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DISCUSSION ON PAPER 8718

Plastic theory of infilled frames with finite interface shear strength

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In the discussion[14] to the previous Paper[12] we suggested that the Authors had not applied limit analysis correctly. We feel that this is also true for this Paper, and the comments made then are valid for the current Paper.

26. We assume the Authors are attempting to apply the lower bound theorem to the analysis of their modes. An investigation of the forces acting on the infill (Fig. 10(a), corresponding to Fig. 3 (mode 1) of the Paper) appears to show that equilibrium has been violated. Could the Authors explain how equilibrium has been preserved? Fig. 10(b) shows the stresses on an infinitesimal element at the corner A of the infill. Could the Authors explain the absence of complementary shear stresses?

27. The normally accepted yield criterion[15,16] used for infill is the square yield criterion in principal stress space (Fig. 11). If an element of material is subjected to a direct stress σ_c and a shear stress of any magnitude, as is the case in all modes, the maximum principal stress will be greater than σ_c. Could the Authors comment on this and give details of the yield criterion used for the infill?

28. The diagonal band action indicated in Fig. 1 implies a direct compressive stress at 45° to the horizontal, together with a direct tensile stress orthogonal to this. Could the Authors explain how this can be reconciled with the distribution of stresses assumed on the frame, as shown in Figs 3–6, and the assumed crushing regions shown in Fig. 2.

29. Further, could the Authors explain both how these regions have been determined and the assumptions applicable to the infill, as no indication has been given that the plane stress equilibrium conditions have been considered in the infill which, in the absence of body forces, are

$$\frac{\partial \sigma_x}{\partial_x} + \frac{\partial \sigma_{xy}}{\partial_y} = 0 \qquad \text{and} \qquad \frac{\partial \sigma_y}{\partial_y} + \frac{\partial \sigma_{xy}}{\partial_x} = 0$$

It should be noted that Wood's[15] stress field definitions automatically satisfy these equations by the selection of constant stress fields. This cannot be done in the case of the Authors' modes as it would not only violate the conditions at the assumed cracks, but would also require the fields to be carried through to the opposing beams or columns.

30. We find it not at all surprising that the case for $s = 0$ reduces to that of the non-integral infilled frames as it is exactly the same analysis! Had there been a

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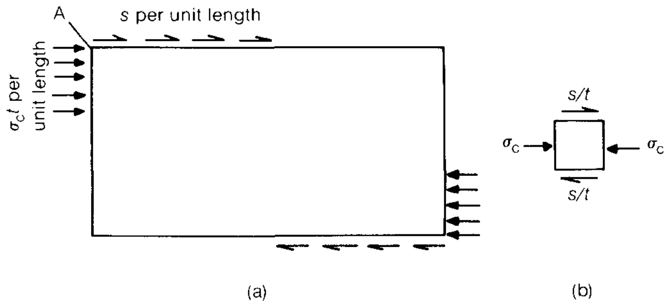

Fig. 10. (a) Loads on infill; (b) stresses at corner A

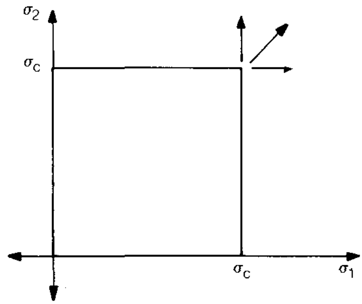

Fig. 11. The 'square' yield criterion

defined yield criterion, such that introduction of a shear stress at the relevant interface would have influenced the magnitude of the allowable direct stress components, then the reduction to the non-shear case would have been an additional check on the theory.

31. It is of interest that modes 1 to 4 all appear to be based upon equilibrium stress considerations, which we have interpreted to mean that the Authors intend to pursue lower bound solutions. The lower bound should be either correct or predict too low a value of the collapse load; consequently, one is looking for the highest lower bound solutions.[17] The Authors state, in § 4, that the collapse mode is the one that leads to the smallest value of collapse shear strength. This minimization is attributable to upper bound solutions and, as a result, the Authors have confused the requirements for valid upper and lower bound solutions. Have the Authors ascertained that the alternative modes violate the requirements of the lower bound theorem? Often Wood[15] found that solutions corresponding to different modes produced larger values for the collapse shear strength, but on checking the moments on the frame members found that some of these members violated the criterion. Thus these solutions were invalid.

32. For the comparison with the experimental results in Table 5 the Authors have taken the lowest value, but as the theoretical values have not been obtained from the upper bound theorem there is no reason to choose the lowest value other than it is closest to the experimental value.

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The present discussion is very similar to the previous discussion[14] on our earlier Paper,[12] and we do not intend, therefore, to repeat the answers which we have already given.

34. In Fig. 3, the panel/beam interface normal stress and the panel/column interface shear stress have been purposely omitted to avoid obscuring the more important details. The interface stresses acting on the infilled panel are shown in full in Fig. 12, which has been determined by simplifying the results of non-linear finite element analysis.[17] The interface stresses shown in Figs 4–6 are similarly incomplete. The Authors wish to apologize for the omission of these details which has led to the misunderstanding. For the purpose of clarity, it would have been more appropriate to mention such an omission in the captions of Figs 3–6.

35. Our plastic theory is based on the pattern of non-linear behaviours as observed in experimental investigation, and non-linear finite element analysis[13,18] in which the square yield criterion in principal stress space has been adopted. The shear stress distribution at the interface, as obtained by non-linear finite element analysis, is shown in Fig. 13. It can be seen that the shear stress near the loaded corners (point A referred to in Fig. 10) is negligible. At the infill/column interface, the shear strength of the connection is not fully mobilized and, in general, the shear stress developed is only a few percent of the crushing strength of the infill. Therefore, the effect of this shear stress on the maximum principal stress is trivial.

36. The actual stress field developed in the panel is very complex because of stress concentration at the loaded corners, and cracking and crushing in the panel. No assumed stress field, though satisfying the equilibrium conditions, would lead to the correct collapse shear strength unless the assumed stress field is a good approximation of the actual one. The Authors deal with this problem by using non-linear finite element method to analyse the infilled frame as a whole. In the analysis, cracking, crushing and non-linear stress–strain relations are taken into

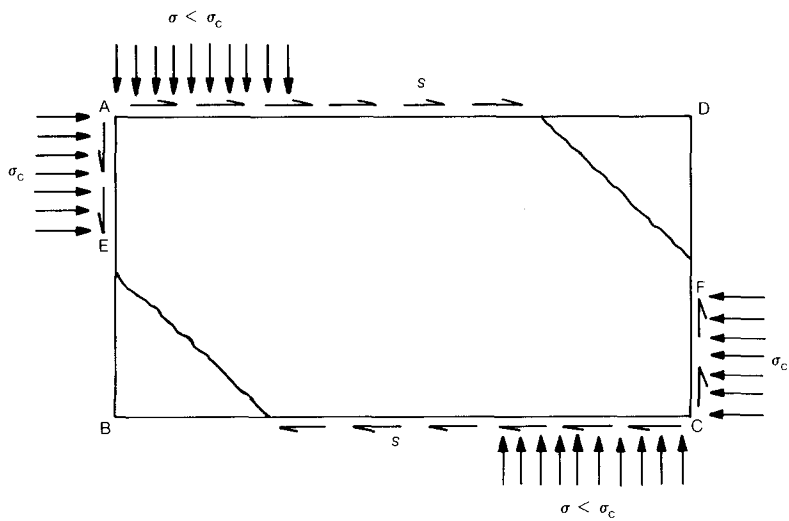

Fig. 12. Simplified interface stress distribution for mode 1 when $l > h$

account. The interface stresses obtained from the analysis automatically satisfy the compatibility, equilibrium and yielding conditions. The interface stresses are then simplified and applied to the frame. Having first obtained and then simplified the interface stresses, the analysis of the frame becomes relatively easy.

37. Considerations of the kinematically admissible collapse mechanisms, equilibrium conditions and yield criterion of the frame, form the central part of our plastic theory. The Authors have chosen to start with the analysis of kinematically admissible collapse modes. The corresponding collapse shears can then be obtained by either the principle of virtual work or by equilibrium considerations leading to the same results. The actual collapse mode is the one that would collapse first, i.e. the one that has the smallest value of collapse shear strength. Hence, the lowest theoretical collapse shear strenghts were compared with the experimental results in Table 5.

38. The upper bound solution using the principle of virtual work is given as follows.

Mode 1—upper bound solution

39. A possible collapse mechanism of mode 1 is shown in Fig. 14. Let the relative horizontal displacement between corners A and C be δ. Then

$$\text{external work done} = H_u \delta$$

$$\text{energy dissipated by the frame} = \frac{(M_{pj} + M_{pc})}{\alpha_c h} \delta$$

$$\text{energy dissipated by the panel} = \tfrac{1}{2}\sigma_c t \alpha_c h \delta$$

$$\text{energy dissipated by the interface connection} = s(l - h/2)\delta$$

Note that the panel/beam vertical normal stress and the panel/column vertical shear stress (which are not shown in Fig. 14) do not produce internal work as the

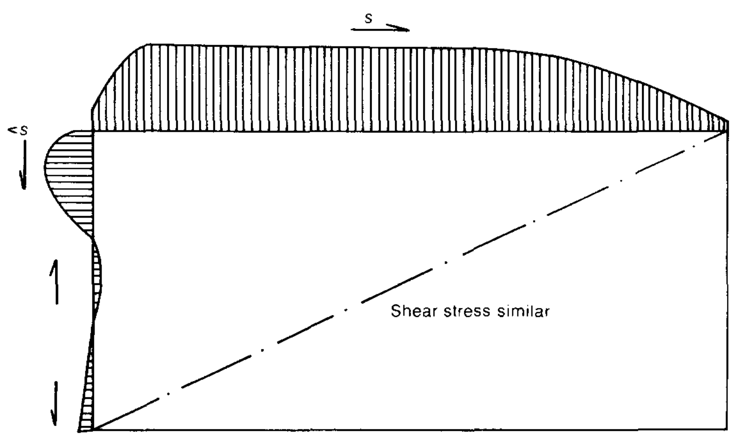

Fig. 13. Interface shear stresses in N/mm – run around third storey panel of model C-2-III at ultimate stage (roof deflexion = 14·4 mm) as obtained by non-linear finite element analysis

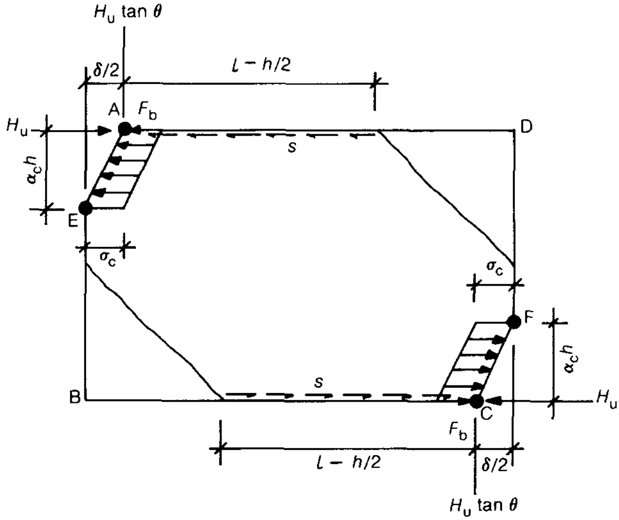

Fig. 14. Mode 1—upper bound solution, l > h case (panel/beam pressure and panel/column shear omitted for clarity)

displacement δ is horizontal. The work equation is

$$H_u\delta = \frac{(M_{pj} + M_{pc})}{\alpha_c h}\left[\delta + \frac{1}{2}\sigma_c t\alpha_c h\delta + s\left(l - \frac{h}{2}\right)\right]\delta \qquad (28)$$

Eliminating δ from the equation and minimizing for H_u yield

$$\alpha_c = \sqrt{\frac{2(M_{pj} + M_{pc})}{\sigma_c th^2}} \qquad (29)$$

and

$$H_u = \sqrt{\left(\frac{2(M_{pj} + M_{pc})}{\sigma_c th^2}\right)}\left[\sigma_c th + s\left(l - \frac{h}{2}\right)\right] \qquad (30)$$

which are identical to equations (4) and (5) respectively.

Mode 2—upper bound solution

40. The analysis of mode 2 is similar to mode 1.

Modes 3 and 4—upper bound solution

41. The analyses of these modes are similar to those presented in the previous discussion.[14]

42. It should be noted that our plastic theory has been developed on the basis of experimental investigation and non-linear finite element analysis with, of course, the necessary assumptions and simplifications. We have also gone one step further in order to verify the theory by comparing the calculated strengths with experimental strengths of many infilled frames tested not only by us, but also by other researchers. With the confidence achieved so far, a design procedure with a

number of design charts have been produced for publication, which would enable a designer to evaluate rapidly the predicted strength of an infilled frame. However, refinements to the proposed theory are possible in the light of further research.

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