DISCUSSION

The influence of the parameter ϕ/ρ_{eff} on crack widths

A. W. Beeby

Structural Concrete, 2004, **5**, No. 2, 71–83.

Professor Beeby's 2004 paper questioned the basis of the widely cited 'classical' model for estimation of crack widths in reinforced concrete elements. It provides a valuable reminder of the need to verify the predictions of theoretical models against experimental data, a task which he has performed to the benefit of our understanding on several occasions in the past. Professor Beeby will therefore not be surprised to learn that others have subjected his own ideas to similar scrutiny. In the several contributions below, members of *fib* Task Group 4.5 'Bond Models' raise questions over the validity of conclusions in the paper, based on a wider range of experimental data and more recent developments in theoretical modelling. (John Cairns, Convenor, *fib* Task Group 4.5 'Bond Models'.)

Discussion

Casper Ålander, *Ruukki Metals*, *Finland*

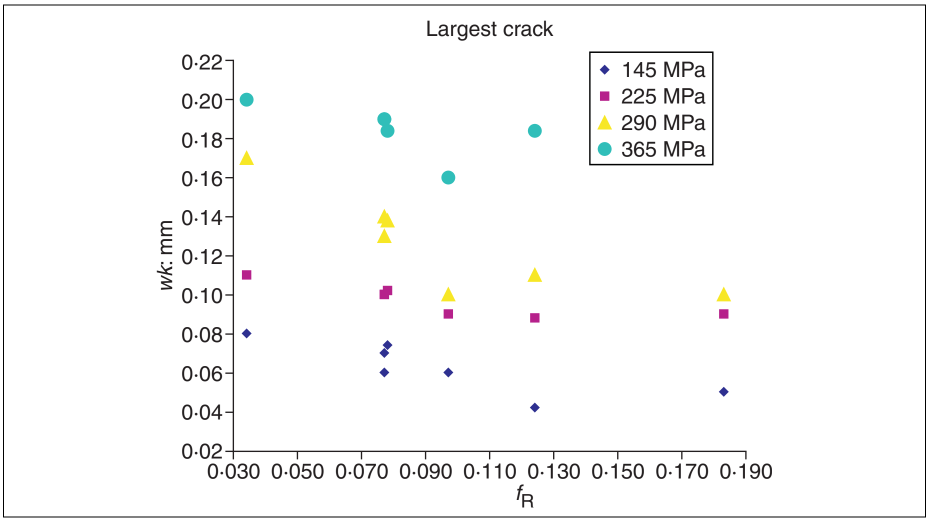
The paper by Professor Beeby gives some reflections concerning the models for calculating crack widths and their correlation with test results. On the basis of test results Beeby questions whether there is a correlation at all between the width of the crack and the ratio ϕ/ρ_{eff} and whether bond characteristics of reinforcement have any influence on crack width.

Figure 15 shows results from tests with shallow beams reinforced with different types of 12 mm steel. All samples were made of the same batch of concrete delivered in a single ready-mix batch. Each point represents the average of the largest crack in tests of five similar test pieces. The two highest stress levels represent the serviceability limit state (SLS) of an optimised ordinary structure.

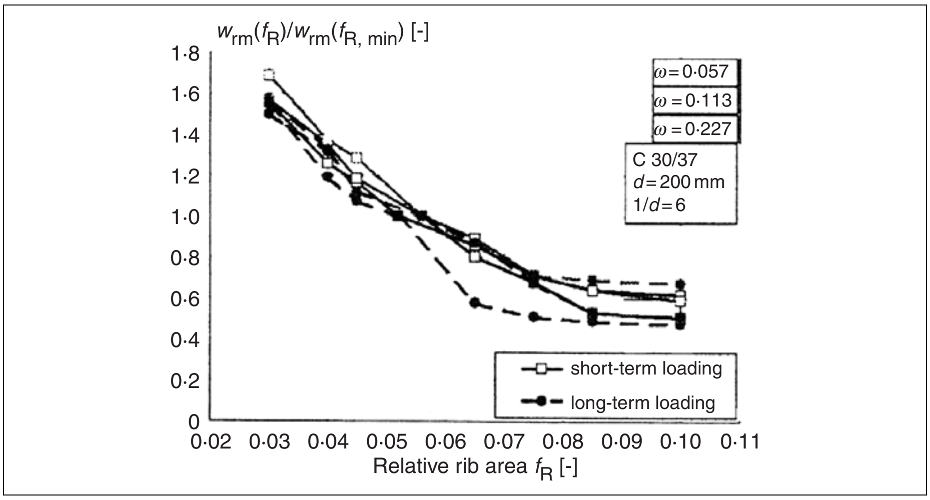
As can be seen the crack widths are significantly reduced when the relative rib area increases at least up to $f_R = 0{\cdot}1$. Figure 15 also indicates that some geometrical configurations could be more effective than others. Otherwise the steel with $f_R = 0{\cdot}097$ should not have given smaller cracks than the steel with $f_R = 0{\cdot}124$.

Also, according to Mayer and Eligehausen,[20] there is a clear relation between relative rib area and crack width, both for long term and short term loading (Figure 16)

Both figures indicate a possible reduction of the crack width of about 30% when increasing the relative rib area from the level 0.056 somewhere close to 0·1, clearly demonstrating that bond characteristics of reinforcement influence cracking.



△ **Figure 15 The mean value of the largest measured crack of five similar tests for different steels at four levels of stress[19]**



△ **Figure 16 Relative crack width $w_{rm}(f_R)/w_{rm}(f_{R,min})$ against relative rib area under short and long term loading for different mechanical reinforcement ratios ω[20]**

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Most design codes, including Eurocode 2,[2] recognise that the calculation model for crack width has to take the bond properties of the steel into account, typically by a bond factor that is not the same bond factor as the one used in ultimate limit state design. For the time being, unfortunately, there is no standard method or even commonly accepted non-standard method for defining this bond factor. The European standard for reinforcing steel (EN10080)[21] is open for different surface geometries of the steel.

Beeby observes no difference in crack width between specimens with a small number of large diameter bars and specimens with the same reinforcement area composed of smaller diameter bars. However, for example if one 16 mm bar is replaced by four 8 mm bars the circumference of the reinforcement is doubled $((4 \times \pi \times 8)/(\pi \times 16) = 2)$. Therefore a certain amount of force can be transferred to the concrete over half the length under the condition that the bond per surface unit is the same. It must be appreciated, however, that the bond per surface unit might not be the same for those two bar sizes; for example the relative rib area typically is smaller for an 8 mm bar than for a 16 mm bar. Unfortunately many crack width tests have been carried out with little or no attention at all to the actual surface geometry of the steel in the test pieces.

John Cairns, *Heriot-Watt University*, *UK*

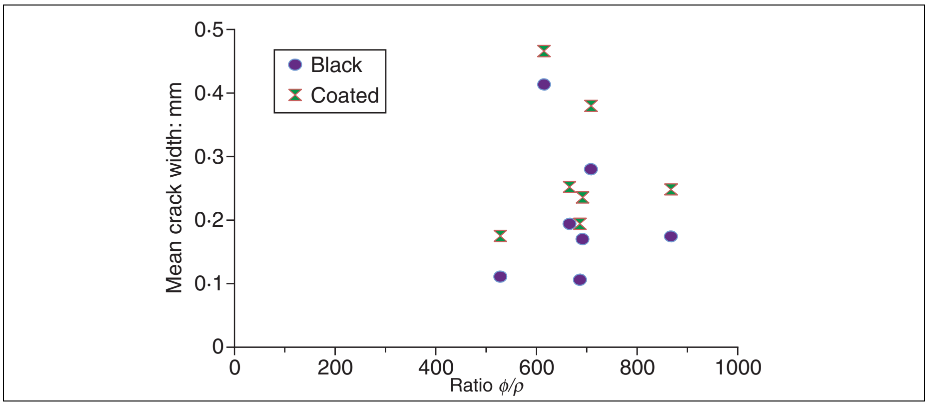
Professor Beeby is to be commended for questioning assumptions that, perhaps by dint of repetition, have become accepted as fact. His paper re-interprets test data to arrive at the conclusions that (a) cover is a better parameter for the prediction of crack width than the ratio ϕ/ρ_{eff}, and that (b) bond conditions of reinforcement exert a nil or small influence on crack widths.

Figures 17 and 18 plot results obtained from an investigation of the performance of fusion-bonded epoxy-coated reinforcement (FBECR) at the serviceability limit state, here re-analysed to test Professor Beeby's conclusions.[22] A total of 14 beams, comprising seven pairs of specimens, were tested in the study. Each pair of specimens was nominally identical, except that one was reinforced with non-coated bars while the other was reinforced with FBECR. Both sets of bars were taken from the same batch of steel. Cracks were measured in the constant moment zone, the length of which was sufficient for between 12 and 20 transverse cracks to form. Cracks were measured at a bar stress of 275 N/mm^2, calculated on the basis of cracked section behaviour. The results shown represent the mean width of cracks observed on the tension face directly over the reinforcing bars; similar trends are observed if maximum crack widths and cracking on side faces is analysed.

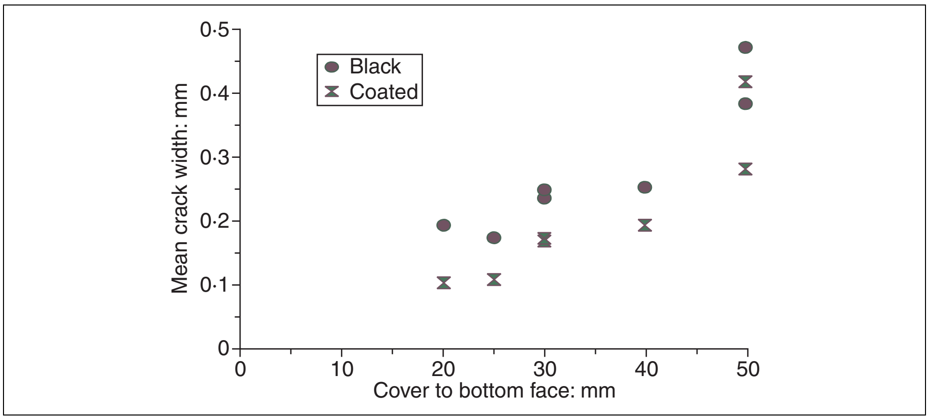
The results tend to support Professor Beeby's first conclusion that cover is a better predictor for crack widths than ϕ/ρ_{eff}. It is also apparent, however, that averaged over the seven pairs of specimens, crack widths were 43% greater in specimens reinforced with FBECR; the largest difference recorded was 80%. Crack spacing and beam deflection were also affected. A total of seven other studies into FBECR all serve to confirm that the smooth coating on the FBECR provides less restraint to opening of transverse cracks than does a black mill scale finish and results in significantly increased crack spacings and crack widths under service loadings.[23]

These results do, however, endorse Professor Beeby's view that ultimate bond strength is not relevant to crack control, although not in the way inferred in the paper. Ultimate bond strength of epoxy-coated bars as measured in a RILEM pullout test is little different from that of black bars.[23] Crack widths were observed to be up to 80% greater with FBECR. The effect on crack widths is therefore greater than might be expected from the respective values for τ_{max}.

To understand the reason for this, reference should be made to Figure 13, which shows crack width at the surface of the bar to be typically of the order of 0·02–0·04 mm. At the corresponding level of slip, the bond



△ **Figure 17 Influence of ratio ϕ/ρ on crack width: re-analysed results[22]**



△ **Figure 18 Influence of cover on crack width: re-analysed results[22]**

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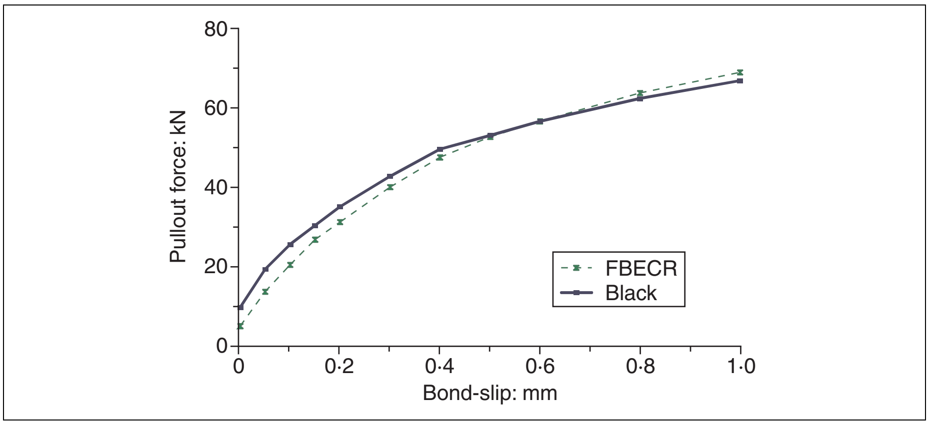

△ **Figure 19 Bond-slip curves for FBECR and ordinary black ribbed bars[23]**

stress ratio of coated to black bars is around 0·6, (see Figure 19), more consistent with the observed difference in cracking.

In summary, research into FBECR provides compelling evidence that bond influences crack widths. Professor Beeby's paper thus points to a need for better understanding of the role of bond in structural performance in addition to test protocols for bond that focus on structural performance.

Rolf Eligehausen, Utz Mayer and Steffen Lettow, *University of Stuttgart*, *Germany*

In the paper by Professor Beeby it is concluded that ϕ/ρ_{eff} has only minimal influence on crack widths and that the effect of the bond conditions of the reinforcement is far less than would be predicted by the classical theories of cracking. Furthermore it is suggested that the consequence of the findings in this paper is that the formula $w = k\,(\phi/\rho)\varepsilon$ to predict crack widths is practically useless and should be abandoned.

We do not agree with these statements and hence we analysed a research project carried out at the Institute of Construction Materials at the University of Stuttgart in the same way as in the above-mentioned article. The test results used in this paper are part of the experimental investigations on 54 large reinforced members (columns) loaded in tension (see Eligehausen, Mayer, Lettow).[24] In the research project several parameters have been varied, in particular the number of bars and the diameter of the reinforcement, while the concrete properties were kept substantially constant. The details of the specimens are given in Table 5. The value used to compare the cracking of the various specimens represents the average value of the crack width measured at all sides on the surface of the specimen. The crack widths were recorded during the whole loading process. The values of the strain at service load and yield load have been calculated from the Young's modulus of the used reinforcement and the respective steel stress value. The

Table 5 Tension specimens tested by Eligehausen, Mayer and Lettow[24]

Specimen notation	Breadth: mm	Width: mm	Bar diameter: mm	Number of bars	Cover: mm	w/ε_{ss}: mm	w/ε_{sy}: mm	w/ε_{su}: mm	ϕ/ρ_{eff}: mm
S02D25V1	400	400	25	2	30	391	714	198	4051
S02D25V2	400	400	25	2	30	486	699	100	4051
S04D12V1	400	400	12	4	38	204	222	125	4234
S04D12V2	300	300	12	4	30	139	227	92	2377
S04D12V3	300	300	12	4	30	143	228	109	2377
S04D25V1	400	400	25	4	30	279	511	159	2013
S04D25V2	400	400	25	4	30	285	459	167	2013
S06D12V1	300	300	12	6	30	222	276	39	1580
S06D12V2	300	300	12	6	30	215	301	57	1580
S06D16V1	400	400	16	6	30	251	310	109	2107
S06D16V2	400	400	16	6	30	204	211	92	2107
S08D06V1	200	200	6	8	30	157	157	17	1056
S08D06V2	200	200	6	8	30	102	150	19	1056
S08D12V1	300	300	12	8	30	146	161	94	1182
S08D12V2	300	300	12	8	30	198	262	97	1182
S08D16V1	400	400	16	8	35	147	173	85	1576
S12D06V1	200	200	6	12	30	142	143	22	702
S12D06V2	200	200	6	12	30	122	152	17	702
S12D12V1	300	300	12	12	30	150	191	86	784
S12D12V2	300	300	12	12	30	185	220	28	784
S16D06V1	200	200	6	16	30	94	105	49	525
S16D06V2	200	200	6	16	30	99	122	77	525

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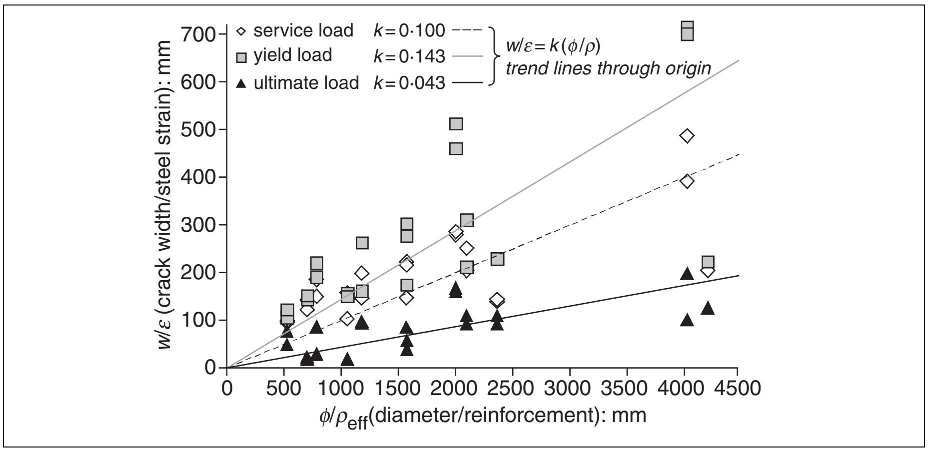

△ **Figure 20 Values of w/ε (crack width/steel strain) against ϕ/ρ_{eff} (diameter/reinforcement ratio) (from Eligehausen, Mayer and Lettow[24])**

strain value at ultimate load represents the value measured in tensile tests on the naked reinforcing bars.

In Figure 20 the values of w/ε against the parameter ϕ/ρ for three different load levels and the respective trend lines through the origin (with a modification factor k) are shown. In the figure it is indicated that, independent of the load stages (service, yield and ultimate load), there is a very strong influence of the parameter ϕ/ρ on the ratio between crack width and steel strain. Furthermore it can be seen that the assumption of the classical theories, which predict a straight line through the origin, agrees sufficiently well (within a common scatter band) with the measured results. From the results at the different load stages it becomes clear that the bond behaviour is significantly influenced by the state of stress in the reinforcement (yielding) (a more detailed description of this phenomenon is presented in Eligehausen, Mayer, Lettow[24]) and the ratio ϕ/ρ.

The consequence for further research is that the development of appropriate models to predict cracking and tension-stiffening phenomena should definitely be continued, but there is no need to neglect the classical theories, especially in practical cases.

Daniele Ferretti and Ivo Iori, *University of Parma*, *Italy*

In this intriguing paper the author questions a classical aspect of reinforced concrete, that is, the importance of bond on cracking and crack width. The problem of bond has been debated for a long time with mixed conclusions. For instance, we remember the provocative title 'Does bond slip exist?' of a paper by Gerstle and Ingraffea,[25] where the complexity of the phenomena involved in cracking is discussed. Because of this complexity, many numerical models based on the bond-slip relationship have been proposed to compute crack width and spacing. Compared to these approaches, the 'classical theory of cracking', referred to by Beeby in equation (1), is actually a very simplified formula. Hence, theoretical conclusions based on this formula suffer from some drawbacks.

Firstly, according to the 'classical theory', the first crack spacing in a tensile member is

$$S_0 = \frac{f_{\text{ct}}}{4\tau_{\text{av}}}\frac{\phi}{\rho} \qquad (6)$$

where $\tau_{\text{av}} = \beta\tau_{\text{max}}$ is the average bond stress, taken as a fraction of bond strength τ_{max}. The problem is how to define β, which depends on a variety of factors, like mechanical properties, geometry of the member and bond-slip law.

To obtain S_0 we can solve the classical second-order differential equation modelling slip, s, in a tensile member. Since the first cracks are thin, we can adopt the initial branch of the bond-slip law proposed in CEB-FIP[1], $\tau = \tau_{\text{max}}\,(s/s_1)^{0\cdot 4}$.

The analytical solution of the problem (for example, Russo and Romano[26]) is

$$S_0 = \frac{1}{6}\left(\frac{5^2 7^5}{2}\frac{E_{\text{s}}^2 s_1^2 f_{\text{ct}}^3}{(1+n\rho)^2\tau_{\text{max}}^5}\frac{\phi^5}{\rho^3}\right)^{1/7} \qquad (7)$$

The spacing S_0 does not depend linearly on ϕ/ρ but on $\phi^{2/7}(\phi/\rho))^{3/7}$. Compared to the 'classical theory', the distinct effects of ϕ and ρ are less important but still present.

Secondly, the tensile members analysed in Beeby's paper have a thick concrete cover. In this case, the contributions of diffusion and bond described by equation (5) are comparable. Since any variation in ϕ/ρ affects only bond contribution, in the end the bond role is overshadowed by diffusion.

Thirdly, as is well known, the tests on tensile members frequently exhibit a sizeable scatter in concrete tensile strength. For this reason, first cracks rarely form at the same load and distance S_0. Even in ideal specimens, the initial crack distance is uncertain, and comprised between S_0 and $2S_0$. The uncertainties in tensile strength and crack spacing lead to a large scatter in crack-width values to the detriment of a clear understanding of bond contribution.

To conclude, the critical analysis accomplished in Beeby's paper is certainly worthy of consideration since the ratio ϕ/ρ in the simplified formula of the 'classical theory' has a limited significance. However, the numerical solutions provided by rigorous models based on bond show that the parameters ϕ and ρ play a not negligible role (for example, Fantilli *et al.*).[27]

Pietro Gambarova and Patrick Bamonte, *Milan University of Technology*, *Italy*

This paper is an intriguing study on some fundamental aspects of bond in reinforced concrete structures, starting from the classical theory of crack formation in a member subjected to normal stresses. However, because of its highly-provocative conclusions on a fundamental problem like bond, the paper lays itself open to criticism, even more since the test results quoted in the paper are not easily found, the references are not always complete and most of the well-documented closely-

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related results on tension stiffening are ignored.

In the following, only two points will be discussed, the first concerning the most relevant assumption adopted in the paper and the second some recent studies on tension stiffening.

(*a*) The ratio between the average crack width and the maximum steel strain (that is, the strain in the cracked section) is introduced as the optimal parameter to investigate the correlation between the crack width and other relevant variables, like concrete cover and diameter/steel ratio. However, proportionality between $(w/\varepsilon_{\max})$ and (ϕ/ρ) cannot be investigated by considering different tests with different values of the factor (k), which is hardly constant, because it depends on a variety of factors (such as f_{ct} or cover). Furthermore, the scanty dependence of the ratio $(w/\varepsilon_{\max})$ on (ϕ/ρ)—and specifically on ρ—has been already observed by a number of scholars. For instance, Giuriani[28,29] shows that the crack width tends to become independent of ρ when bond is very good (that is, when the local bond stress-slip law exhibits a high initial chemical adhesion and a steep ascending branch). Even if it is difficult to carry out tests with different bond properties, any investigation on the $(w/\varepsilon_{\max})/(\phi/\rho)$ relationship should in some way take bond parameters into proper consideration.

(*b*) The introduction of $(w/\varepsilon_{\max})$ with reference to the serviceability limit state is highly questionable. In the experimental studies quoted by Professor Beeby the values of $(w/\varepsilon_{\max})$ fall in the range 88–199. It follows that for crack-spacing values in the range 160–180 mm (as in the first study by Jaccoud[6]) and $w = 0{\cdot}3$ mm (w is generally comprised between 0·1 and 0·3 in the serviceability limit state), the bar stress is close to 300–700 MPa, well above the values usually found under the working loads. These numbers cast some doubts on the way $(w/\varepsilon_{\max})$ was evaluated and/or on the validity of this parameter as a reference parameter.

(*c*) Since bond and cracking in reinforced concrete have been studied for many years and have been treated in innumerable papers, more papers concerning tension stiffening and the role of the steel ratio should have been considered by the author. As a matter of fact, many studies—both experimental and theoretical—clearly show the effects that the steel ratio has on crack spacing (and thus on crack width). Among the many available in the literature, some results presented in a very recent symposium on bond in reinforced concrete may be quoted (Ueda *et al.*[30] and Fantilli *et al.*).[31] Cracks are closer (and thinner) in tension members with higher steel ratios.

To conclude, the paper from Professor Beeby was a nice and intriguing paper that arouses some doubts, and needs some answers, on

(*a*) the soundness of the reference parameter chosen to represent crack width-bond interaction,
(*b*) the setting of the problem (serviceability limit state generally means open, stabilised, multiple cracks, which is hardly the scenario considered by the author), and
(*c*) the actual role of the steel ratio.

Ezio Giuriani, *University of Brescia*, *Italy* and Giovanni Plizzari, *University of Bergamo*, *Italy*

The author should be commended for his comprehensive and highly-provocative paper, with a set of conclusions that are in sharp contrast with commonly-accepted assumptions, namely: that the parameter (bar diameter/effective steel ratio) does not have any practical relevance on surface crack width, and that other parameters, such as concrete cover, are of greater relevance. However, the following points need some explanations.

(*a*) The author asserts that, according to Haqqi's thesis,[4] there is a poor correlation between average strain and crack width. However, the classical theory used by the author is based on the assumption that crack width is the integral of the bar deformation; therefore, the comparison between the classical theory and results based on the local strain (in the cracked section) may not be meaningful. This result can hardly be accepted and should be verified with other results.

(*b*) The control of crack opening is important for serviceability limit state (SLS). For this reason, an approximate value of the crack opening can be accepted since the structural safety is not under discussion and the classical approach is significantly simplified so that its results should be accepted within this approximation. However, theoretical and experimental results on beams show that
   (i) the determination of the crack width should refer to maximum values of 0·3 mm and to bar-to-concrete slips (δ_w) smaller than 0·15 mm; within these values, splitting cracks are not present even for small concrete covers (<20 mm);[32]
   (ii) as underlined by the author, crack width depends on the crack distance but, after cracking ($\tau_b = \tau_{b,cr}$) the bond stress increases (it is not constant) and can be approximated as[28]

$$\tau_{bw} = \tau_0 + \tau_{12}\delta_w \tag{8}$$

According to the classical theory, the crack distance is given by

$$S_0 = (f_{ct}\phi)/(4\tau_{bw}\rho_s) \tag{9}$$

and, considering equation (8), it becomes

$$S_0 = (2f_{ct}\phi)/[4\rho_s(\tau_0 + \tau_{12}\delta_w/2)] \tag{10}$$

By neglecting the tension stiffening, the crack opening is given by

$$w = (\sigma_s/E_s)S_0 \tag{11}$$

and yields

$$w/\phi = \left[-1 + \sqrt{1 + \frac{f_{ct}}{\tau_0}\frac{\tau_{12}\phi}{2\tau_0}\frac{1}{\rho_s}\frac{\sigma_s}{E_s}}\right] \Big/ \left(\frac{\tau_{12}\phi}{2\tau_0}\right)$$

where $\delta_w = w/2$.

Therefore, crack opening is dependent on the parameter (ϕ/ρ_s), although not as markedly as in the classical theory since it depends on $(\phi/\rho_s)^{0{\cdot}5}$. Hence we agree that the classical theory has significant limitations

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in that it is based on the assumption of a constant maximum bond stress.

The dependency on the concrete cover presented by the author may be due to the different confinement and its influence on the bond strength.[33]

The classical theory usually refers to an element under tension or to a beam under constant bending moment. An extension to a beam with a variable bending moment, as it occurs at the internal supports of a continuous beam, has been studied.[34,35] Here the crack distance is constant and equal to l_b which is a function of $\tau_{b,cr}$; further cracks do not develop at the SLS but for higher loads.

In the end the paper is a good, interesting and provocative paper but its conclusions need to be supported by several answers concerning: (a) the treatment of the test results to work out and (b) the precise nature of the test results (mainly concerning first cracking).

Stavroula Pantazopoulou and Souzana Tastani, *Demokritus University of Thrace*, *Greece*

This paper is rather timely in that evaluation of cracking is receiving renewed interest both for durability concerns as well as a design consideration in the use of novel materials. Through evaluation of some of the available tests the author explores the performance of the so-called 'classical theory of cracking'. Through this effort he claims to disprove the relevance of some of the central variables in that theory, namely the bond strength and the ratio of bar diameter to effective steel area, whereas according to his results cover seems to be the critical parameter that controls cracking. Because of the significance of this finding every effort must be made to secure objectivity of the data processing; in this regard, the discusser questions the result for the following reasons.

(*a*) There are additional data in the open literature that have not been included in the study. For example, Polak and co-workers embarked on an experimental program to explore the relevance of bar diameter on cracking, published in the 1990s.[36,37] Their tests were flexural (slabs), and for constant reinforcement ratio different bar sizes were considered. The results showed a clear dependence of cracking on the size of reinforcement, which here is disputed. A follow-up paper by the same investigators[38] expanded the so-called 'classical' model to interpret the test results. Although the algorithm of Polak *et al*. applied the 'classical model' more robustly than is routinely done in practice, its basic premise was not disregarded—rather, it formed the core of the effort. If, by any chance, the present paper is to be used as a vehicle for change of design procedures, the author, or even an independent party, should ensure that all relevant tests be processed. Should a number of tests be excluded, then this ought to be done based on clearly-defined criteria.

(*b*) That the conclusion of the data analysis leads to such an unexpected result, might suggest that the application of the analytical model should be revisited to question the basic steps involved. One is the definition of ρ_{eff}: this is meant to represent the area of concrete mobilised in tension (through bond) by a single bar. It is commonly understood that this is defined as a square or circle centred on the bar, having radius equal to the least distance to the nearest free surface (cover) or to the nearest bar. However the Model Code[1] is vague here and leaves great latitude in how each individual actually performs the calculation. To appreciate the results it would be useful to know how this variable was calculated in the present study. Furthermore, in deriving equation (3) mention is made of 'bond failure' and a correlation of test results against τ_{max} is considered. Care should be exercised in this respect: it is not clear why bond failure is at all anticipated; even if a maximum bond stress is required in deriving equation (3), it is not obvious what its relationship would be with nominal bond strength. The magnitude of bond strength depends on the type of bond failure encountered; here only splitting failure is of relevance. In this case, the dependable bond strength is a function of bar diameter to cover thickness and the presence of stirrups. Using a nominal value for bond strength regardless of cover thickness would be likely to lead to irrelevant results. Which value was used in the correlation study? Similarly the aspect ratio of the beams in addition to the end (anchorage) details of the smooth bars considered have great relevance in comparing with the cracking performance of deformed bars.

Author's reply

Andrew Beeby, *University of Leeds*, *UK*

Firstly, it is very gratifying to find that the paper has been read and that a substantial number of people have been motivated to contribute to a discussion. This has become very rare and the lack of debate on research has become a serious weakness of the research system. I therefore thank all the contributors for their time and effort.

The contributions cover a wide range of topics and it is therefore inevitable that, without writing a further long paper, I will not be able to deal in detail with all the points raised. I hope, however, that I will be found to have covered the most significant points.

Before dealing in turn with the contributions, I think that it will help to deal with two general issues. They will be relevant to my reply to a number of the contributions so it seems helpful to deal with them at the start rather than introducing extensive repetition into my replies.

The first issue is to consider just what crack width we may be interested in. This issue may not have been clear to many readers. Chapter 4.3.2 in the *fib* textbook *Structural Concrete*[39] makes clear that the crack width calculated using bond-slip relationships is that at the steel-concrete interface. The resulting formulae thus do not aim to predict the surface crack width. This does not apply to the formulae in the UK or American Concrete Institute codes where the theory on which they are based are concerned strictly with surface conditions.[40,41] Clearly there has been confusion on this point over the years since almost all crack width formulae

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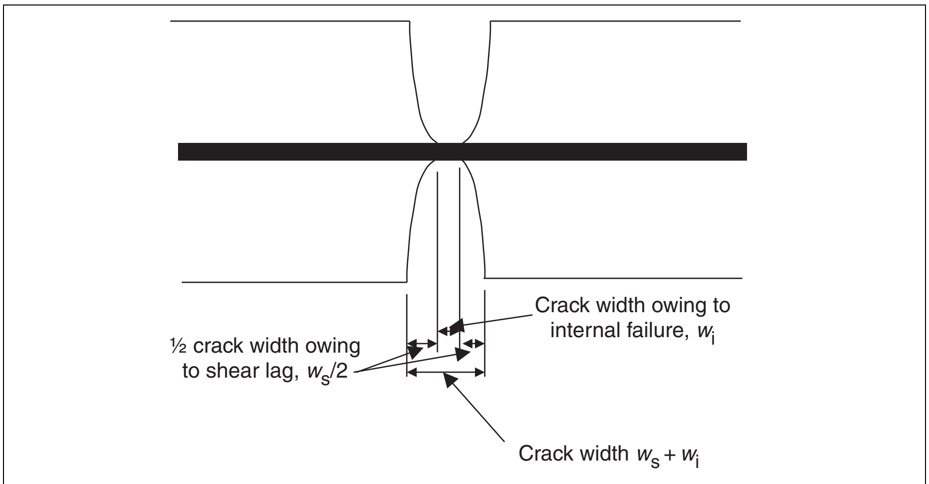
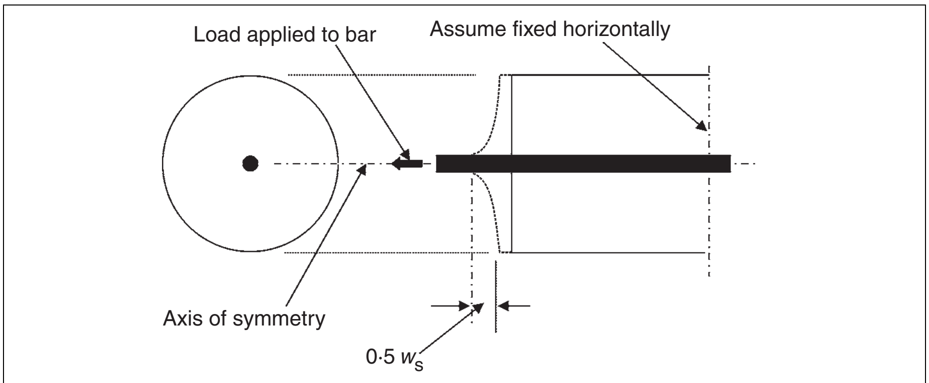
have been tested and validated against surface widths. Surface widths are, after all, the only widths generally visible to measure and I strongly suspect that it is surface widths that code formulae believe they are controlling. It appears to me that most researchers have, without explicit thought, assumed that the surface width will be the same as the width at the bar surface. I tried to demonstrate in the paper that this was not so and that the surface width is actually substantially larger than that close to the bar. It may help to advance discussion to consider this issue further and this will be done below.

The concrete between the bar and the concrete surface in the region of the crack is subjected to a shear stress (which will vary over the length S_0 and the depth of the cover). This shear stress will cause a shear deformation in the cover concrete such that, even if there were no slip, there will be a crack width on the surface. In addition to this crack width owing to shear deformation (in other structural circumstances this is often referred to as 'shear lag') there may be a further crack width owing to either slip or the development of internal cracks of the type originally identified by Goto.[42] To avoid argument about the precise nature of this additional deformation, I shall simply refer to it as 'internal failure'. The result may be illustrated conceptually as shown in Figure 21. The question which is now of interest is the magnitude of the shear lag cracking compared with the internal failure cracking. A rough estimate of this may be obtained by carrying out a simple finite element analysis of a bar embedded axially in a cylindrical concrete specimen. This situation is shown schematically in Figure 22. The analysis carried out was elastic and therefore the crack width will be calculated to be directly proportional to the load applied to the bar. The length of the section analysed was chosen as 150 mm and a bar diameter of 20 mm was chosen. Covers of 50, 40, 30 and 20 mm were used in the analysis. Not surprisingly, the shear lag crack width is found in this analysis to depend on the cover. Farra and Jaccoud[5] tested (*inter alia*) specimens with 20 mm bars and a cover of 40 mm. All cracks were measured using strain gauges along the centreline of the specimens; the crack width was taken as the extension within the strain gauge length within which the crack formed. Maximum crack widths were, unfortunately, not tabulated and they have had to be scaled from graphs. Figure 23 shows the maximum crack widths from three nominally identical specimens plotted against the stress in the reinforcement at a crack. Also drawn on the figure is the shear lag crack width from the finite element analysis. It will be seen that the shear lag crack width accounts for about 2/3 of the experimentally obtained maximum crack widths, suggesting that only about 1/3 of the crack width can be attributed to internal failure. It should be noted that the analysis, being carried out on a circular cross-section while the test was carried out on a square specimen, will have led to some underestimate of the shear lag width. Also, the concrete was assumed to be elastic at all loads. Any plasticity in the concrete in tension will result in a larger shear lag width. Finally, the method of measurement of the cracks will have led to a minor overestimate of the crack width since the crack width will include any tensile strain in the concrete over the gauge length considered. In summary, for the case considered, the width of the surface cracks is dominantly a function of the shear lag and internal failure is a secondary effect. Shear lag would appear to be dominantly a function of the cover.

This consideration of shear lag can be taken further. A series of tension specimens have been reported[43] where the aspect ratio of the cross-section is the main variable. The cross-section is sketched in Figure 24. All crack widths were measured where they crossed lines drawn along the length of the specimens parallel to the reinforcement at points (a) and



△ **Figure 21 Schematic make-up of crack width**



△ **Figure 22 Analysis for shear lag deformation**

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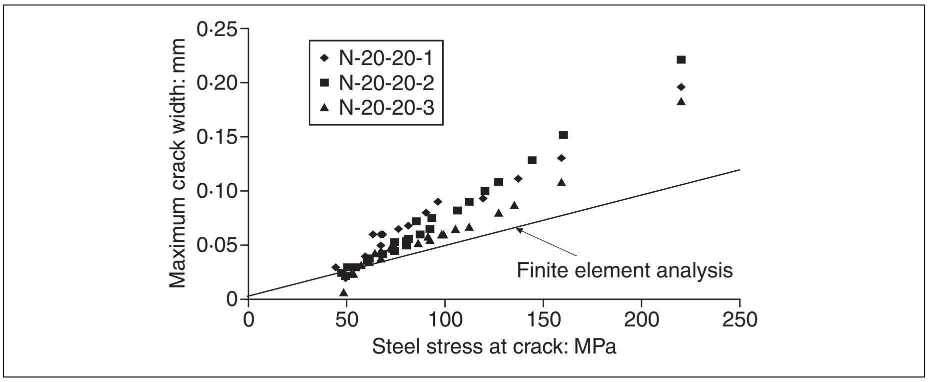
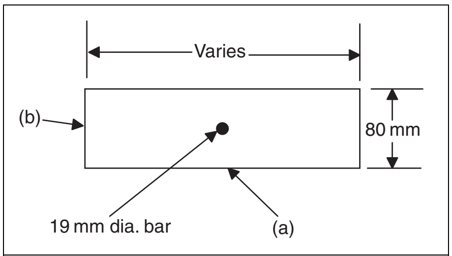


△ **Figure 23 Maximum crack widths of specimens N-20-20 (from Farra and Jaccoud[5])**



△ **Figure 24 Cross-section of specimens of Series Z used in Table 6**

(b) in Figure 24. The specimens were 1.8 m long and six identical specimens were cast at a time. A 19 mm dia. bar was used in all cases. Table 6 gives the average value of the crack width divided by the average strain at points (a) and (b) for the four types of cross-section. In each case, the results from the six nominally identical specimens have been averaged.

The widest cracks will be seen to occur at location (b) and the width increases with increase in the larger dimension of the cross-section. It might reasonably be surmised that the maximum crack width was a function of ϕ/ρ, which clearly also increases with increase in the larger dimension of the cross-section. However, this conclusion is not reasonably sustainable since the crack widths measured on the surface of the cross-section at the point closest to the reinforcement (point (a)) do not vary consistently with ϕ/ρ. Indeed, it would seem reasonable to assume that the crack width remained constant and independent of ϕ/ρ at this point and that any variation was simply owing to general random variation in the crack widths. ϕ/ρ is supposed to reflect the effect of bond phenomena at the bar surface. Logically, this must have at least as large an effect on the crack width at (a) as it does at (b). It would be completely illogical to propose that the effect of slip at (b) was greater than that at (a). It therefore seems that the difference in the crack width at (b) relative to that at (a) was simply owing to the greater distance of the concrete surface from the bar at (b) relative to that at (a) and hence a greater shear lag deformation.

Similar results can be found for other types of test. Table 7 gives results from those slabs from Series P,[44] which have a cover of 25 mm. Various bar sizes, spacings and reinforcement ratios are used. Crack widths were measured (a) on the surface of the slabs directly over the bars and (b) at mid-spacing of the bars. In specimens P4 and P6, more than one spacing and size of bar were used. Each specimen was 1675 mm wide with a 1800 mm constant moment zone. Values of (w/ε_m) directly over the bars have been averaged for all bars in a particular specimen.

Figure 25(a) shows the crack widths at mid-spacing plotted against ϕ/ρ_{eff}. It will be seen that a reasonable relationship is obtained. However, no possible relationship between ϕ/ρ_{eff} and crack width can be proposed for the cracks measured close to the bars where one would expect the effects of slip to be greatest. An alternative approach is to plot the crack widths against the variable a_{cr}, which is the principal variable in the UK code crack prediction formula.[40] a_{cr} is defined as the distance from the point where the crack width is being considered and the surface of the nearest bar. a_{cr} has been calculated for mid-spacing but, for points close to the bars, a_{cr} is equal to the cover of 25 mm. Figure 25(b) shows the crack width plotted against a_{cr} for the situations tabulated in Table 7. It will be seen that the scatter is slightly less than in Figure 25(a) and that the crack widths close to the bar lie on the same line as the points for mid-spacing.

Data from a number of other investigations (for example, references 3 and 4) can be used to show the same result: that the crack width close to the bar is largely independent of ϕ/ρ_{eff} and that the crack width increases with increasing distance from the bar. All suggest that the dominant influence on the surface crack width is the shear lag effect and not any influence of slip.

The various contributions can now be considered individually.

Ålander and Cairns both present results from situations where variations in bond properties have led to an effect on crack width. In the paper, I present two cases where the effect of the difference in bond between plain and ribbed bars is very small. Clearly, there are further areas of doubt here about the factors which affect surface crack width and the mechanisms involved. Cairns' conclusion that cover was a better predictor of crack width than ϕ/ρ_{eff} is useful corroboration of my conclusion. It is particularly interesting to note in Figure 17 that the surface characteristics of the bar have an effect on the crack width even though the bond parameter ϕ/ρ_{eff} has no effect. This should lead to some interesting speculation on the mechanism by which the bar surface characteristics actually influence the surface crack widths.

Eligehausen, Mayer and Lettow present crack width results from a series of columns with varying values of ϕ/ρ from which they conclude that ϕ/ρ is the major variable. Unfortunately, the variations between the various specimens are achieved by varying the dimensions of the specimens and by varying the spacing and diameter of the bars. It is therefore not logically possible to establish what the critical variable actually is. My personal view, from

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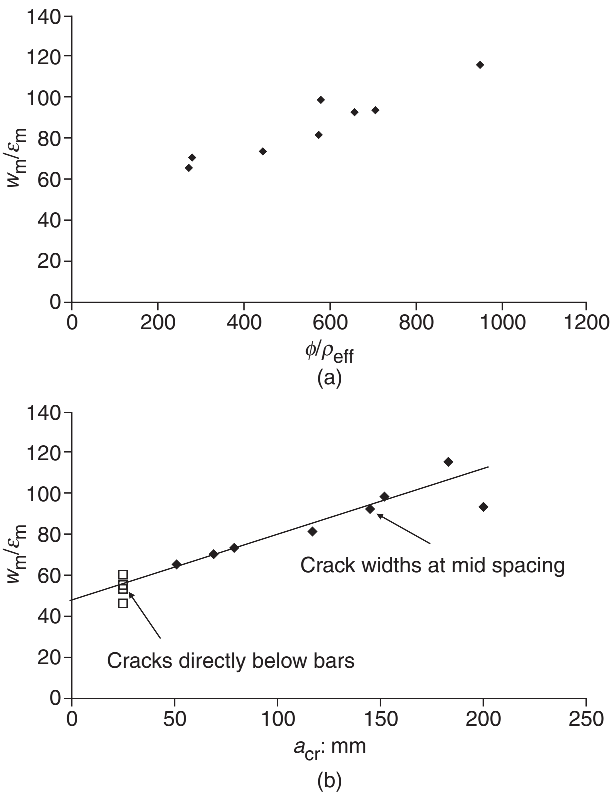
Table 6 Details of test specimens[43]

Specimen type	Minimum dimension: mm	Maximum dimension: mm	$(w/\varepsilon_m)_m$ at (a): mm	$(w/\varepsilon_m)_m$ at (b): mm
Z2	80	80	76	76
Z6	80	130	104	142
Z7	80	180	91	228
Z9	80	230	101	290

Table 7 Details and results from Series P slabs with 25 mm cover[44]

Specimen	Bar diameter: mm	Bar spacing: mm	(w/ε_m) close to bar: mm	(w/ε_m) at mid spacing: mm	ρ_{eff}	ϕ/ρ_{eff}
P1	22	305	54	93	0·0333	660
P4	12	69	56	51	0·0438	274
	12	71		69	0·0426	282
	19	232		152	0·0327	581
	25	372		200	0·0353	709
P5	19	380	61	116	0·0200	952
P6	16	150	47	74	0·0358	446
	19	230		82	0·0330	576



△ **Figure 25 (a) Average crack widths divided by average surface strain plotted against ϕ/ρ_{eff}; (b) average crack widths divided by average surface strain against distance of point considered from bar surface (a_{cr})**

the arguments set out above, is that the critical variable is the bar spacing, possibly reflected by the variable a_{cr}.

I am largely in agreement with Ferretti and Iori. They are thanked for pointing out the paper by Gerstle and Ingraffea,[25] which I had missed. It was fascinating to read of their presentation of the 'no-slip' idea which had been put forward[3] and then developed further[41] where it formed the basis for the development of the UK code method for crack control that has been in use for the last 30 years. The idea has recently been developed further.[45]

I am happy to agree with the view that the 'classical' approach is a simplification of reality. I would simply take the issue slightly further to assert that the simplification has been taken to such an extent that the formula is practically useless. I entirely agree that the predictions of the 'classical' formula are inconsistent with the CEB-FIP bond-slip law, which does not suggest ϕ/ρ as a variable. I did make this point in the paper, though possibly not very clearly. I believe that 'diffusion' is the same as what I have called 'shear lag' and agree that the effect of bond parameters on the surface width will depend on the relative importance of diffusion/shear lag and bond-slip. My personal view is that generally diffusion/shear lag is much more significant than bond-slip.

The issue raised by Ferretti and Iori relating to the variability of tensile strength is, I believe, fundamental to the understanding of cracking and tension stiffening. It has recently been studied in considerable depth.[46]

Gambarova and Bamonte make a number of comments on the way the study has been carried out and that other work has been done in the subject area which has not been mentioned. I will not deny that there are considerable difficulties in treating the question of the effect of ϕ/ρ in an entirely logical way. This issue is discussed to some extent in the paper. To a degree efforts have been made within the study to minimise the problems by carrying out comparisons only within a particular test series so that differences in test method and definition of the variables are not an issue. It is possibly necessary to reiterate my purpose in producing the paper.

It is commented that the 'scanty dependence of the ratio (w/ε_{max}) on ϕ/ρ – and specifically on ρ – has been already observed

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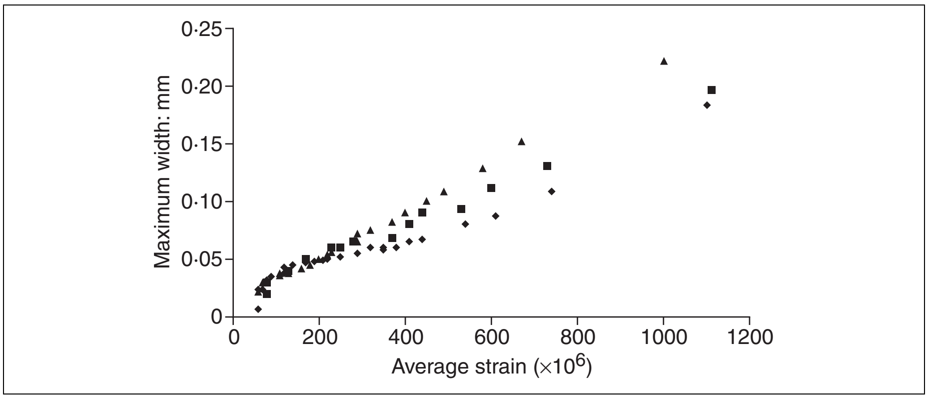

△ **Figure 26 Crack widths used in Figure 23 plotted against average strain**

by a number of scholars.' It is gratifying that there are those who agree with me but a great pity that these scholars have been unable to inform those who develop design rules and draft codes of this fact. Early work (in the 1960s) where this conclusion was reached was, of course, referred to in the paper (for example, Base *et al.*,[3] Broms[12] and Gergerly and Lutz[18]).

Giuriani and Plizzari raise a number of issues. First, they query the use in most cases in my paper of the strain or stress in the reinforcement at a crack rather than the average strain and my statement that the strain at a crack appears to give the better relationship. Figure 23 shows results from one type of specimen used in the tests by Farra and Jaccoud[5] plotted against the stress at a crack. Figure 26 shows the same crack width data plotted against the average strain. On the whole, use of the stress in the reinforcement at a crack appears to give a better straight line through the origin and hence the gradient of the line gives a more convenient single number parameter characterising the cracking. In fact, I doubt if it would make any great difference to the conclusions reached which strain or stress was used.

The modified formula presented by Giuriani and Plizzari is interesting and I certainly agree that, in reality, the bond stress increases after cracking. The reduced influence of ϕ/ρ_{eff} is certainly a move in the right direction but still does not reflect the experimental results. Possibly Balazs' point that formulae based on bond–slip formulations are only valid at the bar-concrete interface[39] is of significance here. As mentioned in the discussion at the start of this reply, I believe that the influence of cover arises from the shear deformation of the concrete between the bar surface and the concrete surface. I doubt if different confinement is a sustainable reason.

Pantazopoulou and Tasani quote work by Polak and co-workers that shows that bar diameter is highly relevant. Most unfortunately, in order to keep the reinforcement ratio constant, the bar spacing is changed from specimen to specimen. It is therefore logically not possible to establish from the tests whether it is the bar diameter or the bar spacing which is critical. In item (b) of their contribution, they point out that the definition of ρ_{eff} is somewhat arbitrary. I entirely agree with this, though, of course, the objection does not apply to axially reinforced tension specimens where ρ_{eff} must be equal to ρ. The introduction of τ_{ult} is an issue to be taken up with some of the derivers of the classical theory. I certainly do not believe that bond failure occurs.

In summary, the purpose of the paper is clearly stated to be the examination of the influence of ϕ/ρ_{eff} on crack width and, in particular, on S_0 as defined in the paper. I conclude that the effect is minimal. It appears from the discussion that four of the contributions agree that ϕ/ρ_{eff} is a poor variable or should be much reduced in importance, two contributors present data which, in my view, do not clearly demonstrate an influence of ϕ/ρ_{eff} and the remaining contributors do not address the issue directly.

A secondary purpose of the paper was to suggest that cover was an important variable. One contributor agrees with this conclusion and one states that cover cannot be a variable but offers no evidence in support of this view. The other contributors do not address this point.

The third issue, which was not seen as particularly important and, indeed, was not even mentioned in my conclusions is the influence of the bond characteristics of the reinforcement on crack width. Good evidence has been submitted here that there are circumstances where the bond characteristics can have an effect.

I believe that there is a strong case for an in-depth re-examination of cracking and the prediction formulae given in some design codes (for example, Model Code 1990[1]) and that the discussion has not weakened this case.

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