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Solutions of the Colebrook–White function for resistance to uniform turbulent flow

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In steady flow problems, the number of calculations involving the use of a friction formula is generally small, and so the savings in computation time that result from the use of the Author's approximation are not very significant. However, in unsteady flow problems, particularly unsteady open channel flow problems, when the Colebrook–White formula is used in connection with the evaluation of the friction slope term,[17,18] the savings in computation time that result from the use of the Author's approximation can be worthwhile. Usually, a uniform open channel flow formula (such as the Manning formula) is used for this purpose with the friction slope in place of the bed slope.

14. The Colebrook–White formula was developed from steady pipe flow experiments[1] and there are marked differences between steady pipe flow and unsteady open channel flow. In the latter case there is a free surface, the flow varies at every section with time and the velocity distribution varies from section to section and with time. Thus there is little or no similarity between the two types of flow. Steady flow concepts of different flow regimes are not readily applicable to unsteady open channel flow. The use of the Colebrook–White formula in the evaluation of the friction slope term of the unsteady open channel flow equations must therefore be seen in perspective. The only resemblance between uniform open channel flow and unsteady open channel flow is the free surface. Steady flow experiments with pipes or channels cannot by themselves validate formulae for the evaluation of the friction slope term of the unsteady open channel flow equations. Heuristic reasoning suggests that these formulae could serve this purpose, but their adequacy for the purpose can be judged only by comparison of the computed and observed results in cases of unsteady open channel flow. A difficulty with unsteady open channel flow is that the effect of friction cannot be isolated. The differences between observed and computed depth hydrographs in unsteady open channel flow are the result, among other factors, of

(*a*) deviations from the gradual depth variation assumption, which invariably occur in practice
(*b*) variations in the velocity and momentum distribution coefficients with distance and time
(*c*) uncertainties of the friction slope formula used
(*d*) errors associated with the difference scheme used.

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15. Adequate agreement between observed and computed depth hydrographs using a particular friction slope formula, in the unsteady open channel flow equations, may therefore only be taken to mean that the mathematical model as a whole (including the friction slope formula used) has proved adequate. The uniform open channel flow formulae have been found to be adequate.[19] Barr and Das[17] have found that the Colebrook–White formula, in conjunction with the Darcy–Weisbach formula, is also suitable. However, there are no grounds for it to be considered superior to other formulae. In unsteady open channel flow, momentum effects are generally predominant and Reynolds' numbers (which would in this case vary with distance and time) are generally in the completely turbulent range, except perhaps for very short durations. Thus, as long as appropriate values have been assigned to the relevant constants, there will be little difference between the computed results whether the Colebrook–White formula or the Manning formula has been used in the evaluation of the friction slope term. The extent to which the friction factor influences the computed results is shown in reference 20. For the problem in reference 21, reference 20 shows the differences caused to the depth hydrographs by a 10% increase in the value of Manning's constant. The depth hydrograph for $x = 50$ ft is the most influenced by this increase and shows an average increase in depth of 3·10% with maximum and minimum increases of 3·57% and 2·43% respectively.

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With regard to steady flow problems where solution of the basic Colebrook–White function is involved, an attempt was made to demonstrate the alternatives now available. Pipe network analyses involve a relaxation procedure towards a steady flow solution. Programmes can be simplified and computation time can be reduced by using approximations. However, the advance in calculating power and capacity for programme retention in hand-held calculators continues. Once entered, an accurate approximation formula gives virtually the basic Colebrook–White solution without concern as to limits of iteration. Simplified usage versions of accurate approximation can then be deduced as appropriate for restricted conditions. In all cases, the assessment of piezometric gradient can readily be made to cover laminar flow, as well as turbulent flow resistance, where median commercial roughness obtains on the surface of the conduit.

17. The basic Colebrook-White function is intrinsically applicable to such turbulent transition flows, just as the Manning formula is intrinsically applicable to rough turbulent flows. The most penetrating examination of the relationship between the Manning type formula and the rough turbulent logarithmic formula is that of Williamson.[22] It is not obvious why the adoption of the one formula or the other should be separated in so far as it concerns the difficulties resulting from the translation from steady to unsteady flow. It is agreed that, in simulation, inadequacy can arise from lack of proper representation of the non-resistance factors outlined by Dr Sivaloganathan where these are dominant. Where resistance effects are dominant, inadequacy can arise from inability of the resistance formula to output the appropriate friction factors for changing flow circumstances.

18. In the circumstances cited by Dr Sivaloganathan, results from a virtually full-scale test rig were available. The value of Reynolds number (on an equivalent pipe basis) was high throughout (minimum value about 7×10^5). Also, the range was small, the value rising to less than twice the minimum. By the need to select a value of Manning's n of 0·011, it was shown that flow was virtually in the rough

turbulent region with a very small range of relative roughness, from k/d of about 3×10^{-4} for the just over half full condition to a minimum of about $2{\cdot}3 \times 10^{-4}$ for any time and place in the flood event tested. Thus there was not even the position to test the adequacy of the Manning formula for rough turbulent flow—far less the position to elucidate assessment of resistance without rough turbulent conditions.

19. There are difficulties in achieving a wide range of Reynolds number. Barr and Das[17,23] met this difficulty in two ways. First, experiments were made under geometrically similar conditions at two different scales. Second, the reflecting dam-burst situation allowed observation of a wide range of values of Reynolds number in respect of the peaks of velocity within the successive surges when the effect of resistance is important. Das[24] had compared predicted profiles for steady state gradually varied flow with different forms of channel roughness. Significant differential effects had been found in the turbulent transition conditions. Thus when the unsteady flow experiments were compared with predictions, only the best available overall assessment of resistance was adopted, using the basic Colebrook–White function for smooth surfaces, and a generalized function, calibrated against uniform flow data, for the rough surfaces. The extent to which this was justified can be disputed. Nothing relevant can be deduced from circumstances where flow was close to rough turbulent throughout, and where, in addition, the relative roughness varied so little throughout that a fixed value of friction factor might well have replaced an assessment of Manning's n without significant effect on the degree of compliance achieved between observation and simulation.

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