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Gradually varied surface profiles in horizontal and adversely sloping channels

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My particular concern is the case of circular channels with horizontal and adverse slopes.[8]

59. In the case of a horizontal channel, equation (39) can be rewritten as

$$dx = \frac{C^2}{g} D \frac{W}{U} \sin\theta \, d\theta - \frac{C^2 D^6}{512 Q^2} W \sin\theta \, d\theta \tag{63}$$

where D is the diameter of the pipe channel

$$W = \frac{(2\theta - \sin 2\theta)^3}{2\theta} = f_1(\theta) \tag{64}$$

$$U = \frac{(2\theta - \sin 2\theta)^3}{\sin\theta} = f_2(\theta) \tag{65}$$

60. Equation (63) can be solved using a step method facilitated by the preparation of tables of W and U or on a desk programmable calculator.

61. The Author did not present the case of an adversely sloping circular channel. One single approach, without the need for alteration of the basic equations, to solve the non-uniform flow equation for any slope is highly desirable. This can be achieved in terms of critical slope as

$$dx = \frac{D}{2} X' \, d\theta - \frac{256 Q^2}{g D^4} \frac{X'}{U} d\theta \tag{66}$$

in which

$$X' = \frac{\sin\theta}{i - (K_c/K)^2 i_c} \tag{67}$$

where K_c and K are the conveyances $AC\sqrt{m}$. The subscript c denotes the critical depth condition.

62. Equation (66) can be solved by a step method by developing appropriate tables.

63. This approach, which is applicable to any bed slope, has a distinct advantage over the Author's method and the methods that depend on the normal depth.[1,2,9]

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I have been interested in the computation of surface profiles in open channels for many years and have seen and participated in the ever-increasing use of digital computers to integrate the differential equations for steady and unsteady flow. It is always possible to integrate the gradually varied steady flow equation numerically with a computer and therefore, in my view, it is very difficult to make a convincing case for continuing to develop highly restrictive analytical methods. Perhaps such a case could have been made when computers were not so readily accessible as they are at present, but with modern multi-access facilities this is no longer possible. Furthermore, the development of inexpensive hand-held programmable calculators is now so advanced that sophisticated numerical solutions can be achieved for open channel profiles by this means.

65. Some critics of numerical integration have wondered about the stability of the numerical procedure adopted and the step size required. My experience suggests that the simple Euler method will always work for steady flow and the step size can easily be found by trial and error for a particular case.

66. What advantages does the Author feel his analytical method has over a computer oriented numerical solution?

Mr Kumar

Step methods have to be used for the computation of profiles in natural and prismatic channels where direct analytical integration of the standard gradually varied flow equation is not possible.

68. The step numerical methods are based on

(*a*) the standard gradually varied flow equation, or
(*b*) the application of Bernoulli's equation between adjacent sections

Method (*a*) may be used for prismatic and non-prismatic channels. There are two types of method (*b*): the direct step method which is suitable for prismatic channels and the standard step method which is used for non-prismatic channels.

69. Direct analytical integration of standard gradually varied flow equations for sloping circular channels has not yet been achieved. Profiles in such channels have to be obtained by numerical step by step integration. Either method (*a*) or method (*b*) may be used. Dr Nalluri has suggested using method (*a*); equation (66) may be useful for step by step integration of profiles in adversely sloping channels.

70. Profiles in horizontal circular channels may be obtained by the direct integration given in equation (41); it is therefore not necessary to use numerical integration in such channels.

71. Analytical integrations of gradually varied flow equations are now available for many cases of interest to hydraulic engineers. These solutions are accurate and should be used where possible as the numerical methods involve inaccuracy or approximation.

72. Some of the other advantages of using the analytical solutions are as follows.

(*a*) It is easier to develop and use programs for digital computation of surface profiles from analytical integrations such as those derived in

the Paper and references 4, 5 and 7. Computation is generally faster than through programs using step methods.

(*b*) Computation of surface profiles from analytical integrations is direct and less cumbersome; calculations with the aid of an electronic scientific pocket calculator usually involve less time and are more accurate than the results obtained from a computer program using step by step numerical integration.

(*c*) The length of a surface profile between two specified depths is the only calculation required in a number of design problems; the analytical solutions provide quick and accurate answers in these cases.

(*d*) Non-dimensional functions occurring in the analytical integrations may be determined once and stored in a library. These functions may then be used to determine specific profiles in specific channels. For example, the solution for horizontal circular channels given by equation (41) may be used with the functions listed in Table 6 to compute surface profiles between given depths (θ_1 and θ_2 known) in a specific channel (R and θ_C given).

(*e*) It is frequently possible to study surface profiles in natural channels by reducing the channel sections to equivalent prismatic sections for which analytic solutions have been derived. In such cases, appropriate analytical integrations may be used to obtain approximate surface profiles in the natural channel under consideration and to check the surface profiles obtained by step by step numerical integration.

73. The methods and the techniques used for the determination of surface profiles in channels, for which analytical solutions are not available, need to be reviewed continuously. In some cases it should be possible to develop more accurate methods than the methods of step by step integration currently in vogue.

74. Attention should also be given to the applicability of analytical solutions available and to the derivation of analytical integrations for cases not covered so far.

References

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