Paper No. 6220

Plastic design of pitched-roof portal frames †

by

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Discussion

The Chairman, opening the discussion, commented on the statement in § 1 that: "growing experience with the practical aspects of design, fabrication, and erection has demonstrated that the economies latent in the plastic method can be realized." It was not the first time that that statement had been made, but he could not see how one could achieve economies by a particular *method.* In § 26 the Author had drawn attention to a possible change in a certain load factor which illustrated his point: whether or not there was an economy in designing a structure in one specified way as compared with another must depend entirely on the values of the load factor and the factor of safety used in the respective designs. The method of design described was obviously valid but the claim that in itself it produced economies seemed to the Chairman to be one which could not be upheld. Would the Author comment on that point?

81. The Chairman's second point concerned deflexions; it seemed to him that in §§ 30 and 31 the emphasis was wrong. Having designed a structure, by whatever method, one wanted to know how it would behave under normal loading. It seemed to him that elastic deflexions could be, and often were, the determining factors in any but the simplest buildings. If, for instance, a portal frame carried a heavy gantry, this could conceivably be displaced by deflexions under wind load. Machinery also could be dislocated if a structure was too flexible. This aspect of design seemed to be dismissed rather summarily in the Paper. Unfortunately, no short cuts seemed to exist for the calculation of working deflexions: the result was that the usual elastic procedure must perforce be followed, which meant that both calculations, plastic and elastic, were necessary to achieve the full result. Would not the result be reached just as economically by choosing a suitable working stress and designing on an elastic basis?

Professor Wilfred Merchant (College of Technology, Manchester) referred to the calculations and experimental evidence of the critical loads of the pitched-roof portal frames, and in particular to a point which had been raised by Dr Bolton when some of the work was presented at the symposium at Cambridge last year.

83. The formula given in the Paper for the critical loads neglected the axial loads in the rafters. At the time of the symposium Dr Heyman had said that it was a small matter and something which could be neglected because one was working to a low value in the rafters.

84. However, the situation was not quite like that. The stiffness of the rafters at the critical load had to be considered. In the experiments carried out by Dr Heyman, with the internal stanchions at the nominal critical load that he calculated, the rafters had lost almost all their stiffness, and the actual critical load was very much reduced compared with the results given in the Paper. That, of course, rendered void the particular empirical discussion of the applicability of the empirical formula allowing for the effect of stability. When Professor Merchant had first put forward formula (21)

† Proc. Instn civ. Engrs, vol. 8, p. 119 (Oct. 1957).

on p. 132 he had modelled it on Rankine's formula for struts and called it the generalized Rankine formula; it was, therefore, no very great surprise that there was a similarity between the two formulae.

85. The stability effects were most important on the type of construction with the pinned internal stanchions, but investigations had been carried out which indicated that for the single-bay portal with pinned feet the original critical load might be only of the order of three times the yield load as given by the plastic method, and when it was as near as that one would also require to take into account the effect of the critical load on the strength of such structures.

86. Professor Merchant's colleagues, Dr Brotton and Mr Bahauddin, would supply graphs giving the critical loads for the types of portal dealt with by the Author.

Mr L. G. Johnson (Assistant Director of Research, Department of Engineering, University of Cambridge) referred to the statical approach in the early part of the Paper where the pitched-roof portal frame was cut at the apex. He could confirm from his own experience that that was a very powerful method indeed and eminently suitable for loading conditions which were rather more complex than had been illustrated that evening.

88. Table 3 should find a place in most design offices, but before it did so, perhaps a little more information ought to be given about the limitations of the Table. For instance it was stated in § 41 that the Table did not take account of deflexions, but no indication was given as to exactly where on the Table deflexions took over and governed the design of the portal frame. If, for example, Table 3 was used to design 100-ft-span frames, 30 ft to eaves, at 15-ft centres, one arrived at a plastic modulus equivalent to a 15-in. × 6-in. × 45-lb R.S.J. Clearly the use of such a section would result in large deflexions under working loads.

89. It was well known that the plastic modulus was directly proportional to the frame spacing provided that the span of the frame and the intensity of loading remained constant, but the moment of inertia increased much more quickly than did the plastic modulus. Therefore, the Table was much more likely to be of value for structures having portal frames which were rather widely spaced. Would the Author give some indications of the limitations of Table 3?

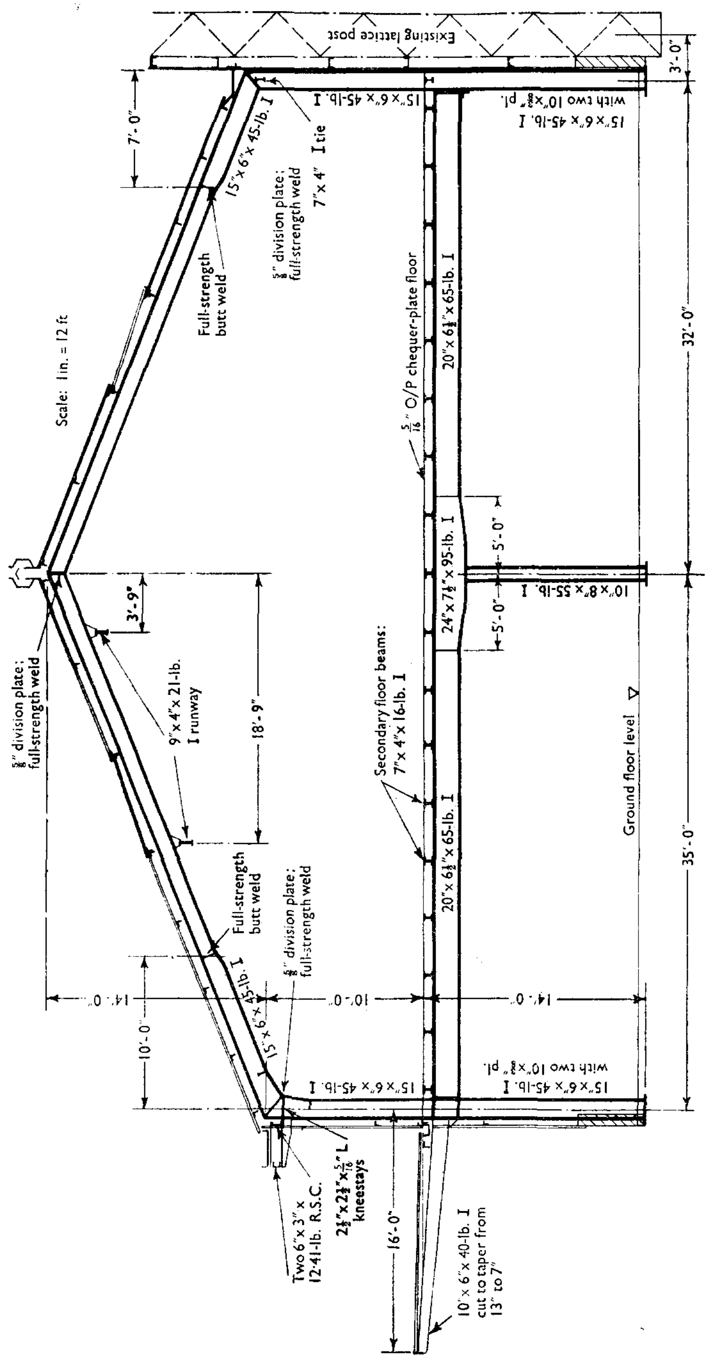
90. With regard to multi-bay frames, it had been shown quite clearly that the use of internal ties at eaves level was far from desirable theoretically, but he thought that there were practical disadvantages too. Failure of one tie or one internal prop could lead in extreme cases to progressive collapse of the whole structure. The internal tie also defeated one of the main objects of the portal frame, namely a completely clear roof space. It could be argued, too, that if the eaves tie was replaced by an external strut at ridge level, this was unsightly and seriously complicated the weatherproofing of the building.

91. In § 32, which dealt with the deflexions of the structure on the point of collapse, it was stated that the hinge at the apex of the portal frame was the last to form. Could the Author say if it was always the last to form, and if this was not always the case, could he give some indication as to when the apex hinge was not the last to form and say what difference it made to the analysis or the final answer?

Mr D. A. Godfrey (Assistant Engineer, I.C.I. Limited, Alkali Division) described a building which was shortly to be erected for his firm and for which two totally different designs were prepared, one by plastic theory and the other elastic.

93. Fig. 21 showed a typical cross-section through the building as it would be erected, this being the result of plastic design. At the moment the job was only in the site-clearance stage, and so he was not able to confound the critics by saying that the structure had not fallen down, but he felt confident that in due course he would be able to add that final evidence.

94. The new structure was of single-bay portal-frame construction, 67-ft span and 24 ft high to the eaves, and had a suspended floor at 14 ft above ground-floor level.



Existing lattice post
3'-0"
32'-0"
35'-0"
7'-0"
15"×6"×45-lb. I
with two 10"×3/8" pl.
15"×6"×45-lb. I
7"×4" I tie
5/8" division plate; full-strength weld
Full-strength butt weld
Scale: 1 in. = 12 ft
5/16" O/P chequer-plate floor
20"×6½"×65-lb. I
24"×7½"×95-lb. I
10"×8"×55-lb. I
5'-0"
5'-0"
3'-9"
18'-9"
9"×4"×21-lb. I runway
5/8" division plate; full-strength weld
Secondary floor beams: 7"×4"×16-lb. I
Ground floor level
14'-0"
10'-0"
14'-0"
10'-0"
15"×6"×45-lb. I
Two 6"×3"×12·41-lb. R.S.C.
2½"×2½"×5/16" L kneestays
16'-0"
10"×6"×40-lb. I cut to taper from 13" to 7"

FIG. 21

It was the suspended floor which he hoped would give particular interest to the job from a design point of view. The roof and side cladding were of lined corrugated sheeting with glazing, the floor load was 150 lb/sq. ft, and there were two runways in the roof, the load on each of which, making allowance for impact and tackle, was 2 tons. The frames were at 15-ft centres.

95. In the first scheme it was intended that the new structure should be tied back at eaves and floor levels to the existing building indicated by dotted lines on the right of Fig. 21. The floor was to be carried by pairs of simply supported beams resting on three stanchions with a two-pin arch sitting on top to carry the roof. The stresses throughout were limited to the working stresses given in B.S. 449.

96. The structure so designed was not very beautiful and Mr Bryan had made an investigation to see if the plastic theory would give a more elegant solution. Mr Bryan, who would speak next, would describe the design technique, but Mr Godfrey pointed out that they had designed for virtually pinned connexions between the floor beams and the stanchions, which might give the impression that they did not know how else to do it. In point of fact, there was probably some small grain of truth in that, because, as had already been hinted in the Paper, the beautiful simplicity of the plastic theory rather disappeared when one had to deal with stanchions carrying considerable axial loads in addition to large bending moments. Nevertheless, they had considered rigid connexions at those points, but had found that the resulting bending moments in the stanchions increased their sections considerably and that the possible saving on the beams was not sufficient to compensate. However, even with the pinned joints, the floor beams tied the stanchions together at that level and had an important influence on the mode of collapse.

97. He compared briefly the main scantlings in the two designs. Designing the floor beams by the plastic theory as continuous over two spans, using a load factor of 1·75, had resulted in 20-in. × 6½-in. × 65-lb. joists for most of the length, with a short piece of 24-in. × 7½-in. joist over the centre support, whereas in the elastic design they had included 20-in. × 12-in. × 135-lb. broad-flange beams. Similarly, the secondary joists running longitudinally were reduced from 8-in. × 4-in. to 7-in. × 4-in. R.S.J. Above the floor, the frame designed by the plastic design consisted of 15-in. × 6-in. light joists and 13-in. × 5-in. joists, whereas in the other scheme they had had 18-in. × 6-in. × 55-lb. joists.

98. It was necessary to apply discretion when comparing the work of two different designers, because one man might be of a more economical turn of mind than the other. However, Mr Godfrey had tried to make allowances for that, and he had calculated that for the job illustrated, with a total weight of steelwork of about 160 tons, the saving in the weight of steelwork which could be accredited to plastic design was roughly 17%, and in these days of steel shortage he thought that that by itself was well worth while.

99. His firm had then asked a friendly contractor to estimate for both schemes, and while the additional workmanship in the second scheme resulted in a rather higher rate per ton, the plastic design still effected a saving of about 10% on the cost.

100. His remarks applied, of course, only to the particular job using a load factor of 1·75. He agreed with the Chairman that there were many cases where deflexion was the main concern, and in such cases his firm would not, of course, use the plastic theory. They regarded the plastic theory as a tool in the designer's tool box, and if it was not the appropriate tool, they did not use it.

Mr E. R. Bryan (Technical Officer, I.C.I. Limited, Alkali Division), before dealing with the design of the structure described by Mr Godfrey made some general remarks about the basic stages of plastic design.

102. The Author had described the effect of stability and deflexions at some length, and also the value of the statical approach, but he had mentioned only briefly the principle of work and the principle of building up the final collapse mode from the basic modes. No doubt that was because these principles had been fully described elsewhere

but to Mr Bryan they seemed a very important stage in plastic design. Sketches of the various collapse modes enabled the designer to see exactly what was happening to the building. Bending-moment diagrams were perhaps rather impersonal things, but to build up the final collapse mode, step by step, gave the designer the feel of the structure.

103. In the Paper it was suggested that the actual loading used in the principle of work might be idealized to simplify the geometry of collapse, but Mr Bryan wondered if the results were not liable to be a little misleading when the loads were simplified to the extent of replacing a uniform distributed load by a point load at the apex. Perhaps the Author could make some comment on that.

104. Fig. 22 showed the three stages in the design of the structure described by Mr Godfrey. Fig. 22a showed the adopted loading per frame and Fig. 22b the collapse mechanism obtained on the basis of the principle of work. It was only approximate because it had been assumed that plastic hinges occurred at the nodes, whereas one would probably occur in the rafter. There had been a little trouble in finding the correct mode of collapse but Dr Heyman had kindly given some help.

105. One puzzling feature about the structure was that although it had only two redundancies, i.e. the shear at the feet and the tension in the first-floor tie, there were four plastic hinges instead of the three expected. That was accounted for by the fact that the shears at the two feet were equal, so that the moments in the stanchions at first-floor level must be the same. So there was a certain degree of symmetry in the frame.

106. The "free" (Fig. 22c) and "reactant" (Fig. 22d) bending-moment diagrams were drawn and the combined bending-moment diagram (Fig. 22e) was derived from them. The effect of the tie caused but little complication.

107. From the work equations, using a 13-in. × 5-in. × 35-lb. R.S.J. for the rafters, the plastic moment required at the haunch was found to be 71·8 tons-feet. This agreed reasonably well with the more exact value of 78 tons-feet obtained from the bending-moment diagram. By measuring the value of Rh_2 and $(T-R)h_1$ on the diagram, first R and then T, the tension in the first-floor tie could be found.

Mr M. W. Low (Research student, University of Cambridge) referred to the Series C tests (§§ 72 *et seq*) and the problem of allowing for the effect of frame instability. Why had it been decided to experiment on such a minute scale? The problem of frame instability was an extremely complex one, and when tests were conducted on such a very small scale, additional complications were far more likely to distort the results.

109. For example, 16-gauge plate was much less reliable than thicker plate, not only because of increased scatter in the full plastic-moment values but also because it was more subject to strain hardening. The Series C material tests gave M_p-values covering a range of 18·5% of the mean value, compared with a range of only 6½% for the ⅛-in. material used in Series A and B. Even in a very long series of tests on material cut from different parts of large ¼-in. plates, he had obtained figures ranging from 8½% to only 11%. When the phenomenon under consideration reduced the collapse load by anything from a mere 4% to a figure of 43%, then such variations became very important, and should be minimized by avoiding very thin plates.

110. Again, the welding of those miniature frames, as might be seen from Fig. 16, had introduced a completely unknown factor into the problem. The welds and weld-affected zones were large compared with the dimensions of the structure. Therefore, since any correction for finite weld-size must be in the nature of an inspired guess, these models provided an unsatisfactory test for checking a design theory. Would it not have been better to adopt a rigid clamping system for all joints? Such a clamp would produce similar effects to a weld, but at least its properties would be known.

111. However, accepting those results for the present, Mr Low turned to the question of modifying the theoretical collapse loads for the effect of frame instability. In view of Professor Merchant's remarks, he pointed out that he had accepted the values of critical load as calculated by the Author. It seemed to him to be the easiest way for the designer to calculate critical loads. Professor Merchant's formula, equation (21), was an extremely simple one for design purposes, but it might be questioned as to

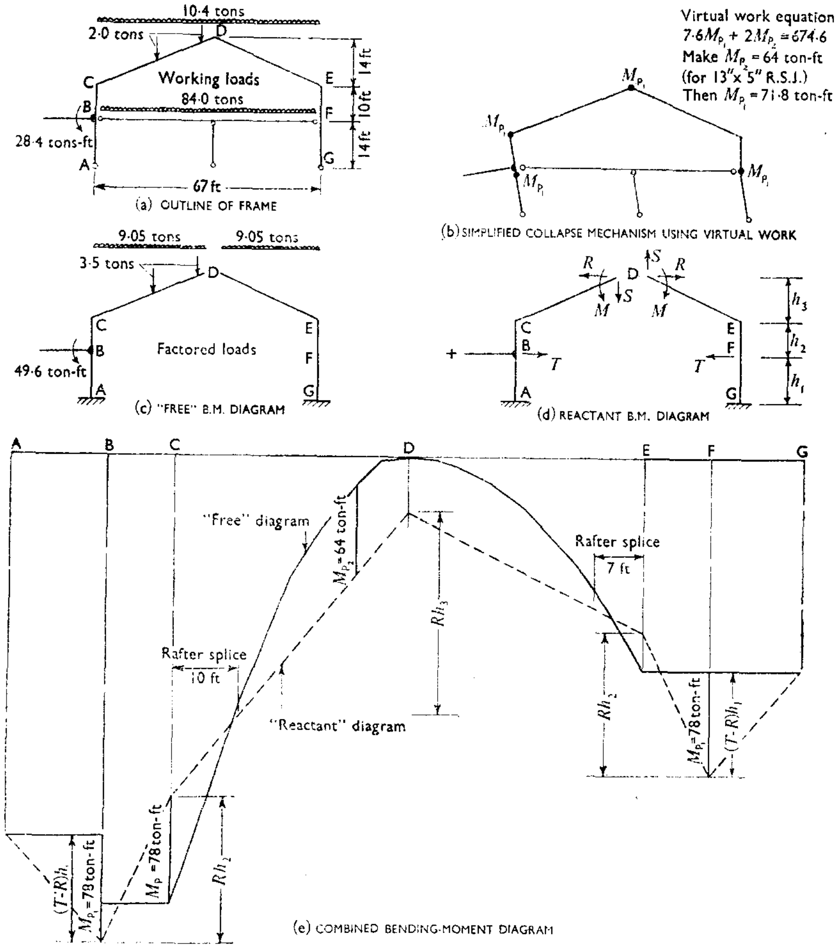


FIG. 22.—STAGES IN THE DESIGN OF THE BUILDING SHOWN IN CROSS-SECTION IN FIG. 21

whether it was an altogether rational one. He suggested an alternative approach to the problem, which, although it was hardly more acceptable on the grounds of pure theory, at least seemed to be a little more rational.

112. The effect of instability in the cases under consideration was a sudden spreading of one of the eaves dimensions—associated, of course, with internal stanchion buckling —causing premature collapse of the structure. What he proposed was that they consider a horizontal disturbing force acting at the top of the internal stanchion (see Table 7) causing it to deflect and the eaves to spread in one bay, inducing collapse.

113. In order to derive a value for that horizontal force, it seemed suitable to postulate an initial eccentricity of the Perry–Robertson type:

$$\epsilon = C.l.\frac{r_y}{a_y}$$

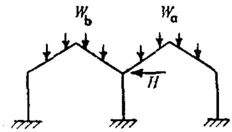


TABLE 7

$$H = \frac{0{\cdot}06 W_p}{1 - W_p/W_c}$$

Frame	Loading	W_L: lb.	W_p: lb.	W_p': lb.	Observed: lb.	Error: %	Error from Table 6: %
C4	$W_b = W_a$	113·6	102·6	104·9	112	− 6	−12
C6	$W_b = \frac{2}{3} W_a$	94·7	82·6	84·4	84	+ 0·5	0
C8	$W_b = \frac{1}{3} W_a$	64·4	54·6	51·0	52	− 2	+ 6
C10	$W_b = 0$	36·4	32·7	33·5	34	− 1·5	+ 3
C5	$W_b = W_a$	113·6	70·7	72·3	92	−21	−21
C7	$W_b = \frac{2}{3} W_a$	72·0	50·0	50·1	41	+22	+27
C9	$W_b = \frac{1}{3} W_a$	38·5	31·3	32·1	26·5	+21	+24
C11	$W_b = 0$	21·2	19·0	19·5	16	+22	+22

where l denoted stanchion length, r_Y the radius of gyration, and a_Y the extreme fibre distance. Since, for a rectangular section, $\left(\frac{r_Y}{a_Y}\right)$ was a constant, it might be said that the initial eccentricity was proportional to the stanchion height. An appropriate magnification factor would seem to be $(1 - W/W_c)$ where W denoted the total vertical load on the structure and W_c the critical load.

Thus:
$$\epsilon_1 = \frac{C_1 h_1}{1 - W/W_c}$$

114. Then, equating stanchion moments due to axial force P and horizontal force H:

$$P\epsilon_1 = Hh_1$$

$$\therefore H = \frac{C_1 P}{1 - W/W_c}$$

It would be sufficiently accurate to take $P = W/2$ at collapse.

Then:
$$H = \frac{C_2 W}{1 - W/W_c} \qquad \text{where } C_2 = \tfrac{1}{2} C_1$$

115. For the purposes of computation Mr Low had taken $C_1 = 0{\cdot}12$.

Thus:
$$H = \frac{0{\cdot}06 W_p}{1 - W_p/W_c}$$

where W_p denoted the predicted collapse load of the frame. He had not used W_f for that quantity in order to avoid confusion with Professor Merchant's equation.

116. He had recalculated Table 6 in the Paper to give the results shown in Table 7. It would be seen that his figures for W_L were lower, with the exception of frame C8, than the values given by the Author. That was because he had made no correction for weld size at that stage. The next column showed the reduced collapse load W_p associated with the force H, while the column marked W_p' attempted to allow for the welds and was found by increasing the W_p-values in the ratio $\frac{W_L(\text{Author})}{W_L\ (\text{Low})}$. The percentage error between the W_p' values and the observed test results was shown, together with the Author's error table. It would be noted that better agreement was obtained than by the use of Merchant's formula.

117. It was unfortunate, from the design point of view, that the formula did not give a direct approach, but it should be of some theoretical interest.

118. He had already mentioned the value of W_L for frame C8. That was inconsistent with the other results, being higher than the Author's value, and he considered that the Author might re-check that calculation. If, as he suspected, the Author's figure for W_L should be approximately 65·8 lb., his own percentage error for the frame would become +7·5% instead of −2%, while the Author's value would become approximately +15%.

119. The bad correlation between the test results on one hand and both the W_f and W_p' values on the other for the hinged stanchion cases could be due to a number of factors of the kind that he had mentioned earlier. In addition, the theoretical mode for C_{11} involved uniform plasticity of one complete rafter, and the consequent large geometry changes would certainly alter the situation. An extensive test programme was required before any general conclusions could be drawn.

Mr T. M. Charlton (Lecturer in Engineering, University of Cambridge) referred to a test which had been carried out recently by the Department of Engineering of the University of Cambridge on a full-scale pitched-roof portal structure.

121. The structure was shown in Fig. 23 and was similar in form to some of the small-scale frames tested by the Author. The rafters and centre stanchion were of 3-in. × 3-in. R.S.J. and the outer stanchions were made rigid by bracing. Loads were applied at the centres of the rafters of one span as shown, until collapse occurred.

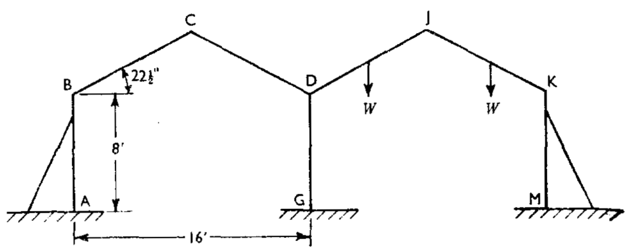


FIG. 23

122. The structure collapsed in the mode shown in Fig. 24 at a total load on each frame of 7·30 tons. The theoretical value obtained by the simple plastic theory ignoring the stability aspect was 6·96 tons. The actual collapse load was, therefore, about 5% greater than predicted but, owing to the effect of strain hardening, an increase of about 10% on the theoretical value was likely. It was probable that the apparent smallness of the strain-hardening effect in this instance was due to instability effects, which, on the basis of the empirical approach suggested by the Author, would cause a reduction in the collapse load of approximately 4%.

123. It was interesting that for such a slender structure, the effect of strain hardening was apparently sufficient to mask the stability aspect so that the behaviour of the structure was in accordance with the simple plastic theory. It should be noted that the centre stanchions were effectively encastré at their feet.

Dr E. H. Brown (Lecturer, Department of Civil Engineering, Imperial College of Science and Technology) welcomed the Author's attempt to assess the modification of the collapse load caused by the deflexions occurring before collapse. It seemed that in some cases this could be a serious effect.

125. The Author had calculated the deflexions at incipient collapse, using small-deflexion theory and a symmetrical collapse mode for symmetrical vertical loading.

He had then made a first approximation to the effect on the collapse load by including a term $-W\delta/2$ in the stanchion equilibrium equation (16). Dr Brown had been interested to take the approximation a step further, and modify the equilibrium equation for the rafter also, taking account of the reduction in the moment arm of the forces H and the sideways movement of the centre of gravity load $W/2$ (see Fig. 25). Equation (15) now became:

$$Hh_2 - \frac{HL\delta}{2h_2} = \frac{WL}{8} + \frac{W\delta}{4} - 2M_p$$

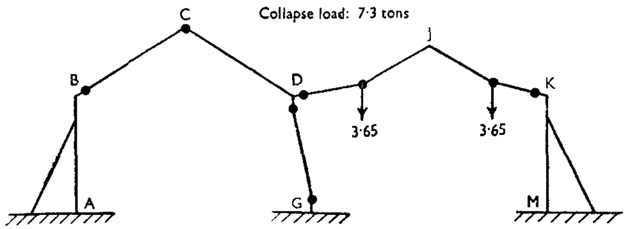


Fig. 24

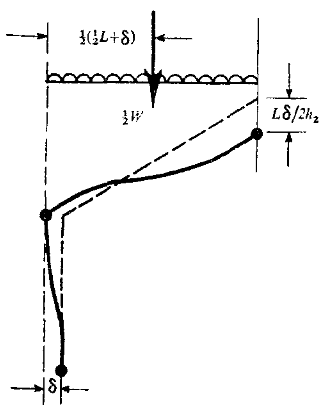


Fig. 25

126. With this modification the percentage corrections of Table 2 had been recalculated for the four extreme values considered by the Author: $h_1 = 10$ ft, 30 ft, $l = 25$ ft, 100 ft. The new figures for the four corner values in Table 2 were:

5·5 . . .	36·4
2·5 . . .	9·0

Since the correction formula assumed a prior knowledge of M_p (which was taken from Table 1) the high value 36·4 was inaccurate, and a better approximation could be found by multiplying the Table 1 value by 1·364 and substituting again in the correction formula, repeating the process until no further correction arose.

127. This had not been considered worthwhile, because intuition, supported by the photograph of a collapsed frame shown in Fig. 16, insisted that the true deflexions at

incipient collapse would form an *asymmetrical* mode, for which the correction calculations would be a little more complex, and of course completely different. Carrying these calculations out, Dr Brown had arrived at the following figures for the four corners of Table 2:

6·0 . . .	44·0
0·5 . . .	10·6

In this case the figure 44·0 had been found by the iterative process described above, the initial value having been 52·7. The details of the calculations leading to these figures would form part of a short Paper which it was hoped to publish in the near future.

128. Although the effect described was only a part of the second-order behaviour of the structure it was clear from these results that at least for frames of certain proportions this type of effect would have to be taken into account in designing by the plastic method.

TABLE 8.—EAVES DEFLEXIONS FOR FRAMES IN TABLE 3

Height to eaves, h_1: feet	Span l: feet						
	25	30	40	50	60	80	100
10	inch 0·49	inch 0·53	inch 0·56	inch 0·56	inch 0·53	inch 0·45	inch 0·35
12·5	0·58	0·62	0·69	0·71	0·72	0·66	0·58
15	0·64	0·71	0·80	0·86	0·88	0·87	0·81
20	0·75	0·84	0·98	1·08	1·16	1·24	1·21
25	0·83	0·94	1·12	1·26	1·38	1·51	1·58
30	0·91	1·02	1·25	1·41	1·55	1·75	1·88

Dr M. R. Horne (Assistant Director of Research, Department of Engineering, University of Cambridge) referred to the Author's statement in § 41 that deflexions at working loads might be troublesome. It was possible from the data given to calculate the elastic deflexions at working loads for the frames discussed in §§ 36 to 44. Table 8 showed the deflexions at eaves level tabulated in inches for frames at 25-ft centres. Despite the Author's fears, these were not generally excessive, being in practically all cases less than 1/200 of the height (unless a light crane was present, in which case the deflexions would usually be in excess of the crane maker's requirements). The results were shown graphically in Fig. 26, for various spans l. The maximum vertical deflexion in relation to the span occurred for a tall narrow frame, of span 25 ft and height to eaves 30 ft, the deflexion then being 1/330 of the span.

130. In general, it was not found that deflexions governed the design for frame spacings of 25 ft, but they were more likely to control the design if smaller frame spacings were adopted. The probable deflexions for any other frame spacing L could be derived by multiplying the given deflexion by $\left(\frac{25}{L}\right)^{0\cdot431}$, which gave the following factors for frame spacings varying between 10 and 40 ft.

L	10	12·5	15	20	25	30	35	40
$\left(\frac{25}{L}\right)^{0\cdot431}$.	1·49	1·35	1·25	1·10	1·00	0·92	0·86	0·82

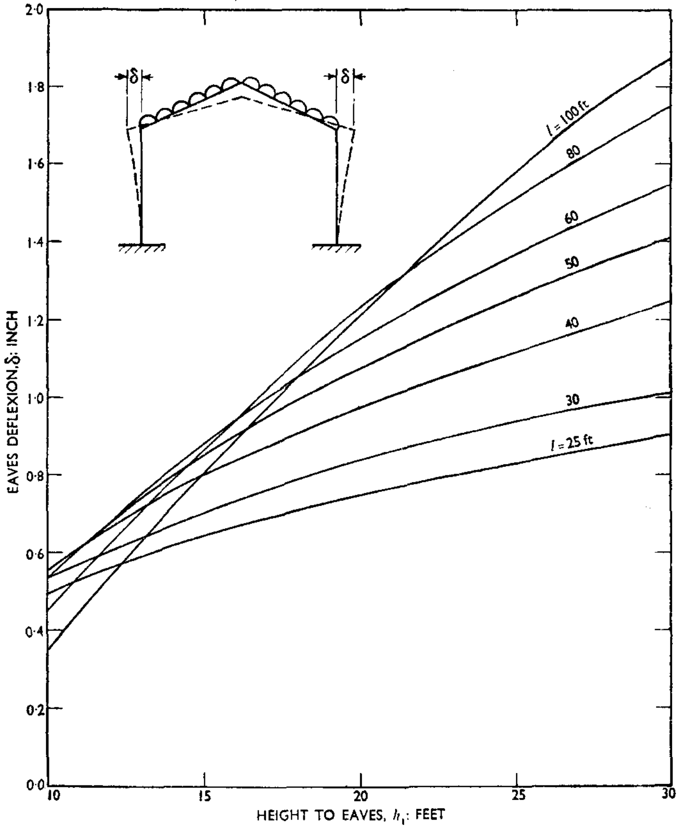


FIG. 26.—EAVES DEFLEXIONS FOR FRAMES IN TABLE 3

131. In § 42, the Author had stated that "for frames at centres other than 25 ft, Table 3 may be scaled linearly to give the value of M_p and hence the section required". That was not strictly true, since the percentage correction for deflexions varied with the frame spacing. The true correction was obtained by multiplying the appropriate value obtained from Table 2 by the factors already quoted for obtaining the elastic deflexions. The correct procedure was thus to scale the M_p-values given in Table 1 in proportion to the frame spacing, and then to apply the true correction obtained from Table 2 as described. It was also necessary to multiply by 1·03 to allow for the effect of a uniformly distributed load on the hinge position near the apex. Thus a frame with $l = 80$ ft, $h_1 = 15$ ft, with frames spaced at 12·5-ft centres, would give a corrected M_p of:

$$\left[893 \times \frac{12 \cdot 5}{25}\right]\left[\frac{100 + 4 \cdot 8 \times 1 \cdot 35}{100}\right]\left[1 \cdot 03\right] = 491 \text{ tons-in.}$$

(compared with 482 from Table 3 by direct proportion).

132. That was a fairly unfavourable case, and the error obtained by simple proportion from Table 3 would not usually be noticeable. However, it might have been

better to have ensured values on the safe side by preparing Table 3 for a small frame spacing such as 12·5 ft instead of 25 ft.

The following contributions were received in writing.

Mr B. J. Vickery (Research Student, University of Sydney, New South Wales) observed that the analysis presented in § 35 omitted the effect of rafter displacement, and as a result led to a low estimate of the reduction in collapse load due to deformation. Rafter displacement produced reductions in collapse load of the same order of magnitude as those caused by stanchion displacement. This might be demonstrated by the following analysis, in which the notation employed was identical with that in the Paper but with the addition of the symbol δ' to denote vertical displacement of apex.

134. From Fig. 27b, equation (15) became:

$$H(h_2-\delta') = \frac{W}{2}\left(\frac{l}{2}+\delta\right)-\frac{W}{2}\left(\frac{l}{4}+\frac{\delta}{2}\right)-2M_P$$

i.e.

$$H(h_2-\delta') = \frac{Wl}{8}+\frac{W\delta}{4}-2M_P \quad \ldots \ldots \ldots \ldots \quad (26)$$

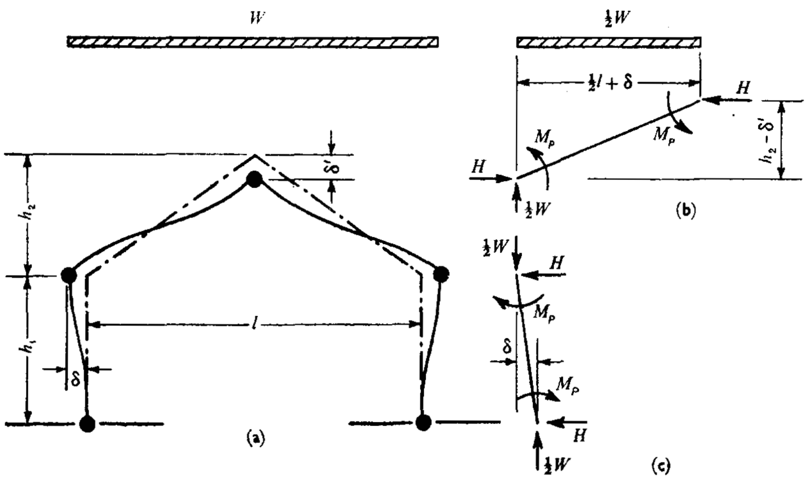


FIG. 27.—FRAME AT COLLAPSE

and equation (16) was unchanged, namely:

$$Hh_1 = 2M_P-\frac{W}{2}\delta \quad \ldots \ldots \ldots \ldots \quad (27)$$

Eliminating H:

$$M_P\left(k+1-\frac{\delta'}{h_1}\right) = \frac{Wl}{16}\left\{1+\frac{4\delta(k+\frac{1}{2})}{l}\right\} \quad \ldots \ldots \ldots \ldots \quad (28)$$

neglecting terms of the second order.
But from the geometry:

$$\frac{\delta'}{\delta} = \frac{l}{2h_2} \quad \ldots \ldots \ldots \ldots \quad (29)$$

Hence:
$$M_P = \frac{\frac{Wl}{16}\left\{1+\frac{4\delta(k+\frac{1}{2})}{l}\right\}}{(k+1)\left(1-\frac{\delta l}{2(k+1)h_1h_2}\right)} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Thus, if $\frac{4\delta(k+\frac{1}{2})}{l}$ and $\frac{\delta . l}{2(k+1)h_1h_2}$ were small compared with unity, the value of the collapse load was reduced by $r\%$, where:

$$r = 100\delta\left\{\frac{l}{2(k+1)h_1h_2}+\frac{4(k+\frac{1}{2})}{l}\right\} \quad . \quad . \quad . \quad . \quad . \quad (31)$$

which should be compared with the Author's result:

$$r = \frac{400k\delta}{l} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

On substituting equation (14) into equation (31), and putting $\frac{l}{h_1}=K$, the expression for r became:

$$r = \frac{100M_Ph_1}{6EI}k(2+k)\sqrt{\frac{K^2}{4}+k^2}\left\{\frac{K}{2k(k+1)}+\frac{4(k+\frac{1}{2})}{K}\right\} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

or:
$$r = \alpha\cdot\frac{M_Ph_1}{EI} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where α was a function of frame geometry, the values of which were shown in Fig. 28 for a range of fixed-base pitched-roof portal frames subjected to vertical loading only.

135. For pinned-base frames an equation similar in form to (34) applied, but α was modified to α' where:

$$\alpha' = \frac{100}{6}k\left(\frac{k}{2}+2\right)\sqrt{\frac{K^2}{4}+k^2}\left\{\frac{K}{2k(k+2)}+\frac{4(k+\frac{1}{2})}{K}\right\} \quad . \quad . \quad . \quad . \quad (35)$$

136. In Table 9 new values for the range of fixed-base frames given in § 37 had been prepared from equation (34) and corresponded to those in Table 2. It would be noted that the values quoted here were considerably higher than those given by the Author.

TABLE 9.—PERCENTAGE CORRECTION TO ALLOW FOR DEFLEXIONS

Height to eaves h_1: feet	Span l: feet						
	25	30	40	50	60	80	100
10	5·8	6·9	9·8	13·5	16·0	32	51
12·5	4·6	5·2	8·1	10·1	13·5	17·5	32
15	4·3	4·8	6·7	8·4	10·1	15·4	20·2
20	3·4	4·0	5·1	6·4	7·7	10·8	14·9
25	2·8	3·8	4·4	5·3	6·3	8·7	10·9
30	2·8	2·9	3·9	4·7	5·4	7·6	9·1

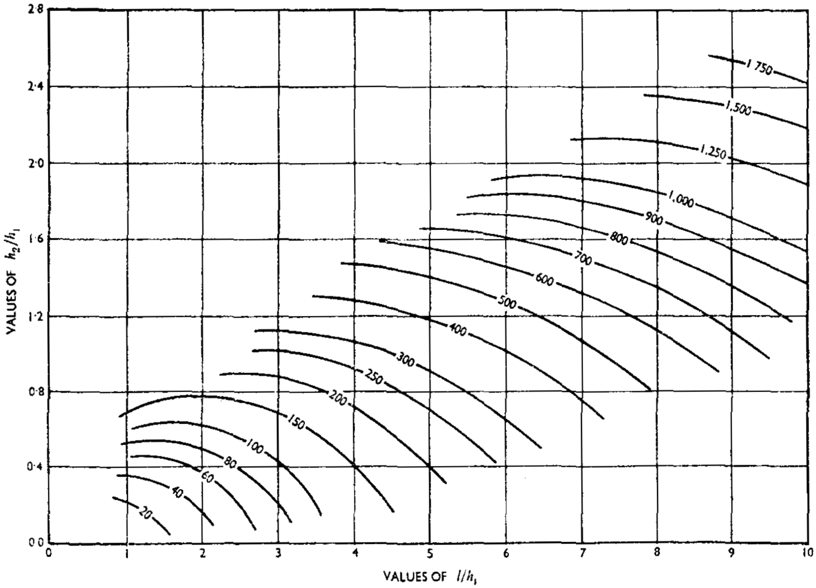


FIG. 28

The analysis used in the derivation of Table 9 neglected a number of second order effects, for instance the influence of deformation moments on deflexions, which became quite appreciable for reductions in collapse load of say 25% or more, corresponding approximately to a value of α of 500. Where the approximate method indicated corrections of this order of magnitude it would be necessary to adopt a more refined approach to the problem.

137. With reference to the Author's experimental results, Table 4, it was noted that the correction applied to the simple collapse load W_0 to take account of the welded joints was in general of a much greater magnitude than the deflexion correction. The Author had made no mention of the method employed in the determination of the joint correction, which would need to be determined with considerable accuracy to avoid disguising the reduction in collapse loads due to deformation. During recent months at the University of Sydney, Mr Vickery, under the direction of Professor J. W. Roderick, had carried out a number of tests on model pinned-base pitched-roof portal frames. The overall dimensions of these portal frames were considerably greater than those of the Author's models and as a result the joint correction had been reduced to about 1 or 2% of the collapse load. Control for the frame tests had been provided by loading a series of beam and joint specimens. The results obtained from this test programme provided confirmation of the analysis presented in this discussion and in addition indicated that appreciably greater reductions in collapse load would occur when frames were subject to both vertical and side load.

Dr Arthur Bolton *(Department of Engineering, University of Liverpool) commented* on the Author's calculation of critical loads and some details of the experimental results. He thought that the calculation of elastic critical loads was not a tedious operation if modern methods[9, 10] were used, and would usually be easier than an ordinary elastic analysis. He had demonstrated[11] two examples when pointing out that the

Author's expressions (§ 56) underestimated the critical load by ignoring the axial load in the rafters. (See discussion by Dr Brotton and Mr Bahauddin in §§ 149–170.)

139. With regard to all the elastic stability calculations in the Paper, an important piece of information, the value of *EI*, had been omitted. The absence of that value made it virtually impossible to check the critical loads. In comparing experimental and theoretical values of critical load for model structures it had been Dr Bolton's experience that the *EI*-values for the members must be determined experimentally. That was because the *I*-value for thin members was extremely sensitive to slight variations in thickness and warping of the cross-section. He would expect the *EI*-value to have a greater range than, for instance, the value of the fully plastic moment. For the Series C experiments, the Author had found the latter to vary over a range of 90 to 108% of the mean value. Would he give the appropriate figures for *EI*?

140. From the nominal dimensions of the strip a value of *EI* of 153 lb/sq. in. was to be expected but that did not correspond with the Author's expressions and the W_C figure given in Table 6 for frames C5, C7, C9, C11. It seemed unlikely that an *EI*-value as low as 102 lb/sq. in. had been obtained for the strip but that would be required to obtain $W_C = 193$ lb. Dr Bolton wondered if that was to be explained by an error in completing Table 6. The parameter used to describe the fully plastic load W_L was the total load applied to the frame. Hence the parameter used to describe W_C should also be the total load applied to the frame. The formula in § 56, however, used W_C to refer to the load on the centre stanchion only, which was only a half of that required. If by mischance only half the proper value for W_C had been used in Table 6 then a great deal of the discrepancy between calculated and observed failure loads was immediately removed. The value of *EI* required in the Author's formula was now 203 lb/sq. in. and that would be obtained if the nominal 16-gauge strip was 0·006 in. too thick (or slightly distorted).

141. Using that value of 203 lb/sq. in. for *EI* and making allowance for the axial load in the rafters gave the value of W_C as 123 lb. Table 10 showed the results of recalculating Table 6 using this value.

TABLE 10

Frame	W_L: lb.	W_C: lb.	W_F: lb.	Observed: lb.	Error: %
C5	116	123	60	92	−35
C7	72	123	45	41	+ 9·7
C9	39·5	123	30	26·5	+12·8
C11	22	123	18·5	16	+15·6

142. Table 11 shows the results of correcting Table 6 to the value of critical load 93 lb. which would occur if *EI* had its nominal value.

143. With regard to other uncertainties, the calculated failure loads might, perhaps, be regarded as satisfactory, except for frame C5. Here the error was of a different order from that of all other frames tested.

TABLE 11

Frame	W_L: lb.	W_C: lb.	W_F: lb.	Observed: lb.	Error: %
C5	116	93	51·5	92	−44
C7	72	93	40·5	41	− 1·2
C9	39·5	93	27·7	26·5	+ 3·8
C11	22	93	17·8	16	+11·2

144. It had often been found that an apparently insignificant "propping force" was able to inhibit a failure in the lowest critical mode and allow deformations in a higher mode to develop. In the present experiment it would be seen that in the lowest critical mode, which was a sway mode, vertical displacements would require considerable movement of string over all the pulleys shown in Fig. 17. Although the strings showed equal tensions when at rest, any relative displacement of the load points would call into play frictional forces which overloaded points tending to buckle upwards and relieved the load on points tending to buckle downwards. This might have been sufficient to cause the next higher critical load to be the important one. In Dr Bolton's own tests on model portals he had found this effect even with ball-bearing pulleys when using light loading chains.

145. If it was assumed that that possibly happened, the next higher mode for the frame under consideration would be that in which the centre valley joint rotated without displacement. This critical load could easily be calculated; it occurred when $P/P_e = 1{\cdot}29$ for the rafters, i.e. $W_C = 440$ lb. If this value were to be used in the calculation of W_F it was found that

$$W_F = \frac{116 \times 440}{116 + 440} = 90 \text{ lb.}$$

which was within 3% of the value observed by the Author.

146. A final comment on the generalized Rankine formula might be appropriate. As propounded by Merchant, an important requirement was that instability could magnify the characteristic deflexion of the plastic collapse load. This should be borne in mind when considering local collapse loads in which there was no overall collapse of the structure. The sway critical mode was thus unlikely to be appropriate with "no sway" plastic collapse modes.

147. It remained to review the application to practical design of the experimental evidence contained in the Paper. There was a clear warning which ought to be taken to heart by every steelwork designer. Pin joints drastically reduced the strength of ductile mild-steel structures and should be avoided if at all possible. They survived only as a hang-over from difficult elastic analysis and irrational allowable values in design codes like B.S.449. At the best they were a poor design expedient. The natural outcome of rational plastic design was surely that every connexion must be as rigid as possible under the circumstances so as to take advantage of all redundancies.

148. The experiments carried out by the Author were of the type which could lead to more efficient full-size structures and were a way of buying experience at bargain prices. Many full-scale tests to failure were, of course, required but much more information would be obtained if they were supplemented by a carefully chosen series of model tests.

Dr D. M. Brotton and Mr Bahauddin (Manchester College of Science and Technology) gave results from an investigation of the critical loads on portal frames to supplement the information given by the Author. The results would be considered in two parts:—

(*a*) Single-bay frames.

(*b*) Multi-bay frames with rigid external stanchions.

Single-bay frames of constant cross-section

150. The critical mode of all practical single-bay pitched-roof portal frames which were unrestrained was the "sway mode" and the critical condition[12] was given by:

$$m_1 s_1 (l + c_1) - 2s_1 = 2s_2''\lambda \text{ for frames with fixed feet}$$

and

$$s_1''\left(\frac{1}{n_1} - 1\right) = s_2''\lambda \quad \text{for frames with pinned feet}$$

where s, c, m, s'', n are stability functions as defined by Livesley and Chandler[13] and $\lambda = \frac{k^2}{k_1}$ = ratio of stiffnesses of rafter and stanchion. Suffix 2 referred to the rafter and suffix 1 to the stanchion.

151. The exact solution of these equations depended on an iterative process, since the axial forces in the members and the bending moments were interdependent.

152. If the above calculation was based on the non-stability axial loads (i.e. those obtained without using the iterative process), it gave an error which varied from approximately 15% for low values of k_2/k_1 to zero for $k_2/k_1 > 1 \cdot 0$ for portals with fixed feet. There was no significant difference for portals with pinned feet.

153. The Author had shown in Fig. 13b the load disposed equally on the stanchions, thereby neglecting the reduction in stiffness due to the axial load in the rafters. Critical-load calculations based on this assumption applied to portal frames with fixed feet and $\lambda = 0 \cdot 25$ gave a value of critical load which was approximately three times the actual value. The corresponding discrepancy when $\lambda = 4$ was negligible.

154. Experiments and calculations had been carried out to determine:—

(*a*) The effect of application of the load between the joints.
(*b*) Variation in angle of the rafter.
(*c*) Variation in span/height ratio.
(*d*) Effect of lateral loads.

155. *Application of the load between the panel points.*—Experiments showed no measurable difference between the elastic critical loads of a model when the load was applied at the joints, including the apex, and when the load was applied between the joints, the latter simulating purlin loads.

156. *Variation of the angle of the rafter.*—Fig. 29 showed the variation in the sway critical load of fixed-feet portal frames with different slope of rafter for different values of k_2/k_1. The axial load in rafters contained two components, one which increased and the other which decreased when the slope of the rafter was increased; an increase in the slope of the rafter therefore resulted in a greater critical load for frames with low values of k_2/k_1 and a smaller critical load for frames with high values of k_2/k_1.

157. *Variation of the span-to-height ratio.*—For all frames with the same slope of rafter subjected to the same value of the load parameter, the component of the axial load in the rafter due to the vertical reaction at the foot would be the same. The component due to the horizontal reaction, however, increased when the k_2/k_1-ratio decreased, the critical load therefore decreased with the k_2/k_1-ratio.

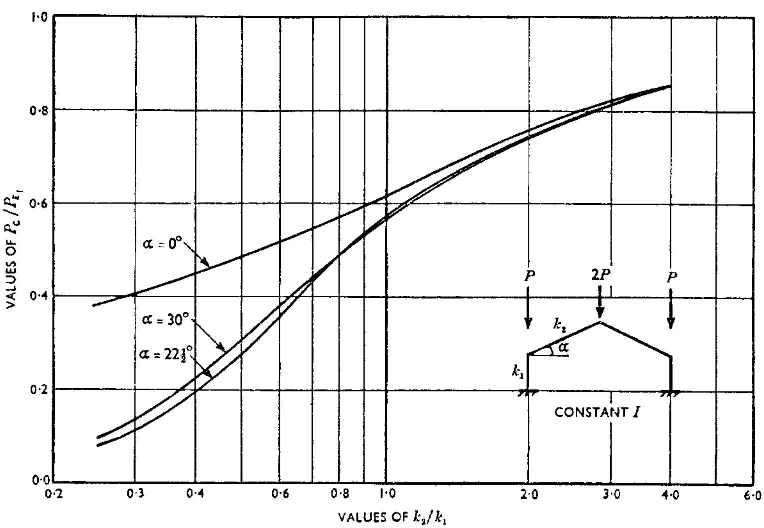


FIG. 29.—SWAY BUCKLING MODE—FIXED FEET

158. For comparison, the critical loads of rectangular portal frames were shown in Fig. 29.

159. *Effect of lateral loads.*—The exact calculation of elastic critical loads for frames subjected to horizontal as well as vertical loads was very tedious. Experiments showed, however, that for practical ratios of horizontal to vertical loads the effect of the horizontal loads could be ignored in the calculations since the axial loads in some members were increased while the loads in other members were decreased.

160. *Frames with pinned feet.*—Fig. 30 showed the variation in sway critical load with the k_2/k_1-ratio for pinned-feet portal frames having a rafter slope of 22½°. The curve was of similar shape to that for fixed-feet frames. The ratio of critical loads for frames with pinned and fixed feet varied from ½ for $\lambda = 0{\cdot}25$ to ¼ for $\lambda = 1{\cdot}0$.

161. *Comparison of plastic collapse loads and elastic critical loads.*—Table 12 listed the plastic collapse loads W_L and the elastic critical loads W_C for single-bay frames of different spans and heights. The Table had been based on sections obtained from the

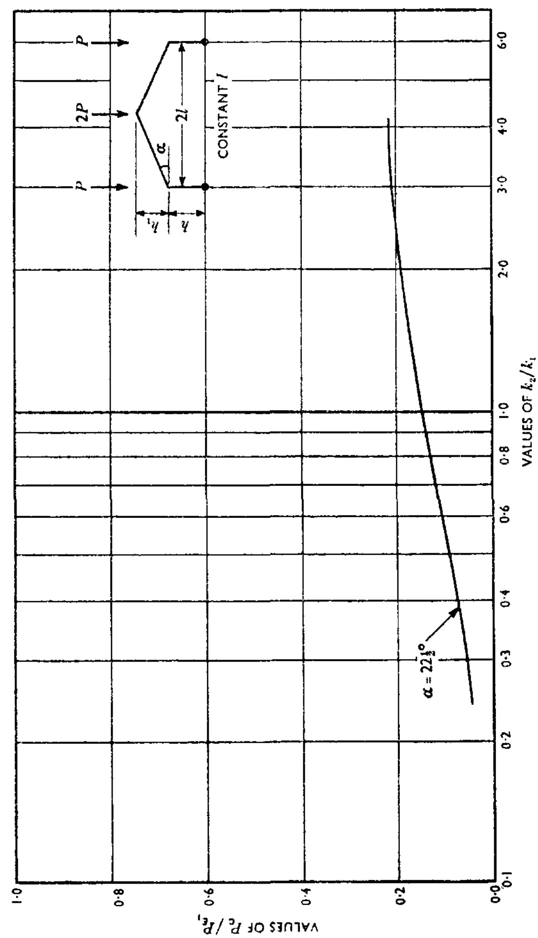


FIG. 30.—SWAY BUCKLING MODE—HINGED BASE

M_P values given in the Author's Table 3. The magnitude of the deflexions had not been considered in choosing the sections and as the Author had stated, deflexions might be excessive in some cases.

162. It would appear from the graph in Fig. 29 that, since the P_C/P_{E1}-value increased as the k_2/k_1-ratio increased, the critical load would also increase, but it must be remembered that for frames with high values of the k_2/k_1-ratio the sections required would be more slender and would consequently have smaller values of Euler load in the stanchion and smaller values of the critical load.

163. It would be seen from Tables 12 and 13 that the significance of instability as indicated by the value of the ratio W_L/W_C was generally least for frames with large spans and small height to eaves and greatest for frames with small spans and large height to eaves.

164. It was noteworthy that in some cases the value of W_L was $0{\cdot}20W_C$; the corresponding value of W_L for frames with pinned feet was $0{\cdot}75W_C$ (Table 13).

165. The deterioration of critical loads with the formation of plastic hinges[14] had not been evaluated for this type of portal but evidence from other structures showed a significant reduction in failure load for ratios of original W_L to W_C as low as 0·15. The apparent load factor of 1·75, based on pure plastic collapse, would not therefore be attained.

Multi-bay frames with rigid external and pinned internal stanchions

166. As with single-bay frames, the Author had ignored the axial loads in the rafters. Here, however, the effect was much more serious, since the critical condition arose when the overturning effect caused by a displacement at the top of an internal column was equal to the restoring force (see Fig. 31). The restoring force might be

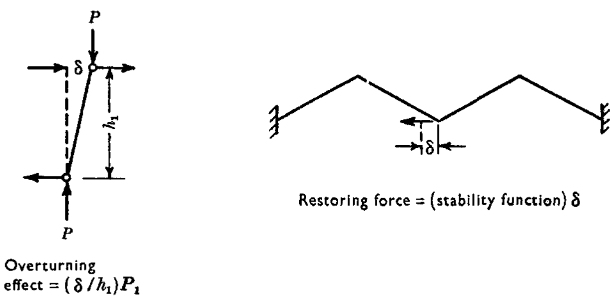


FIG. 31

termed the "spring stiffness" of the rafters. Neglect of the axial loads in the rafters was equivalent to the assumption of constant spring stiffness, whereas, when allowance was made for stability effect in the rafters, the spring stiffness reduced quite steeply as the load was increased and in fact became zero when P/P_E for the rafter$=0{\cdot}51$ (Fig. 32).

167. Graphs of the disturbing effect for a series of values of span/height ratio were also shown in Fig. 32. They were straight lines and it could be seen that the critical loads of a frame would be given by the intersection of the particular disturbing-effect line and the spring-stiffness curve. It was apparent that the true critical load might be only 1/5 or 1/6 of the critical load based on constant spring stiffness. The sway mode gave the lowest critical load for frames with span/height ratio larger than 1·6, but for frames with span/height ratio less than 1·6 the critical mode would be buckling of the stanchion and the critical load was given by the intersection of the curve of column buckling with the corresponding straight line.

TABLE 12.—FRAMES WITH FIXED BASE

h_1: feet	Span 25 ft					Span 30 ft				
	I-section: in. × in. × lb.	M_P	W_L	W_C	$\frac{W_L}{W_C}$	I-section: in. × in. × lb.	M_P	W_L	W_C	$\frac{W_L}{W_C}$
10·0	6×3×12	124	9·6	168	0·057	7×4×16	197	13·6	260	0·058
12·5	7×4×16	197	14·35	244	0·059	7×4×16	197	12·55	212	0·059
15·0	7×4×16	197	13·68	188	0·073	7×4×16	197	11·95	170	0·07
20·0	7×4×16	197	12·8	119·6	0·107	8×4×18	244	13·7	156	0·088
25·0	7×4×16	197	12·3	82	0·15	8×4×18	244	13·1	109	0·12
30·0	7×4×16	197	12·0	60	0·2	8×4×18	244	12·6	80·6	0·156

h_1: feet	Span 40 ft					Span 50 ft				
	I-section: in. × in. × lb.	M_P	W_L	W_C	$\frac{W_L}{W_C}$	I-section: in. × in. × lb.	M_P	W_L	W_C	$\frac{W_L}{W_C}$
10·0	9×4×21	316	18·2	348	0·053	10×4½×25	428	21·6	358	0·06
12·5	9×4×21	316	16·7	310	0·054	10×4½×25	428	19·65	334	0·059
15·0	9×4×21	316	15·6	274	0·57	10×5×30	512	21·9	370	0·059
20·0	10×4½×25	428	19·4	298	0·085	10×5×30	512	19·8	294	0·063
25·0	10×4½×25	428	18·25	213	0·09	12×5×30	603	21·8	320	0·068
30·0	10×4½×25	428	17·6	160	0·11	12×5×30	603	20·8	246	0·085

h_1: feet	Span 60 ft					Span 80 ft				
	I-section: in. × in. × lb.	M_P	W_L	W_C	$\frac{W_L}{W_C}$	I-section: in. × in. × lb.	M_P	W_L	W_C	$\frac{W_L}{W_C}$
10·0	10×5×30	512	23·5	300	0·079	12×6×44	925	36·3	368	0·10
12·5	12×5×30	603	25·0	412	0·06	12×6×44	925	32·6	364	0·09
15·0	12×5×32	645	24·7	420	0·059	15×5×42	1,000	32·4	500	0·065
20·0	13×5×35	765	26·4	468	0·057	12×6×54	1,100	31·8	404	0·079
25·0	13×5×35	765	24·4	380	0·064	14×6×57	1,335	35·1	514	0·069
30·0	12×6×44	925	27·8	342	0·082	14×6×57	1,335	32·8	448	0·073

h_1: feet	Span 100 ft				
	I-section: in. × in. × lb.	M_P	W_L	W_C	$\frac{W_L}{W_C}$
10·0	15×6×45	1,150	39·8	482	0·083
12·5	14×6×57	1,335	42·0	394	0·107
15·0	16×6×50	1,353	39·2	450	0·087
20·0	16×6×62	1,590	40·0	532	0·075
25·0	14×8×70	1,765	40·6	484	0·084
30·0	16×8×75	2,120	45·2	616	0·074

TABLE 13.—FRAMES WITH PINNED BASE

h_1: feet	Span 25 ft			Span 30 ft			Span 40 ft			Span 50 ft		
	W_L	W_C	$\frac{W_L}{W_C}$	W_L	W_C	$\frac{W_L}{W_C}$	W_L	W_C	$\frac{W_L}{W_C}$	W_L	W_C	$\frac{W_L}{W_C}$
10·0	7·6	44·8	0·17	11·0	74	0·149	14·1	123	0·12	16·1	148	0·109
12·5	12·3	63	0·195	10·5	76	0·138	13·4	92	0·15	15·2	118	0·129
15·0	11·9	27	0·254	10·2	44	0·232	12·8	74	0·173	17·5	114	0·154
20·0	11·0	30	0·367	12·1	40	0·303	16·6	76	0·218	16·4	78	0·21
25·0	11·2	21	0·54	11·8	27	0·44	16·0	52	0·308	18·6	82	0·227
30·0	11·0	15	0·75	11·5	20	0·575	15·7	40	0·392	18·0	62	0·29

h_1: feet	Span 60 ft			Span 80 ft			Span 100 ft		
	W_L	W_C	$\frac{W_L}{W_C}$	W_L	W_C	$\frac{W_L}{W_C}$	W_L	W_C	$\frac{W_L}{W_C}$
10·0	17·0	144	0·108	25·0	226	0·11	26·4	262	0·101
12·5	18·8	163	0·114	23·3	180	0·13	28·9	242	0·12
15·0	19·0	148	0·129	23·9	208	0·12	27·8	244	0·114
20·0	21·4	133	0·161	24·6	142	0·174	29·8	220	0·136
25·0	20·4	101	0·202	28·2	152	0·186	31·4	172	0·183
30·0	23·7	88	0·27	27·0	122	0·222	36·0	189	0·19

168. *Multi-bay frames with rigid external and internal stanchions.*—A stability analysis of these frames showed a similar difference in the elastic critical load.

169. *Multi-bay frames with eaves tie.*—As the Author had stated, the provision of an eaves tie would reduce the value of M_P required to less than half. This would result in frames with more slender members and the sway critical might become significant.

170. Dr Brotton and Mr Bahauddin remarked that their contribution was limited to a general consideration of the elastic critical loads of multi-bay frames, since discussion of the experiments and the results shown in Table 6 had already been given by Dr Bolton in §§ 138-148.

The Author, in reply, thanked Dr Horne for having presented the Paper in his absence in the United States.

172. The Chairman (§ 80) had commented on "the economies latent in the plastic method". The Author had intended this phrase to cover all aspects of steelwork construction, including "design, fabrication, and erection". It was perfectly true, as the Chairman had said, that material saving was a function of the load factor chosen for design, but the Author did not agree that such saving "must depend entirely on the values of the load factor and the factor of safety". The working stresses in bending specified in B.S.449 implied a load factor of 1·75, and experience had shown that portal frames designed to collapse at this load factor would show some material saving compared with frames to carry the same working loads designed elastically. In this connexion, the Author thanked Mr Godfrey for the details of the new design of a building for I.C.I. Limited (§ 92 *et seq.*), which showed a material saving of 17% and a cost saving of 10%. While welding costs were still high, there were signs such as these that the price of welded erected steelwork was approaching that of riveted steelwork.

173. Perhaps more important than savings in material and cost, however, were the design economies made possible by the use of plastic theory. The simple plastic

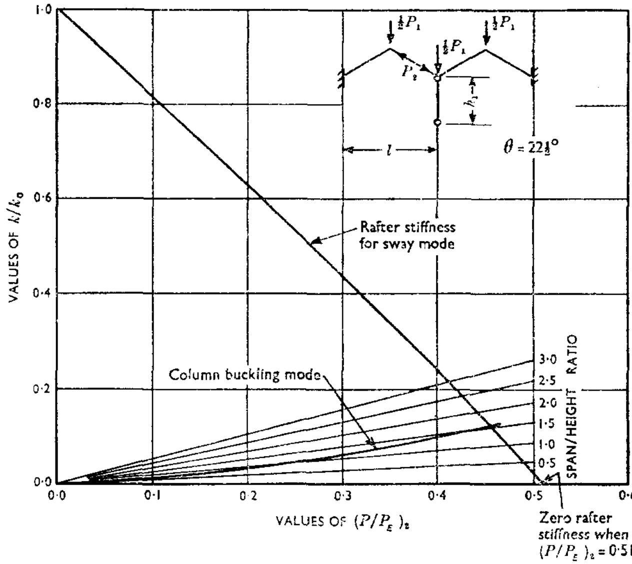


FIG. 32

theory was essentially *simple*; it was also rational. Once the theory had been mastered, design time was saved in the office and several trials could be made at the design cost of a single elastic analysis. A dramatic illustration of this was furnished by the new raft in the Cromwell Road for London Transport Executive. The site was such that a grillage of virtually only one form could be used. This grillage had so many redundancies that it was in effect almost unanalysable by elastic methods. However, the plastic design showed that the structure was the simplest possible, namely, sets of intersecting continuous beams.

174. Turning then to some of the design aspects mentioned by the contributors, the Author was interested to hear of Mr Godfrey's experiences with pinned and fixed-ended stanchions (§ 96). In general, the Author believed that fixed-ended stanchions were in most cases the best to use for all types of portal frame, providing that suitable foundations could be provided at reasonable cost. For multi-bay frames, they seemed almost essential, particularly for internal columns, although the theory predicted that these columns should be only lightly loaded. Strong internal columns prevented the onset of frame instability, and the test result quoted by Mr Charlton (§ 121 *et seq.*) was remarkable for such a slender section.

175. The Author agreed with Mr Bryan (§ 102) that combining mechanisms gave great understanding of the behaviour of frames at collapse, and for this purpose, the replacement of distributed load by point loads increased the speed of calculation. Both the mechanism method, due to Professors Neal and Symonds, and the moment-distribution method, due to Dr Horne, were invaluable tools in the designer's tool box (to use a phrase of Mr Godfrey's). Both methods, however, required judgement on the part of the designer, whereas the statical approach, while one of trial and error, enabled a direct solution to be obtained. The Author was working on an automatic method of plastic design, in which no judgement was required at any stage, and he hoped that this method would be published shortly.

176. Working deflexions had been mentioned by many contributors, and Dr Horne was to be thanked particularly for his curves (Fig. 26). The Author agreed with

Dr Horne that the recommendation on scaling from Table 3 (§ 42) was slightly unsafe, and Dr Horne's proposal gave a more rational approach.

177. It would have been appreciated that the deflexions at collapse calculated by the Author were at best approximate; Mr Vickery's more refined calculations, summarized in Table 9, showed that greater corrections should be applied for deflexions. Dr Brown (§ 124) had arrived at similar modified corrections by a slightly different process. These deflexions had been calculated, presumably, again on the assumption that the apex hinge was the last to form; the Author would not reply unequivocally to Mr Johnson's question (§ 91), but he believed that this hinge was usually the last to be formed. Qualitatively, that was because there was a long region near the apex at which the bending moment was high, leading to large curvatures and to increase of moment at the other critical sections.

178. The series of model tests had been designed to obtain orders of magnitude of frame instability effects for single-storey frames. The Author agreed with Mr Low (§ 108) that tests on larger frames were very desirable; one had indeed already been made, as reported by Mr Charlton. The results of the model tests appeared alarming for those frames with pinned internal columns, and a more careful and comprehensive series of tests should be made for that type of frame. However, the results on frames with strong internal stanchions were not alarming, either theoretically or experimentally, and this type of structure should be used whenever possible, as recommended by Professor Bolton (§ 147). The Author thanked Professor Bolton for his analyses, and for his detection of a possible source of experimental error.

179. Dr Brotton and Mr Bahauddin (§ 149 *et seq.*) had presented valuable results on elastic frame stability, and both they and Professor Merchant (§ 82) had commented on the effect of axial load in the rafters. Again, the neglect of that effect had far more serious consequences for pinned rather than fixed-base frames.

References

9. A. Bolton, "The critical load of portal frames when sidesway is permitted" and references. Structural Engineer, vol. 33 (1955), p. 229.
10. R. E. Bowles and W. Merchant, "Critical loads of tall building frames" and references. Structural Engineer, vol. 34 (1956), p. 324.
11. J. Heyman, "Plastic design of single-storey frames". Brit. Welding J., vol. 3 (1956), p. 332. See discussion by A. Bolton, vol. 4 (1951), p. 3.
12. K. M. Bahauddin, "The prediction of critical loads of symmetrical, single bay pitched roof portals". M.Sc. Thesis.
13. R. K. Livesley and D. B. Chandler, "Stability functions for structural frameworks". Manchester Univ. Press, 1956.
14. W. Merchant, "Frame instability in the plastic range". Brit. Welding J., vol. 3 (1956), p. 366.