Online Appendix

"On the Benefit of Developing Customers Profile Analysis to Implement Personalized Pricing in a Supply Chain"

A. Description about Model U.

The sequence of events under model U is as follows. First, the manufacturer decides on a wholesale price ω to maximize his profit Π_M. Then, the platform determines p to implement uniform pricing with the objective of maximizing her profit Π_P.

The manufacturer's problem is given by

$$\begin{aligned} \max_{\omega^U} \quad & (\omega - c)(1-p), \\ s.t. \quad & \omega \geq c. \end{aligned} \tag{A1}$$

The platform's problem is given by

$$\begin{aligned} \max_{p} \quad & (p - \omega)(1-p), \\ s.t. \quad & \omega \leq p \leq 1. \end{aligned} \tag{A2}$$

B. Proofs.

Proof of Lemma 1. We solve the uniform pricing model using backward induction. First, given ω, the platform decides a retail price to maximize her profit $\Pi_P = (p-\omega)(1-p)$. Due to $\frac{\partial^2 \Pi_P}{\partial p^2} = -2 < 0$, solving the first order condition $\frac{\partial \Pi_P}{\partial p} = 1 - 2p - \omega = 0$ yields $p^U = \frac{1+\omega}{2}$. Then, plugging p^U into the profit of the manufacturer, we have $\Pi_M = (\omega - c)(1-p^U) = \frac{(1-\omega)(\omega-c)}{2}$. By solving the first order condition (i.e., $\frac{\partial \Pi_M}{\partial \omega} = \frac{1+c-2\omega}{2} = 0$) within $\omega \geq c$, we get $\omega^U = \frac{1+c}{2}$, so $p^U = \frac{3+c}{4}$, $\pi_M^U = \frac{(1-c)^2}{8}$ and $\pi_P^U = \frac{(1-c)^2}{16}$. $\square$

Proof of Lemma 2. Similarly, we solve the personalized pricing model using backward induction. Given ω, the platform decides a customer profile error to maximize her profit. By solving the first order condition (i.e., $\frac{\partial \Pi_P}{\partial \Delta} = -1 + \Delta + \omega + 2\beta(\Delta_0 - \Delta) = 0$), we get $\Delta_1 = \frac{1-\omega-2\beta\Delta_0}{1-2\beta}$. Due to $\frac{\partial^2 \Pi_P}{\partial \Delta^2} = 1 - 2\beta$, we have the following two cases.

(i) When $0 < \beta \leq \frac{1}{2}$, Π_P is a convex function of Δ as $\frac{\partial^2 \Pi_P}{\partial \Delta^2} > 0$. Due to $c \leq \omega \leq 1 - \Delta_0$, i.e., $0 < \Delta_0 \leq \Delta_1$, Π_P decreases in $\Delta \in [0, \Delta_0]$, so $\Delta^P = 0$.

(ii) When $\beta > \frac{1}{2}$, Π_P is a concave function of Δ as $\frac{\partial^2 \Pi_P}{\partial \Delta^2} < 0$.

(ii-a) If $c \leq \omega \leq 1 - 2\beta\Delta_0$, i.e., $\Delta_1 < 0 < \Delta_0$, Π_P decreases in $\Delta \in [0, \Delta_0]$, so $\Delta^P = 0$.

(ii-b) If $1-2\beta\Delta_0 < \omega \leq 1-\Delta_0$, i.e., $0 < \Delta_1 \leq \Delta_0$, Π_P first increases in $\Delta \in [0,\Delta_1]$ and then decreases in $\Delta \in (\Delta_1,\Delta_0]$, so $\Delta^P = \Delta_1$.

Then, the desired result follows as shown in Lemma 2. □

Proof of Proposition 1. Plugging Δ^P in Lemma 2 into the profit of manufacturer.

(i) When $0 < \beta \leq \frac{1}{2}$, then

$$\Pi_M = (\omega - c)(1-\omega), c \leq \omega \leq 1-\Delta_0 \tag{A3}$$

By solving the first order condition (i.e., $\frac{\partial \Pi_M}{\partial \omega} = 1+c-2\omega = 0$), we get $\omega_1 = \frac{1+c}{2} > c$. Therefore, we have the following cases.

(i-a) If $0 \leq c \leq 1-2\Delta_0$, i.e., $c < \omega_1 \leq 1-\Delta_0$, Π_M first increases in $\omega \in [c,\omega_1]$ and then decreases in $\omega \in (\omega_1, 1-\Delta_0]$, so $\omega^P = \omega_1 = \frac{1+c}{2}$ and $\Delta^P = 0$.

(i-b) If $1-2\Delta_0 < c \leq 1-\Delta_0$, i.e., $\omega_1 > 1-\Delta_0$, Π_M increases in $\omega \in [c, 1-\Delta_0]$, so $\omega^P = 1-\Delta_0$ and $\Delta^P = 0$.

(ii) When $\beta > \frac{1}{2}$, we have the following cases.

(ii-a) If $0 \leq c \leq 1-2\beta\Delta_0$, then

$$\Pi_M = \begin{cases} (\omega-c)(1-\omega), & c \leq \omega \leq 1-2\beta\Delta_0 \\ (\omega-c)(1-\omega-\Delta_1), & 1-2\beta\Delta_0 < \omega \leq 1-\Delta_0 \end{cases} \tag{A4}$$

By solving the first order condition of the first line (i.e., $\frac{\partial \Pi_M}{\partial \omega} = 1+c-2\omega = 0$), we get $\omega_1 = \frac{1+c}{2} > c$. By solving the first order condition of the second line (i.e., $\frac{\partial \Pi_M}{\partial \omega} = \frac{2\beta(2\omega-c-1+\Delta_0)}{1-2\beta} = 0$), we get $\omega_2 = \frac{1+c-\Delta_0}{2} \leq 1-\Delta_0$. Then, comparing ω_1, ω_2 and three endpoints, we have

1) if $0 \leq c \leq 1-4\beta\Delta_0$, Π_M first increases in $\omega \in [c,\omega_1]$ and then decreases in $\omega \in (\omega_1, 1-\Delta_0]$, so $\omega^P = \omega_1 = \frac{1+c}{2}$ and $\Delta^P = 0$;

2) if $1-4\beta\Delta_0 < c \leq 1-(4\beta-1)\Delta_0$, Π_M first increases in $\omega \in [c, 1-2\beta\Delta_0]$ and then decreases in $\omega \in (1-2\beta\Delta_0, 1-\Delta_0]$, so $\omega^P = 1-2\beta\Delta_0$ and $\Delta^P = 0$;

3) if $1-(4\beta-1)\Delta_0 < c \leq 1-2\beta\Delta_0$, Π_M first increases in $\omega \in [c,\omega_2]$ and then decreases in $\omega \in (\omega_2, 1-\Delta_0]$, so $\omega^P = \omega_2 = \frac{1+c-\Delta_0}{2}$ and $\Delta^P = \Delta_1 = \frac{1-c+(1-4\beta)\Delta_0}{2(1-2\beta)}$.

(ii-b) If $1-2\beta\Delta_0 \leq c \leq 1-\Delta_0$, then

$$\Pi_M = (\omega-c)(1-\omega-\Delta_1), c \leq \omega \leq 1-\Delta_0 \tag{A5}$$

By solving the first order condition (i.e., $\frac{\partial \Pi_M}{\partial \omega} = \frac{2\beta(2\omega-c-1+\Delta_0)}{1-2\beta} = 0$), we get $\omega_2 = \frac{1+c-\Delta_0}{2}$ and

$c \le \omega_2 \le 1-\Delta_0$. Π_M first increases in $\omega \in [c,\omega_2]$ and then decreases in $\omega \in (\omega_2, 1-\Delta_0]$, so $\omega^P = \omega_2 = \frac{1+c-\Delta_0}{2}$ and $\Delta^P = \Delta_1 = \frac{1-c+(1-4\beta)\Delta_0}{2(1-2\beta)}$.

We define the following sets according to the analysis of the above cases.

$$
\begin{aligned}
I =&\{0<\beta\le \tfrac{1}{2}, 0\le c\le 1-2\Delta_0\}\cup\{\beta>\tfrac{1}{2}, 0\le c\le 1-4\beta\Delta_0\},\\
II =&\{0<\beta\le \tfrac{1}{2}, 1-2\Delta_0<c\le 1-\Delta_0\},\\
III =&\{\beta>\tfrac{1}{2}, 1-4\beta\Delta_0<c\le 1-(4\beta-1)\Delta_0\},\\
IV =&\{\beta>\tfrac{1}{2}, 1-(4\beta-1)\Delta_0<c\le 1-\Delta_0\}.
\end{aligned}
$$

Then the desired result follows as shown in Proposition 1. □

Proof of Lemma 3. First, we establish the monotonicity of the equilibrium wholesale price and profile error with respect to c and β case by case according to the results in Proposition 1.

(i) When $(\beta,c)\in I$, then $(\omega^P,\Delta^P)=\left(\frac{1+c}{2},0\right)$. It is easy to check that ω^P increases with c while Δ^P is irrelevant with c; moreover, both ω^P and Δ^P are irrelevant with β.

(ii) When $(\beta,c)\in II$, then $(\omega^P,\Delta^P)=(1-\Delta_0,0)$. It is easy to check that both ω^P and Δ^P are irrelevant with c and β.

(iii) When $(\beta,c)\in III$, then $(\omega^P,\Delta^P)=(1-2\beta\Delta_0,0)$. It is easy to check that both ω^P and Δ^P are irrelevant with c; moreover, ω^P decreases with β while Δ^P is irrelevant with β.

(iv) When $(\beta,c)\in IV$, then $(\omega^P,\Delta^P)=\left(\frac{1+c-\Delta_0}{2},\frac{1-c+(1-4\beta)\Delta_0}{2(1-2\beta)}\right)$. It is easy to check that both ω^P and Δ^P increases with c; moreover, ω^P is irrelevant with β, while Δ^P increases with β as $\frac{\partial\Delta^P}{\partial\beta}=\frac{4(1-c-\Delta_0)}{4(1-2\beta)^2}>0$.

Next, we analyze the impact of c on ω^P and Δ^P.

(i) When $0<\beta\le\frac{1}{2}$, the path of the equilibrium solutions is $I\to II$. Therefore, ω^P first increases and then keeps irrelevant with c, but Δ^P keeps irrelevant with c.

(ii) When $\frac{1}{2}<\beta\le\frac{1}{4\Delta_0}$, the path of the equilibrium solutions is $I\to III\to IV$. Therefore, ω^P first increases, then keeps irrelevant and finally increases with c, but Δ^P first keeps irrelevant and then increases with c.

(iii) When $\frac{1}{4\Delta_0}<\beta\le\frac{1+\Delta_0}{4\Delta_0}$, the path of the equilibrium solutions is $III\to IV$. Therefore, ω^P first keeps irrelevant and then increases with c, but Δ^P first keeps irrelevant and then increases with c.

(iv) When $\beta>\frac{1+\Delta_0}{4\Delta_0}$, the path of the equilibrium solutions is IV. Therefore, both ω^P and Δ^P increases with c.

In summary, $\frac{\partial \omega_P}{\partial c} \geq 0$ and $\frac{\partial \Delta_P}{\partial c} \geq 0$ always exists.

Finally, we analyze the impact of β on ω^P and Δ^P.

(i) When $0 < c \leq 1 - 2\Delta_0$, the path of the equilibrium solutions is $I \to III \to IV$. Therefore, ω^P first keeps irrelevant, then decreases and finally keeps irrelevant with β, but Δ^P first keeps irrelevant and then increases with β.

(ii) When $1 - 2\Delta_0 < c \leq 1 - \Delta_0$, the path of the equilibrium solutions is $II \to III \to IV$. Therefore, ω^P first keeps irrelevant, then decreases and finally keeps irrelevant with β, but Δ^P first keeps irrelevant and then increases with β.

In summary, $\frac{\partial \omega_P}{\partial \beta} \leq 0$ and $\frac{\partial \Delta_P}{\partial \beta} \geq 0$ always exists. □

Proof of Proposition 2. Similarly, we establish the monotonicity of the equilibrium demand and profits with respect to c and β case by case according to the results in Proposition 1.

(i) When $(\beta, c) \in I$, then $D^P = \frac{1-c}{2}$, $\Pi_M^P = \frac{(1-c)^2}{4}$ and $\Pi_P^P = \frac{(1-c)^2}{8} - \beta\Delta_0^2$. It is easy to check that D^P, Π_M^P and Π_P^P decrease with c; moreover, D^P and Π_M^P are irrelevant with β while Π_P^P decrease with β.

(ii) When $(\beta, c) \in II$, then $D^P = \Delta_0$, $\Pi_M^P = (1 - c - \Delta_0)\Delta_0$ and $\Pi_P^P = \frac{(1-2\beta)\Delta_0^2}{2}$. It is easy to check that Π_M^P decreases with c while D^P and Π_P^P are irrelevant with c; moreover, D^P and Π_M^P are irrelevant with β and Π_P^P decreases with β.

(iii) When $(\beta, c) \in III$, then $D^P = 2\beta\Delta_0$, $\Pi_M^P = (1 - c - 2\beta\Delta_0)2\beta\Delta_0$ and $\Pi_P^P = \beta\Delta_0^2$. It is easy to check that Π_M^P decrease with c while D^P and Π_P^P are irrelevant with c; moreover, Π_M^P decreases with β as $\frac{\partial \Pi_M^P}{\partial \beta} = 2\Delta_0(1 - c - 4\beta\Delta_0) < 0$, D^P and Π_P^P increase with β.

(iv) When $(\beta, c) \in IV$, then $D^P = \frac{\beta(1-c-\Delta_0)}{2\beta-1}$, $\Pi_M^P = \frac{\beta(1-c-\Delta_0)^2}{2(2\beta-1)}$ and $\Pi_P^P = \frac{\beta(1-c-\Delta_0)^2}{4(2\beta-1)}$.

It is easy to check that D^P, Π_M^P and Π_P^P decrease with c; D^P, Π_M^P and Π_P^P decrease with β.

Next, we analyze the impact of c on Π_M^P and Π_P^P.

(i) When $0 < \beta \leq \frac{1}{2}$, the path of the equilibrium solutions is $I \to II$. Therefore, Π_M^P decreases with c, but D^P and Π_P^P first decrease and then keep irrelevant with c.

(ii) When $\frac{1}{2} < \beta \leq \frac{1}{4\Delta_0}$, the path of the equilibrium solutions is $I \to III \to IV$. Therefore, Π_M^P decreases with c, but D^P and Π_P^P first decrease, then keep irrelevant, then decrease with c.

(iii) When $\frac{1}{4\Delta_0} < \beta \leq \frac{1+\Delta_0}{4\Delta_0}$, the path of the equilibrium solutions is $III \to IV$. Therefore, Π_M^P decreases with c, but D^P and Π_P^P first keep irrelevant and then decrease with c.

(iv) When $\beta > \frac{1+\Delta_0}{4\Delta_0}$, the path of the equilibrium solutions is IV. Therefore, D^P, Π_M^P and Π_P^P decrease with c.

In summary, $\frac{\partial \Pi_M^P}{\partial c} < 0$, $\frac{\partial \Pi_P^P}{\partial c} \leq 0$ and $\frac{\partial D^P}{\partial c} \leq 0$.

Finally, we analyze the impact of β on D^P, Π_M^P and Π_P^P.

(i) When $0 < c \leq 1-2\Delta_0$, the path of the equilibrium solutions is $I \to III \to IV$. Therefore, Π_M^P first keeps irrelevant and then decreases with β, Π_P^P first decreases,then increases and finally decreases with β, D^P first keeps irrelevant, then increases and finally decreases with β.

(ii) When $1-2\Delta_0 < c \leq 1-\Delta_0$, the path of the equilibrium solutions is $II \to III \to IV$. Therefore, Π_M^P first keeps irrelevant and then decreases with β, Π_P^P first decreases, then increases and finally decreases with β, D^P first keeps irrelevant, then increases and finally decreases with β. □

Proof of Proposition 3. Recall that $\pi_M^U = \frac{(1-c)^2}{8}$ and $\pi_P^U = \frac{(1-c)^2}{16}$. Then we compare the profits case by case according to the results in Proposition 1.

(i) When $(\beta, c) \in I$, then $\Pi_M^P = \frac{(1-c)^2}{4}$ and $\Pi_P^P = \frac{(1-c)^2}{8} - \beta\Delta_0^2$. $\pi_M^U - \Pi_M^P = -\frac{(1-c)^2}{8} < 0$ and $\pi_P^U - \Pi_P^P = -\frac{(1-c)^2}{16} + \beta\Delta_0^2$. Define $f_1(x) = -\frac{x^2}{16} + \beta\Delta_0^2$, where $x = 1-c \geq \max\{2\Delta_0, 4\beta\Delta_0\}$.

(i-a) If $0 < \beta \leq \frac{1}{4}$, then $f_1(x) < 0$ when $x \in [2\Delta_0, +\infty)$.

(i-b) If $\frac{1}{4} < \beta \leq \frac{1}{2}$, then $f_1(x) > 0$ when $x \in [2\Delta_0, 4\sqrt{\beta}\Delta_0)$ and $f_1(x) < 0$ when $x \in (4\sqrt{\beta}\Delta_0, +\infty)$.

(i-c) If $\frac{1}{2} < \beta \leq 1$, then $f_1(x) > 0$ when $x \in [4\beta\Delta_0, 4\sqrt{\beta}\Delta_0)$ and $f_1(x) < 0$ when $x \in (4\sqrt{\beta}\Delta_0, +\infty)$.

(i-d) If $\beta > 1$, then $f_1(x) < 0$ when $x \in [4\beta\Delta_0, +\infty)$.

(ii) When $(\beta, c) \in II$, then $\Pi_M^P = (1-c-\Delta_0)\Delta_0$ and $\Pi_P^P = \frac{(1-2\beta)\Delta_0^2}{2}$. $\pi_M^U - \Pi_M^P = \frac{(1-c)^2}{8} - (1-c)\Delta_0 + \Delta_0^2$. Define $f_2(x) = \frac{x^2}{8} - x\Delta_0 + \Delta_0^2$, where $\Delta_0 \leq x < 2\Delta_0$. Therefore, $f_2(x) > 0$ when $x \in [\Delta_0, (4-2\sqrt{2})\Delta_0]$ and $f_2(x) < 0$ when $x \in ((4-2\sqrt{2})\Delta_0, 2\Delta_0)$.

Similarly, $\pi_P^U - \Pi_P^P = f_3(x) = \frac{x^2}{16} - \frac{(1-2\beta)\Delta_0^2}{2}$, where $\Delta_0 \leq x < 2\Delta_0$.

(ii-a) If $0 < \beta \leq \frac{1}{4}$, then $f_3(x) < 0$ when $x \in [\Delta_0, 2\Delta_0)$.

(ii-b) If $\frac{1}{4} < \beta \leq \frac{7}{16}$, then $f_3(x) < 0$ when $x \in [\Delta_0, 2\sqrt{2-4\beta}\Delta_0]$ and $f_3(x) > 0$ when $x \in (2\sqrt{2-4\beta}\Delta_0, +\infty)$.

(ii-c) If $\frac{7}{16} < \beta \leq \frac{1}{2}$, then $f_3(x) > 0$ when $x \in [\Delta_0, 2\Delta_0)$.

(iii) When $(\beta, c) \in III$, then $\Pi_M^P = (1-c-2\beta\Delta_0)2\beta\Delta_0$ and $\Pi_P^P = \beta\Delta_0^2$. $\pi_M^U - \Pi_M^P = f_4(x) = \frac{x^2}{8} - 2\beta\Delta_0 x + 4\beta^2\Delta_0^2$, where $(4\beta-1)\Delta_0 \leq x < 4\beta\Delta_0$.

(iii-a) If $\frac{1}{2} < \beta \leq \frac{1+\sqrt{2}}{4}$, then $f_4(x) > 0$ when $x \in [(4\beta-1)\Delta_0, (8-4\sqrt{2})\beta\Delta_0)$ and $f_4(x) < 0$ when $x \in ((8-4\sqrt{2})\beta\Delta_0, 4\beta\Delta_0)$.

(iii-b) If $\beta > \frac{1+\sqrt{2}}{4}$, then $f_4(x) < 0$ when $x \in [(4\beta-1)\Delta_0, 4\beta\Delta_0)$.

Similarly, $\pi_P^U - \Pi_P^P = f_5(x) = \frac{x^2}{16} - \beta\Delta_0^2$, where $(4\beta-1)\Delta_0 \leq x < 4\beta\Delta_0$.

(iii-a) If $\frac{1}{2} < \beta \leq 1$, then $f_5(x) < 0$ when $x \in [(4\beta-1)\Delta_0, 4\beta\Delta_0)$.

(iii-b) If $1 < \beta \leq \frac{3+2\sqrt{2}}{4}$, then $f_5(x) < 0$ when $x \in [(4\beta-1)\Delta_0, 4\sqrt{\beta}\Delta_0)$ and $f_5(x) > 0$ when $x \in (4\sqrt{\beta}\Delta_0, 4\beta\Delta_0)$.

(iii-c) If $\beta > \frac{3+2\sqrt{2}}{4}$, then $f_5(x) > 0$ when $x \in [(4\beta-1)\Delta_0, 4\beta\Delta_0)$.

(iv) When $(\beta, c) \in IV$, then $\Pi_M^P = \frac{\beta(1-c-\Delta_0)^2}{2(2\beta-1)}$ and $\Pi_P^P = \frac{\beta(1-c-\Delta_0)^2}{4(2\beta-1)}$. $\pi_M^U - \Pi_M^P = f_6(x) = \frac{x^2}{8} - \frac{\beta(x-\Delta_0)^2}{2(2\beta-1)}$, where $\Delta_0 \le x < (4\beta-1)\Delta_0$.

(iv-a) If $\frac{1}{2} < \beta \le \frac{1+\sqrt{2}}{4}$, then $f_6(x) > 0$ when $x \in [\Delta_0, (4\beta-1)\Delta_0)$.

(iv-b) If $\beta > \overline{\beta}$, then $f_6(x) > 0$ when $x \in [\Delta_0, \frac{A}{A-1}\Delta_0)$ and $f_6(x) < 0$ when $x \in (\frac{A}{A-1}\Delta_0, (4\beta-1)\Delta_0)$, where $A = \sqrt{\frac{4\beta}{2\beta-1}}$.

Due to $\pi_P^U - \Pi_P^P = \frac{1}{2}f_6(x)$, the analysis is similar and we omit the detail.

In summary, as far as the comparison of π_M^U and Π_M^P, we have $\pi_M^P \ge \pi_M^U$ if $0 < c \le \min\{1-(4-2\sqrt{2})\Delta_0, 1-(8-4\sqrt{2})\beta\Delta_0, 1-\frac{A\Delta_0}{A-1})$ and $\pi_M^P < \pi_M^U$ otherwise.

Regarding the comparison of π_P^U and Π_P^P, we can use the following Figure A to show the results.

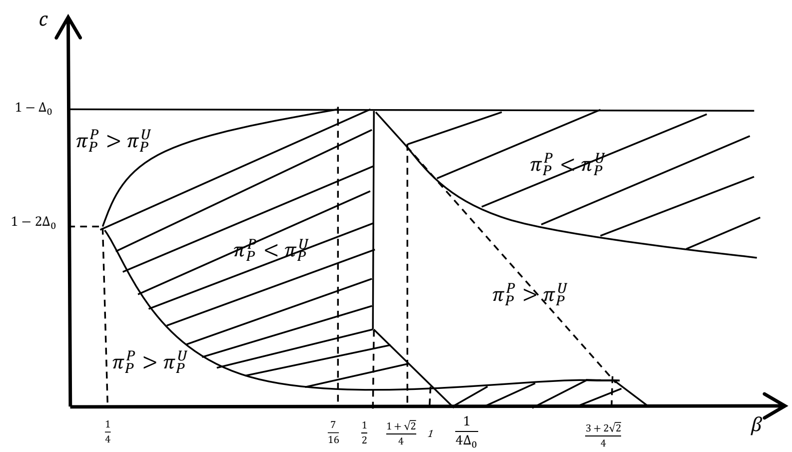

Figure A The comparison of platform's profits

Source(s): Figure created by authors

We define the following sets according to the analysis of the above cases.

$$
\begin{aligned}
A =& \{1-(4-2\sqrt{2})\Delta_0 < c \le \min\{1-2\sqrt{2-4\beta}\Delta_0, 1-\Delta_0\} \cup \max\{1-\tfrac{A\Delta_0}{A-1}, 1-(8-4\sqrt{2})\beta\Delta_0\} < c \le 1-\Delta_0\}, \\
B =& \{0 \le c \le \min\{1-2\Delta_0, 1-4\sqrt{\beta}\Delta_0, 1-4\beta\Delta_0\}\} \cup \{\max\{1-2\Delta_0, 1-2\sqrt{2-4\beta}\Delta_0\} < c \le 1-(4-2\sqrt{2})\Delta_0\} \\
& \cup \{\max\{1-4\beta\Delta_0, 1-4\sqrt{\beta}\Delta_0\} \le c < \min\{1-(8-4\sqrt{2})\beta\Delta_0, 1-\tfrac{A\Delta_0}{A-1}\}\}, \\
C =& \{\max\{1-(4-2\sqrt{2})\Delta_0, 1-2\sqrt{2-4\beta}\Delta_0\} \le c \le 1-\Delta_0\} \cup \{1-(8-4\sqrt{2})\beta\Delta_0 \le c < 1-(4\beta-1)\Delta_0\}, \\
D =& \{1-4\sqrt{\beta}\Delta_0 \le c < \min\{1-2\sqrt{2-4\beta}\Delta_0, 1-(4-2\sqrt{2})\Delta_0\}\} \cup \{1-4\sqrt{\beta}\Delta_0 \le c < 1-4\beta\Delta_0\} \\
& \cup \{0 \le c < \min\{1-4\sqrt{\beta}\Delta_0, 1-(4\beta-1)\Delta_0\}\}.
\end{aligned}
$$

Then, the final results are shown in Proposition 3.