo is dynamic in nature; it is important that the choice of parameters such as K_0 reflects this fact. In other words, a pressure calculation based on Rankine, i.e.

$$P_h = K_0\gamma h$$

$$K_0 = \frac{1-\sin\mu}{1+\sin\mu}$$

is not adequate for the design of a silo, unless the pressures are increased by an overpressure factor or the method is used only for comparison with other methods.

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