

Appendix A. Budget-uncertainty set robust optimization

The goal of converting deterministic models into robust ones is to find a solution that remains feasible and optimal when the uncertain demand decreases relative to its nominal (deterministic) value. This reformulation involves three main steps.

Step 1 - Definition of the uncertainty set

Following the approach in Bertsimas & Sim (2004), we consider the uncertain demand of each retailer, denoted by $\tilde{d}_i$, to be a bounded and symmetric random variable within the interval $[d_{i,t} - \hat{d}_{i,t}, d_{i,t} + \hat{d}_{i,t}]$, where $d_{i,t}$ is the nominal demand and $\hat{d}_{i,t}$ is the possible deviation of it. A scale deviation $\epsilon_{i,t} = \frac{(\tilde{d}_i - d_i)}{\hat{d}_{i,t}}$ is associated with the uncertain parameter and belongs to the interval $[-1, 1]$. Therefore, we draw the uncertain demand as follows: $\tilde{d}_{i,t} = d_{i,t} + \hat{d}_{i,t} \cdot \epsilon_{i,t}$, where ϵ belongs to the following uncertainty set $\mathcal{U}$:

$$\mathcal{U} = \{|\epsilon| \leq 1, \epsilon_{i,t} \leq \Gamma_{i,t}, \forall i \in \{1, 2\}, t \in \mathcal{T}\} \quad (\text{A.1})$$

This uncertainty set, $\mathcal{U}$, is designated the budget-uncertainty set due to parameter Γ , which is known as the budget of uncertainty. The parameter Γ is user-defined and limits the variation of the uncertain parameter. In other words, Γ limits the number of possible deviations. For instance, setting $\Gamma_{i,1} \in [0, 1]$ means that, at most, only one worst-case value (i.e., the largest possible deviation) of the demand may occur for each retailer throughout the entire planning horizon.

Step 2 - Definition of the robust counterpart

In this step, the deterministic demand $d_{i,t}$ is replaced by the uncertain demand $\tilde{d}_i$. Therefore, all constraints that include $d_{i,t}$ are reformulated. These are constraints (13) and (14) for both the VMI and RMI models, and constraints (24) for the RMI model. The reformulated constraints are as follows:

$$\sum_{a \in \mathcal{A}} W_{a,t,i}^L \leq \alpha \cdot (d_{i,t} - \hat{d}_{i,t} \cdot \epsilon_{i,t}) \quad \forall t \in \mathcal{T}, i \in \mathcal{R} \quad (\text{A.2})$$

$$\sum_{a \in \mathcal{A}} W_{a,t,i}^F \leq (1 - \alpha) \cdot (d_{i,t} + \hat{d}_{i,t} \cdot \epsilon_{i,t}) \quad \forall t \in \mathcal{T}, i \in \mathcal{R} \quad (\text{A.3})$$

$$Q_{t,i} \geq (d_{i,t} + \hat{d}_{i,t} \cdot \epsilon_{i,t}) - \sum_{a \in \mathcal{A} \setminus sl} I'_{a,t,i} \quad \forall t \in \mathcal{T}, i \in \mathcal{R} \quad (\text{A.4})$$

Note that the withdrawal constraints are equalities in the deterministic models. However, in the robust counterpart, they must be inequalities, since the demand is now uncertain; equalities could lead to infeasible solutions.

Step 3 - Reformulation for the robust counterpart

The constraints obtained in Step 2 are intractable. To obtain a tractable robust counterpart, the dual formulation must be incorporated into the original model, where $\lambda_{t,i}$ and $\mu_{t,i}$ are the dual variables. The reformulation procedure yields the following set of constraints:

$$\sum_{a \in \mathcal{A}} W_{a,t,i}^L \leq \alpha \cdot (d_{i,t} - \Gamma_{i,t} \cdot \lambda_{t,i} - \mu_{t,i}) \quad \forall t \in \mathcal{T}, i \in \mathcal{R} \quad (\text{A.5})$$

$$\sum_{a \in \mathcal{A}} W_{a,t,i}^F \leq (1 - \alpha) \cdot (d_{i,t} - \Gamma_{i,t} \cdot \lambda_{t,i} - \mu_{t,i}) \quad \forall t \in \mathcal{T}, i \in \mathcal{R} \quad (\text{A.6})$$

$$Q_{t,i} \geq (d_{i,t} + \Gamma_{i,t} \cdot \lambda_{t,i} + \mu_{t,i}) - \sum_{a \in \mathcal{A} \setminus sl} I'_{a,t,i} \quad \forall t \in \mathcal{T}, i \in \mathcal{R} \quad (\text{A.7})$$

$$\lambda_{t,i} + \mu_{t,i} \geq \hat{d}_{i,t} \quad \forall t \in \mathcal{T}, i \in \mathcal{R} \quad (\text{A.8})$$

$$\lambda_{t,i}, \mu_{t,i} \geq 0 \quad \forall t \in \mathcal{T}, i \in \mathcal{R} \quad (\text{A.9})$$

Appendix B. Average profits and relative waste for instance groups B and C

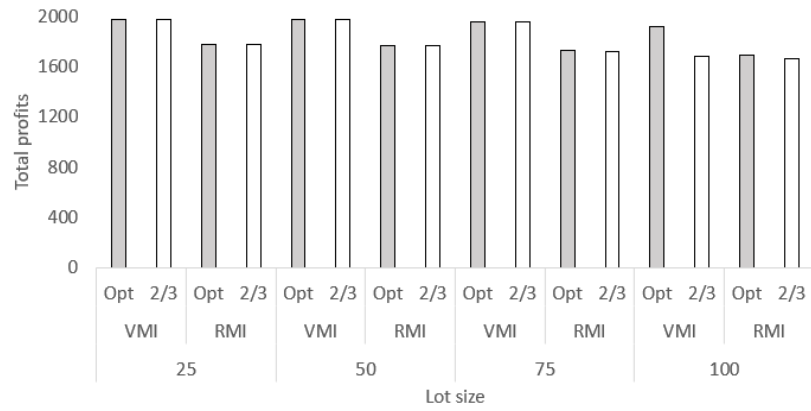


Figure B.8: Total profits obtained with the optimal MLOR and the 2/3 rules for the VMI and the RMI, for instance group B

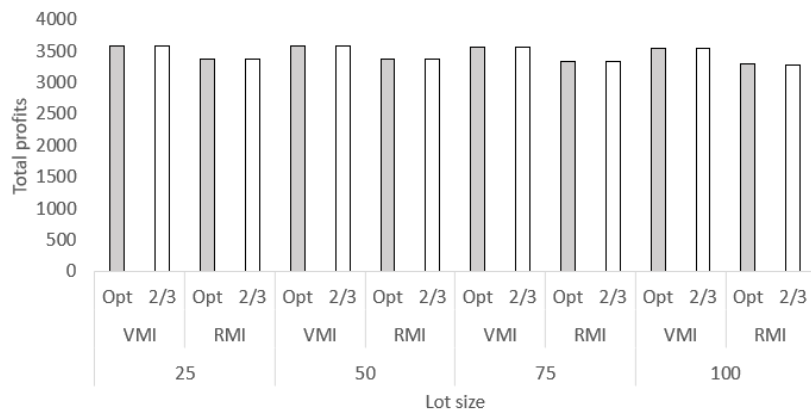


Figure B.9: Total profits obtained with the optimal MLOR and the 2/3 rules for the VMI and the RMI, for instance group C

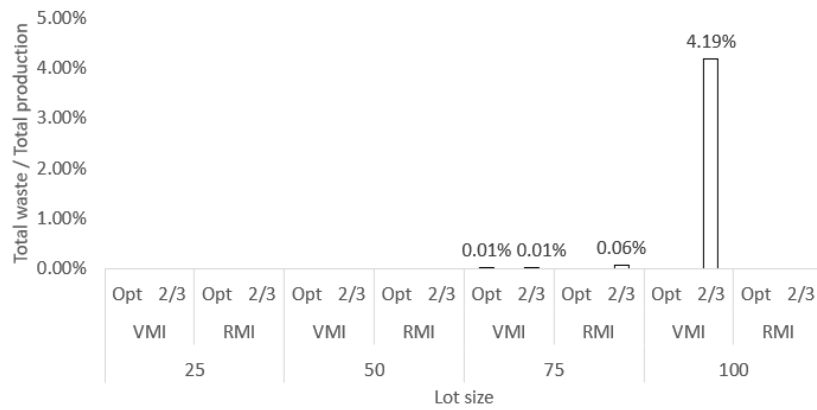


Figure B.10: Relative waste obtained with the optimal MLOR and the 2/3 rules for the VMI and the RMI, for instance group B

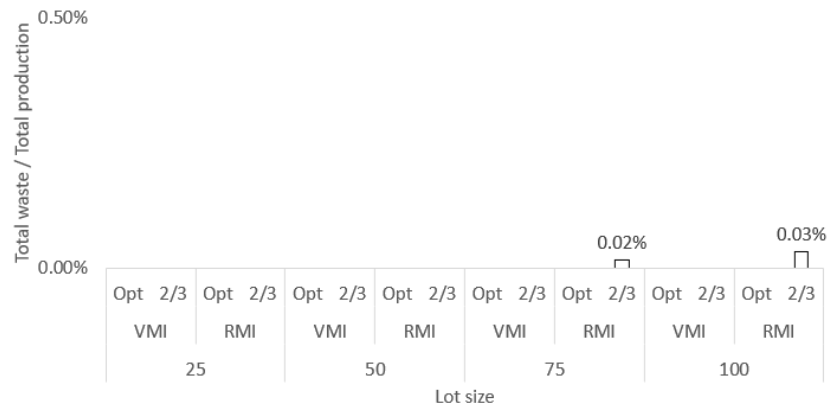


Figure B.11: Relative waste obtained with the optimal MLOR and the 2/3 rules for the VMI and the RMI, for instance group C

Appendix C. Optimal MLOR and profits in a setting with heterogeneous retailers

The following results assume a setting in which Retailer 2 has twice Retailer 1's demand. Table C.10 reports the average optimal MLOR obtained for each retailer with each strategy.

Table C.10: Average MLOR (and standard deviation) (in %)

Group	Lostize	VMI (%)		RMI (%)	
		Retail 1	Retail 2	Retail 1	Retail 2
A	25	84 (8)	84 (8)	92 (16)	80 (13)
	50	80 (0)	80 (0)	72 (24)	52 (10)
	75	68 (10)	60 (0)	36 (15)	40 (13)
	100	36 (8)	40 (0)	32 (10)	20 (0)
B	25	84 (8)	84 (8)	100 (0)	88 (10)
	50	80 (0)	80 (0)	80 (18)	72 (10)
	75	52 (24)	64 (20)	36 (20)	44 (20)
	100	40 (18)	40 (18)	40 (18)	32 (16)
C	25	100 (0)	100 (0)	100 (0)	88 (10)
	50	100 (0)	100 (0)	92 (10)	76 (8)
	75	72 (27)	88 (10)	68 (16)	68 (16)
	100	80 (0)	84 (8)	52 (20)	64 (15)

Appendix D. Relative waste for different budgets of uncertainty for instance groups B and C

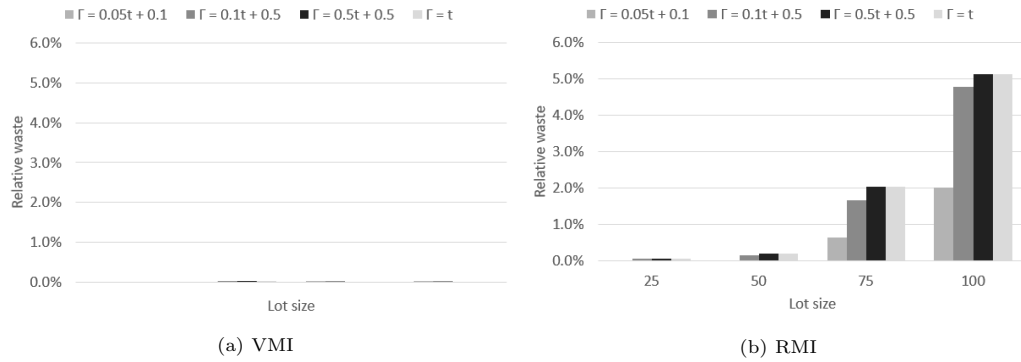


Figure D.12: Relative waste in the supply chain generated for different budgets of uncertainty in instance group B

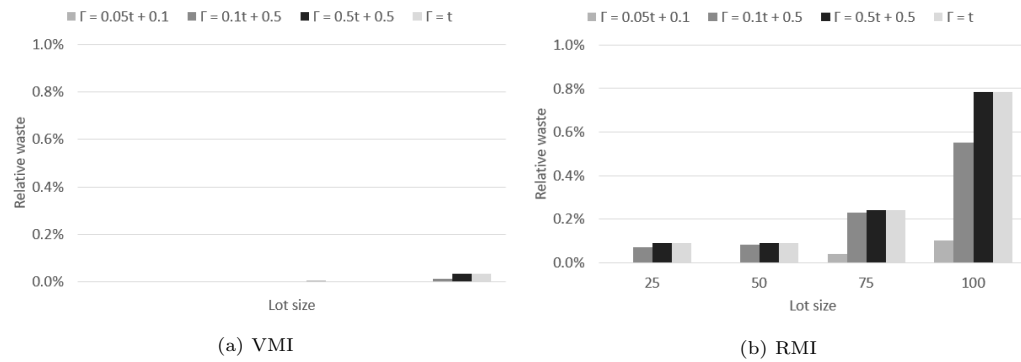


Figure D.13: Relative waste in the supply chain generated for different budgets of uncertainty in instance group C