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Discussion: New and improved four-parameter non-linear Muskingum model

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Notation

A_i	dimensionless inflow interval i ($i = 1, 2, 3$)
a_1, a_2	upstream and downstream depth-discharge characteristics, respectively
b_1, b_2	upstream and downstream mean depth-storage characteristics, respectively
I	value for inflow
I_j	inflow for time interval j
$I_{\max}$	maximum inflow during the routing period
j	time interval
K	storage parameter
K_1, K_2	storage parameters for upstream and downstream sections, respectively
L	specified number of inflow levels
Q	value for outflow
S	storage in the reach
S_{in}	storage at upstream section of river
S_{out}	storage at downstream section of river
u_j	dimensionless inflow variable
v_1, v_2	dimensionless inflow dividing values
w	relative weight of inflow
α	flow parameter
β	exponent parameter
γ_1, $\gamma 2$	parameters for upstream and downstream sections, respectively
χ	indicator function of time interval

Contribution by Reza Barati, Hossein Shahheydari, Ehsan Jafari Nodoshan and Tahereh Barati

The original paper by Easa (2014) developed a new storage equation for the Muskingum flood routing model to improve the fit to observed outflows. The original research is both useful and interesting. In this discussion, some notes and comments will be presented to extend and uphold the results of the original paper.

Storage–discharge relationship

In general, the storage of unsteady flow in a natural river depends primarily on the inflow and outflow and on the geometric and hydraulic characteristics of the river and its control features (Akbari and Barati, 2012; Chow, 1959; Viessman and Lewis, 2003). Easa (2014) developed a new four-parameter non-linear Muskingum model by assuming that the flow and storage characteristics at the upstream and downstream cross-sections are *constant*. If this assumption is not established, the storages at the upstream and downstream sections can be expressed as

$$S_{\text{in}} = b_1(I/a_1)^{m_1/n_1},\ S_{\text{out}} = b_2(Q/a_2)^{m_2/n_2} \tag{24}$$

where I and Q are simultaneous values of inflow and outflow; S_{in} and S_{out} mirror storage at the upstream and downstream sections, respectively; a_1 and n_1 denote the depth-discharge characteristics; b_1 and m_1 denote the mean depth-storage characteristics at the upstream cross-section; and a_2 and n_2, b_2 and m_2 are corresponding values at the downstream cross-section.

By substituting S_{in} and S_{out} from the above equation into Equation 16 of the original paper by Easa (2014), the *general form* of storage–discharge relationship (or storage equation) of the Muskingum model can be expressed as

$$S = [\gamma_1 I^{\alpha_1} + \gamma_2 Q^{\alpha_2}]^{\beta} \tag{25}$$

where β, $\gamma_1 = wb_1/(a_1)^{m_1/n_1}$, $\alpha_1 = m_1/n_1$, $\gamma_2 = (1-w)b_2/(a_2)^{m_2/n_2}$, and $\alpha_2 = m_2/n_2$ are flood routing parameters; w defines the relative weights given to inflow and outflow for the river reach.

The model has five parameters, γ_1, α_1, γ_2, α_2 and β, which must be calibrated using available flood events. The available storage equations of the Muskingum model including linear and non-linear models along with routing parameters are presented in Table 7. For the linear model, a finite-difference method is available for the numerical solution of the governing equations (i.e. storage equation together with the one-dimensional continuity equation) (Barati, 2013), while for each form of non-linear model, similar to Easa (2014) or Barati (2011), the Euler method can be adopted for the numerical solution. In order to avoid negative discharges in the routing procedure of the non-linear models, the penalty approach which was proposed by Karahan *et al.* (2013) can be considered.

Row	Equation	Number of parameters	Routing parameters
1	$S = K[wI + (1 - w)Q]$	2	K and w
2	$S = K[wI^{\alpha} + (1 - w)Q^{\alpha}]$	3	K, w and α
3	$S = K[wI + (1 - w)Q]^{\beta}$	3	K, w and β
4	$S = K[wI^{\alpha_1} + (1 - w)Q^{\alpha_2}]$	4	K, w, α_1 and α_2
5	$S = K[wI^{\alpha} + (1 - w)Q^{\alpha}]^{\beta}$	4	K, w, α and β
6	$S = [\gamma_1 I^{\alpha_1} + \gamma_2 Q^{\alpha_2}]^{\beta}$	5	γ_1, α_1, γ_2, α_2 and β

Table 7. Summary of the available storage equations of Muskingum flood routing model

Interestingly, a better solution can be obtained by using the four-parameter storage equation for example 3 of Easa (2014) as $K = 0{\cdot}4608$, $w = 0{\cdot}1665$, $\alpha = 0{\cdot}9215$ and $\beta = 1{\cdot}5679$, with sum of the square of the deviation (SSQ) equal to 73 379·35.

For a five-parameter model, the flood routing parameters and SSQ values along with the percentage of the improvement over the best results of the four-parameter model for the three case studies of Easa (2014) are presented in Table 8. The results indicated that the five-parameter model yielded better results than the four-parameter one. The significant improvement is achieved for the first and latter case studies, although the improvement for the second case is not considerable.

Conclusion

Different weighted-flow and storage volume relationships in natural rivers may be occurring around the world. By considering the available storage equations of Table 7, it is possible to select the proper storage equation for a given natural river. As discussed by Barati (2014), the proper storage–discharge relationship for each river must be selected by considering the relationship between weighted-flow and storage volume of the flood events. However, it should be stated that the calibration procedure of the routing parameters may be more challenging; also the dependence of the model's results on the calibration data may increase by increasing the number of parameters of the storage equation. Therefore, by considering the *degree of non-linearity* of the flood event, the engineers must select a storage equation with lower routing parameters, as far as possible, without losing accuracy in the flood routing simulation.

Author's reply

The author thanks the discussers for their interest in the paper. Their extension to include separate flow parameters for the upstream and downstream sections is useful for the cases in which the sections have different characteristics. There are just a few points that are worthy of note.

The discussers' five-parameter model should naturally improve model performance as it has more parameters, compared with the four-parameter model. As the discussers correctly pointed out, a trade-off should be made between the extra parameters and the resulting improvement in model performance. However, a relatively large number of parameters no longer presents an issue given modern computers and superior optimisation software.

The discussers have obtained a slightly improved solution of SSQ = 73 379·35 for example 3 with the following optimal parameters $K = 0{\cdot}0768$, $w = 0{\cdot}1665$, $\alpha = 0{\cdot}9215$ and $\beta = 1{\cdot}5679$. This is indeed a better solution. The author obtained a solution with identical SSQ and similar optimal parameter values, except that the optimal value of K was 0·4609 (now corrected by discussers). It is worth noting that the flood routing optimisation problem is highly non-linear and non-convex. Generally, it is possible to improve the solution by running the model several times, each time starting with the previous 'optimal' solution. The process is stopped when the improvement in the optimal solution is not large.

Although the discussers' model is theoretically correct, a better model that preserves the physical significance of the original parameters is given by

Case studies	Parameter vector (γ_1, α_1, γ_2, α_2 and β)	SSQ	Improved: %
Smooth single-peak hydrograph	(0·0722, 0·6964, 0·8825, 0·4252, 3·8166)	5·44	29·07
Non-smooth single-peak hydrograph	(0·3378, 1·1263, 0·2979, 1·1944, 1·3455)	32 055·46	0·75
Multiple-peak hydrograph	($6{\cdot}74\times10^{-9}$, 3·3262, 0·0873, 1·3655, 1·0383)	69 726·75	4·98

Table 8. Results of the new model for three case studies

Example	SSQ[c]				Parameters of five-parameter model with VEP					
	Four-parameter model (Easa, 2014)	Discussers' five-parameter model	Five-parameter model with VEP	Improvement:[b] %	K	w	α	β_1	β_2	v_1
Example 1	7·67	5·44	5·35	2	0·971	0·264	0·356	-[a]	–	–
Example 2	32 299	32 056	23 069	28	0·942	0·338	1·283	1·133	1·154	0·289
Example 3	73 379	69 727	59 709	14	0·064	0·267	1·037	1·439	1·426	0·393

[a] For this example, a continuous function of the exponent parameter provided better results, $\beta(u_j) = a + b \log(1{\cdot}1 - u_j)$, where $a = 4{\cdot}830$ and $b = -0{\cdot}052$.
[b] Improvement of the author's five-parameter model with VEP, compared with the discussers' five-parameter model.
[c] All results include SSQ for $t = 0$, which equals $(I_0 - Q_0)^2$.

Table 9. Performance of the four-parameter non-linear Muskingum model with VEP

$$S_j = [wK_1 I_j^{\alpha_1} + (1 - w)K_2 Q_j^{\alpha_2}]^{\beta} \quad (26)$$

where K_1 and K_2 = storage parameters for the upstream and downstream sections, respectively, given by $b_1/a_1^{\alpha_1}$ and $b_2/a_2^{\alpha_2}$. For constant characteristics of the upstream and downstream sections, $K_1 = K_2$, $a_1 = a_2$, and Equation 26 reduces to the four-parameter model of the original paper by Easa (2014).

The discussers presented in Table 7 a summary of the available storage equations of the Muskingum model. There is, however, another storage equation that should be included in this table. It is a three-parameter non-linear Muskingum model with variable exponent parameter (VEP), which takes the form (Easa, 2013)

$$S_j = K[wI_j + (1–w)Q_j]^{\beta(u_j)} \quad (27)$$

where $\beta(u_j)$ = exponent parameter for time interval j and u_j = dimensionless inflow variable (0 to 1) for time interval $j = I_j/I_{\max}$, where I_j = inflow for time interval j and $I_{\max}$ = maximum inflow during the routing period. The number of exponent parameters depends on the number of specified inflow levels L and equals $2 + L$. The exponent parameter was modelled using a step function which is given by (for $L = 3$ for example)

$$\beta(u_j) = \sum_{i=1}^{3} \beta_i \chi_{A_i}(u_j) \quad (28)$$

where A_i = dimensionless inflow interval i ($i = 1, 2, 3$) and $\chi_{A_i}(u_j)$ = indicator function of A_i, which is defined as

$$\chi_{A_i}(u_j) = \begin{cases} 1 & \text{if} \quad u_j \in A_i \\ 0 & \text{if} \quad u_j \notin A_i \end{cases} \quad (29)$$

Note that there are three exponent parameters (β_1, β_2 and β_3) that are defined for three dimensionless inflow ranges (0, v_1), (v_1, v_2), and (v_2, 1), where v_1 and v_2 are dimensionless inflow dividing values.

Extending the idea of VEP to the four-parameter model presented in the paper yields the following storage equation

$$S_j = K[wI_j^{\alpha} + (1–w)Q_j^{\alpha}]^{\beta(u_j)} \quad (30)$$

The total number of parameters of this model is $3 + L$. For $L = 2$, for example, the number of parameters is 5. Note that this five-parameter model with VEP is obtained by using two exponent parameters, whereas the discussers' five-parameter model is obtained by using two flow parameters. To allow comparison of the two models, the five-parameter model with VEP was applied to the three examples and the results are shown in Table 9. As noted, the reductions in SSQ, compared with the discussers' model, are 2, 28 and 14% for examples 1, 2, and 3, respectively (for example 1, which already has an almost perfect fit, a continuous function of the exponent parameter was used). In addition, a six-parameter model ($L = 3$) was evaluated and SSQ was reduced further to 31 and 33%, for examples 2 and 3 respectively. Clearly, the extra parameter has more than doubled SSQ reduction for example 3 (multiple-peak hydrograph), but produced a small improvement for example 2 (non-smooth, single-peak hydrograph). Thus, the number of inflow levels may vary from one situation to another.

Based on these and the discussers' results, it appears that the exponent parameter β has a more significant effect on model performance than the flow parameter α. This was perhaps the main reason most researchers in the past have focused on the non-linear model with β (NL2) than the one with α (NL1), as mentioned in the paper. The encouraging results of the VEP presented in this response have provided the motivation to extend the paper's four-parameter Muskingum model to account for the variability of all four model parameters with flood characteristics. A forthcoming work by the author, which has been submitted for publication, demonstrates that the model would be quite useful for all types of hydrographs.

In conclusion, over the past several decades researchers have attempted to improve the performance of the non-linear Muskingum model by adopting different solution algorithms. This strategy, however, has resulted in a very slight improvement. To promote alternative thinking, the author has recently focused on changing the structure of the model itself. The author was pleased to see the discussers' contribution as it followed the same thinking.

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