Proc. Instn Civ. Engrs, Part 2, 1984, **77**, June, 279–286

DISCUSSION ON PAPER 8635

Plastic theory of non-integral infilled frames

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This Paper professes to use plastic analysis to determine collapse loads for infilled frames. In general we cannot agree that this is so. It is a basic tenet of the theory that both upper and lower bounds to the collapse load exist. The Authors appear to have mixed up both to produce estimates of the collapse load.

26. In § 3, the Authors state that the over-estimation of collapse load using Wood's theory is due to the assumption of full interaction or excessive friction at the boundary and the neglecting of separation. While agreeing that full interaction is one of the contributory factors, it is important to note that by using discontinuous stress fields, Wood was effectively allowing for separation. Our studies[11] have shown that the lack of ductility of the infill is as important a factor as friction.

27. We assume that for the analysis of their collapse mode 1 (Fig. 2), the Authors intend to use the lower bound theorem. This requires that

(a) equilibrium is satisfied,
(b) the yield criterion is not violated.

28. A valid lower bound, which is based on the Authors' mode 1, is shown in Fig. 17. xl is determined by consideration of equilibrium of the infill.

29. Further calculations show that

(*i*) F_b (Fig. 2) = 0, for all cases. (As is to be expected as friction is ignored and the shear in members AB is zero below E);
(*ii*) the maximum moment within the spans AD and BC is less than M_p
(*iii*) significant tensile forces occur in AB and DC which may have an effect on the value of M_p that should be used for the columns.

It only remains to check that yield is not violated in the infill. This is difficult to do as no simple stress fields can be postulated for the infill, except in the case of square panels. Furthermore, no failure criterion for the infill is given by the Authors.

30. A similar approach can be used to determine a valid lower bound for mode 2.

31. Mode 3 suffers from the same problem in that the forces shown to be acting on the panel are not in equilibrium.

32. The Authors' mode 3 also suffers the failings of previous methods in that it requires the insertion of an assumed contact length. Furthermore, it is noteworthy

Paper published: *Proc. Instn Civ. Engrs*, Part 2, 1983, **75**, Sept., 379–396.

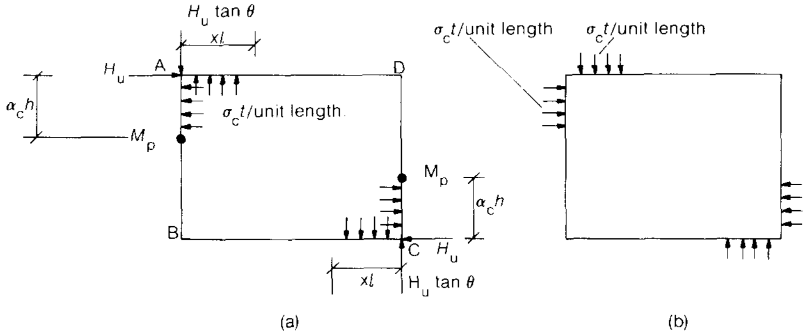

Fig. 17. Liauw and Kwan's mode 1 amended to give valid lower bound; (a) forces acting on frame; (b) forces acting on infill

that the transition from mode 1 to mode 3 implies a sudden jump in hinge position, as the frame becomes stronger, from about 2/3 the way up the column to the lower column–beam junction. The stress field in the infill also changes dramatically in the transition from mode 1 to mode 3 failure.

33. The agreement between the experimental and theoretical results needs further clarification. The Authors correctly state that the assumption of no friction at the frame–interface boundary is conservative in the design of panels. Therefore, one would expect the experimental results to lie above the theoretical predictions. Although this is generally so for Barua and Mallick's results (Fig. 5), it is not so for the other results (Figs 6–8).

34. The analysis of the multi-storey frames appears to be an upper bound analysis. It is difficult to envisage the occurrence of mode 2 (Fig. 10(b)) as the infill would act as a constraint on the formation of hinges in the beams.

35. The calculation of an upper bound relies on the calculation of the external work done and the internal energy dissipation. For the calculation of the internal energy dissipation, the Authors have used the load obtained from the single-storey analysis. This appears to be incorrect as the single-storey analysis is a lower bound. To do as the Authors have done would be possible if the single-storey analysis were an upper bound or an exact solution.

36. A major advantage of Wood's method was that it could be used to include the effects of openings and panels of various shapes. The present Paper appears to suggest a step back to the semi-empirical approach of earlier methods.

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The discussors appear to have misunderstood Figs 2–4, which are only schematic for clarity as stated in § 24. More details of these collapse modes are shown in Figs 14–16 and further clarification in connection with §§ 28–31 is as follows.

38. In Figs 2 and 14, showing mode 1 collapse, the contact pressure between the beams and the panel has been purposely omitted to avoid obscuring the more important details. In § 28 the point is made that beam–panel interaction is necessary in order to keep the panel in equilibrium. However, it has no bearing on the formulation of the collapse shear because the latter is derived from the column–panel interaction which is given in § 6.

39. In Figs 3 and 15, showing mode 2 collapse, the contact pressure between the columns and the panel has also been purposely omitted. The column–panel interaction keeps the panel in equilibrium, but does not affect the collapse shear. Similar conclusions can be drawn from Figs 4 and 16, showing mode 3 collapse.

40. With regard to F_b in § 29 (*i*), the assumption that $F_b = 0$ in mode 1 is only an approximation, as it has been found[4,6] that F_b actually has a small negative value. It can be seen from Fig. 14 that the column and the panel are still in contact along a small length beyond the plastic hinge referred to as E in Fig. 2. This small reaction below E accounts for the negative column shear.

41. The point in § 29 (*iii*) is well taken. Although not explicitly stated, the effect on the theoretical values of M_p of axial forces in the frame members is already accounted for in the comparisons with the experimental results. The existence of these axial forces depends on the support conditions. The effect of axial forces on the value of M_p is generally trivial in single storey infilled frames, but can become significant in high rise multi-storey infilled frames.

42. Turning to the question of upper and lower bounds (§ 25, § 27, § 34 and § 35), the analysis of the collapse modes of single storey infilled frames given in our Paper is based on the equilibrium of forces on the frame yielding the lower bound solutions. In actual fact, the same solution can also be obtained by kinematically admissible collapse modes yielding the upper bound solution. Such an analysis is as follows.

Mode 1—upper bound solution

43. A possible collapse mechanism of mode 1 is shown in Fig. 18, in which the panel can be viewed to be replaced by the contact pressure on the interface boundary. Let the relative horizontal displacement between corners A and C be δ. Then

$$\text{external work done} = H_u \delta$$

$$\text{energy dissipated by the frame} = \frac{(M_{pj} + M_{pc})}{\alpha_c h} \delta$$

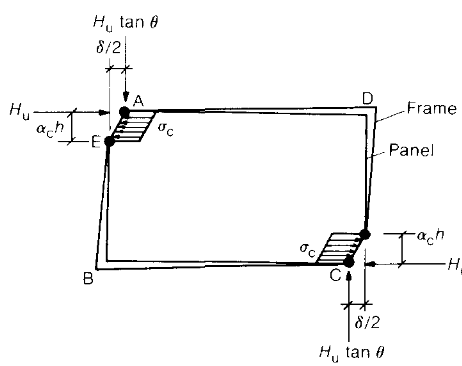

Fig. 18. Mode 1—upper bound solution (panel–beam pressures omitted for clarity)

The energy dissipated by the panel is equivalent to the work done on the panel by the contact pressure on the boundary. Thus

$$\text{energy dissipated by the panel} = \tfrac{1}{2}\sigma_c t \alpha_c h\delta$$

The work equation is

$$H_u \delta = \frac{(M_{pj} + M_{pc})}{\alpha_c h} \delta + \tfrac{1}{2}\sigma_c t\alpha_c h\delta \tag{43}$$

Eliminating δ from the equation and minimizing for H_u yields

$$\alpha_c = \sqrt{\frac{2(M_{pj} + M_{pc})}{\sigma_c th^2}} \tag{44}$$

and

$$\frac{H_u}{\sigma_c th} = \sqrt{\frac{2(M_{pj} + M_{pc})}{\sigma_c th^2}} \tag{45}$$

which are identical to equations (3) and (4).

Mode 2—upper bound solution

44. The analysis of mode 2 is similar to that of mode 1.

Mode 3—upper bound solution

45. The collapse mechanism of mode 3 is shown in Fig. 19 when the span is greater than the height. Let the relative horizontal displacement between corners A and C be δ. Then

$$\text{external work done} = H_u\delta$$

$$\text{energy dissipated by the frame} = \frac{4M_{pj}}{h}\delta$$

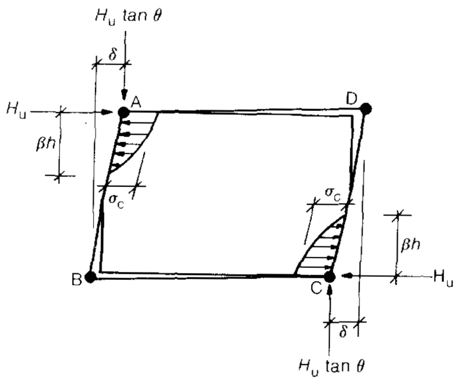

Fig. 19. Mode 3—upper bound solution (panel–beam pressures omitted for clarity)

The energy dissipated by the panel is equivalent to the work done on the panel by the contact pressure on the boundary. Thus

$$\text{energy dissipated by the panel} = \sigma_c th(\tfrac{2}{3}\beta - \tfrac{1}{2}\beta^2)\delta$$

The work equation is

$$H_u \delta = \frac{4M_{pj}}{h} \delta + \sigma_c th(\tfrac{2}{3}\beta - \tfrac{1}{2}\beta^2)\delta \tag{46}$$

Eliminating δ from the above equation gives

$$H_u = \frac{4M_{pj}}{h} + \sigma_c th(\tfrac{2}{3}\beta - \tfrac{1}{2}\beta^2) \tag{47}$$

which is identical to equation (11). The analysis of mode 3 when span is less than height is similar to the above.

46. With regard to the contact length in mode 3 (Fig. 4) raised in § 32, we have observed by comparison with the experimental results in Figs. 5–8 that β varies from about $\frac{1}{3}$ to $\frac{1}{2}$. It can be seen from the theoretical curves that the overall effect of the variation of β on the strength prediction is small, and we conclude that the assumption of $\beta = \frac{1}{3}$ is reasonably accurate.

47. As to the transition between mode 1 and mode 3, it is shown clearly in Fig. 20 that the transition does exist. When the frame is strong relative to the panel, the plastic hinges are formed at the joints A and B of mode 3 (Fig. 20(a)). The corresponding value of F_b can be obtained from equations (9) and (10) as given by

$$F_b = \frac{2M_p}{h} - 0{\cdot}028\ \sigma_c th \tag{48}$$

It is noted that the value of F_b decreases with the decrease in bending strength of the frame, until F_b becomes zero when $M_p = 0{\cdot}014\ \sigma_c th^2$. At this transition mode (Fig. 20(b)), one plastic hinge occurs at the joint A while the other occurs within the length with constant moment. With a weaker frame, the collapse mode becomes mode 1. Hence, equation (48) is no longer applicable. In this case (Fig. 20(c)), the plastic hinges occur at joint A and point E which is the point of maximum moment. In this sense, therefore, the position of the plastic hinge does 'jump' in the transition from mode 1 to mode 3 (Fig. 21).

48. With regard to the contribution of friction raised in § 33, it is negligible.[4] The assumption of no friction is, therefore, only slightly conservative. In view of the material variations and experimental errors which are unavoidable, such as in the experiments using concrete or brickwork for the infill, it is not surprising that the small contribution of friction could have been over-ridden by the scatter of the experimental results.

49. With reference to § 26, we are pleased to note that Dr Ma[11] has made a study on the lack of ductility of the infill and we hope that his results will be published. In our study[4,6] on the interaction at the frame–panel interface, the contributions of friction to the strength of the non-integral infilled frames are generally unimportant. Incidentally, there should be plastic hinges at joints A and C in Fig. 17, and the beam–panel contact pressure should be less than σ_c, except in square panels. The latter, however, is unimportant in the evaluation of the collapse shear as given in § 6 and explained in § 38.

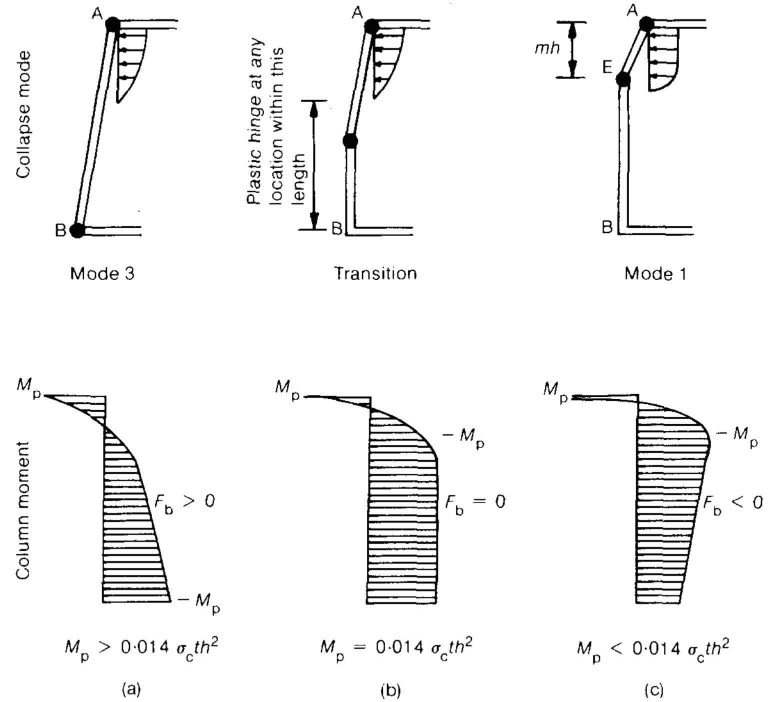

Fig. 20. Changeover from one collapse mode to the other when $M_p = M_{pb} = M_{pc}$

50. With regard to Wood's method[5] raised in § 36, we were inspired by the contribution in which the importance of the bending strength of the frame and the various modes of failure were explicitly shown. However, we are not aware of whether or not the method could be used in relation to panels with openings and of various shapes as no reference is made to this. Be that as it may, we do not share the view expressed in the last comment, as the present Paper was developed in a relatively simple and clear manner with the support of non-linear analysis and experimental results. Our method has been further developed[12] to deal with infilled frames with finite shear strength at the frame/infill interface when shear connectors/bonding are provided.

References

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12. LIAUW, T. C. and KWAN, K. H. Plastic theory of infilled frames with finite interface shear strength. *Proc. Instn Civ. Engrs*, Part 2, 1983, **75**, Dec., 707–723.

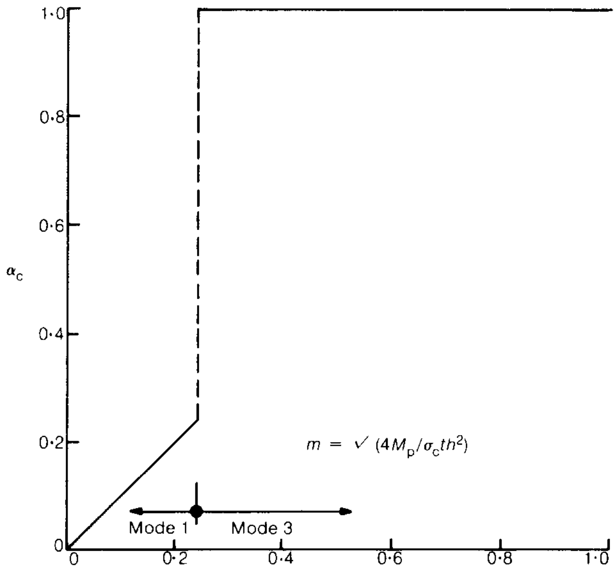

Fig. 21. Relation between α_c *and m*

Corrigenda to Paper 8635

Figures 11 and 12 should be as shown in the following.

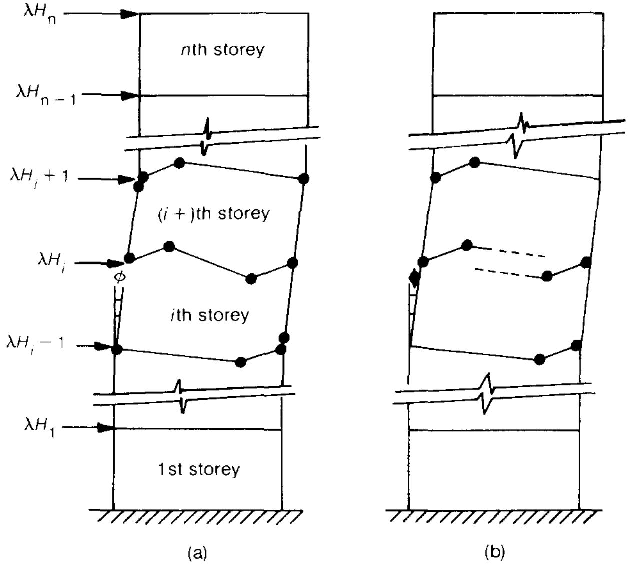

Fig. 11. Corner crushing with failure in beams (multi-storey): (a) actual collapse mechanism; (b) approximation for evaluation of internal energy absorption

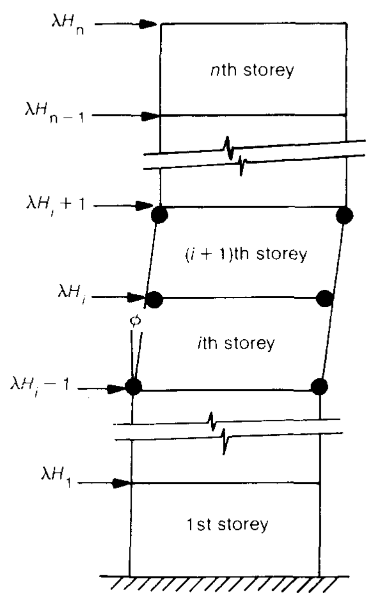

Fig. 12. Diagonal crushing (multi-storey)