Supplementary Material

.1. Proof of Lemma 1

Apparently, a customer would choose restaurant i, rather than restaurant $j \neq i$, only when his/her utility $U_i(x)$ is non-negative and no less than the utility $U_j(x)$ of choosing restaurant (i.e., $U_i(x) \geq 0, U_i(x) \geq U_j(x)$). By solving $U_1 = U_2$, we find a customer is indifferent between two restaurants at x_* as follows:

$$x_* = \frac{1}{2} + \frac{\delta(\lambda_1 - \lambda_2)}{2t_s}.$$

Moreover, by solving $U_i = 0$, we find the customer at x_i^0 has zero utility, where x_i^0 is as follows:

$$x_1^0 = \frac{V - p - \delta(1-\lambda_1)}{t_s}, x_2^0 = 1 - \frac{V - p - \delta(1-\lambda_2)}{t_s}. \tag{1}$$

Next, we discuss the customer demands for two restaurants as follows.

(a) When $x_1^0 < x_* < x_2^0$, which is equivalent to $p > V - \delta - \frac{t_s}{2} + \frac{\delta(\lambda_1+\lambda_2)}{2}$, customers located at $[0, x_1^0]$ will choose restaurant 1, and those located at $[x_2^0, 1]$ will choose restaurant 2. In other words, the demand functions are given by:

$$D_1(p) = x_1^0 = \frac{V - p - \delta(1-\lambda_1)}{t_s}, D_2(p) = 1 - x_2^0 = \frac{V - p - \delta(1-\lambda_2)}{t_s}, \tag{2}$$

which satisfies $D_1(p) + D_2(p) < 1$.

(b) When $x_1 \geq x_*$ and $x_2 \leq x_*$, which is equivalent to $p \leq V - \delta - \frac{t_s}{2} + \frac{\delta(\lambda_1+\lambda_2)}{2}$, the customers located at $[0, x_*]$ choose restaurant 1, and those located at $[x_*, 1]$ choose restaurant 2. In this case, the demand functions are given by:

$$D_1(p) = x_* = \frac{1}{2} + \frac{\delta(\lambda_1 - \lambda_2)}{2t_s}, D_2(p) = 1 - x_* = \frac{1}{2} - \frac{\delta(\lambda_1 - \lambda_2)}{2t_s}, \tag{3}$$

which satisfies $D_1(p) + D_2(p) = 1$.

Based on these results, we can obtain the total demands for chain restaurants directly. □

.2. Proof of Proposition 1

From the demand function shown in (16), we find that the profit function becomes $\Pi_F^B(p) = (p - c)$, which is increasing in p for $p \leq V - \frac{1}{2}t_s - \delta\frac{\gamma}{2}$, and the profit function becomes $\Pi_P^B(p) = (p-c)(\frac{-\delta\gamma - 2(p-V)}{t_s})$, which is concave in p for $p > V - \frac{1}{2}t_s - \delta\frac{\gamma}{2}$. In this case, the structural property of the profit function depends on the interior solution $p_b = \frac{1}{4}(2c - \delta\gamma + 2V) = \arg\max_p \Pi_P^B(p)$, and $p^\dagger = V - \frac{1}{2}t_s - \delta\frac{\gamma}{2}$. We discuss as follows.

(a) when $p_b > c$ and $p_b > p^\dagger$, which is equivalent to $V < \frac{1}{2}(2c + \delta\gamma + 2t_s)$, the profit function is quasi-concave with the global maximum point p_b (see Figure 1 (a)).

(b) when $p_b > c$ and $p_b \leq p^\dagger$, which is equivalent to $V \geq \frac{1}{2}(2c + \delta\gamma + 2t_s)$, the profit function is concave with the global maximum point $p^\dagger$ (see Figure 1 (b)). □

.3. Proof of Proposition 2

We first fix θ and maximize the objective function by optimizing p. From the demand function shown in (17), we find that the profit function becomes $\Pi_F^M(p) = (p - c) - b\frac{\theta^2}{2} - F_k$, which is increasing in p, when $p \leq V - \frac{1}{2}t_s + \delta\frac{-\gamma+2\theta}{2}$, and the profit function becomes $\Pi_P^M(p) = (p-c)(\frac{\delta(-\gamma+2\theta)-2(p-V)}{t_s}) - \frac{b\theta^2}{2} - F_k$, which is concave in p, when $p > V - \frac{1}{2}t_s + \delta\frac{-\gamma+2\theta}{2}$.

In this case, the structural property of the profit function depends on the interior solution $p_m = \frac{1}{4}(2c - \delta\gamma + 2\delta\theta + 2V) = \arg\max_p \Pi_P^M(p)$, and $p^\dagger = V - \frac{1}{2}t_s + \delta\frac{-\gamma+2\theta}{2}$. We discuss as follows.

(a) when $p_m > p^\dagger$, which is equivalent to $V < \frac{1}{2}(2c + 2t_s + \delta\gamma - 2\delta\theta)$, the profit function is quasi-concave with the global maximum point p_m (see Figure 1 (a)).

(b) when $p_m \le p^\dagger$, which is equivalent to $V \ge \frac{1}{2}(2c + 2t_s + \delta\gamma - 2\delta\theta)$, the profit function is concave with the global maximum price $p^\dagger$ (see Figure 1 (b)).

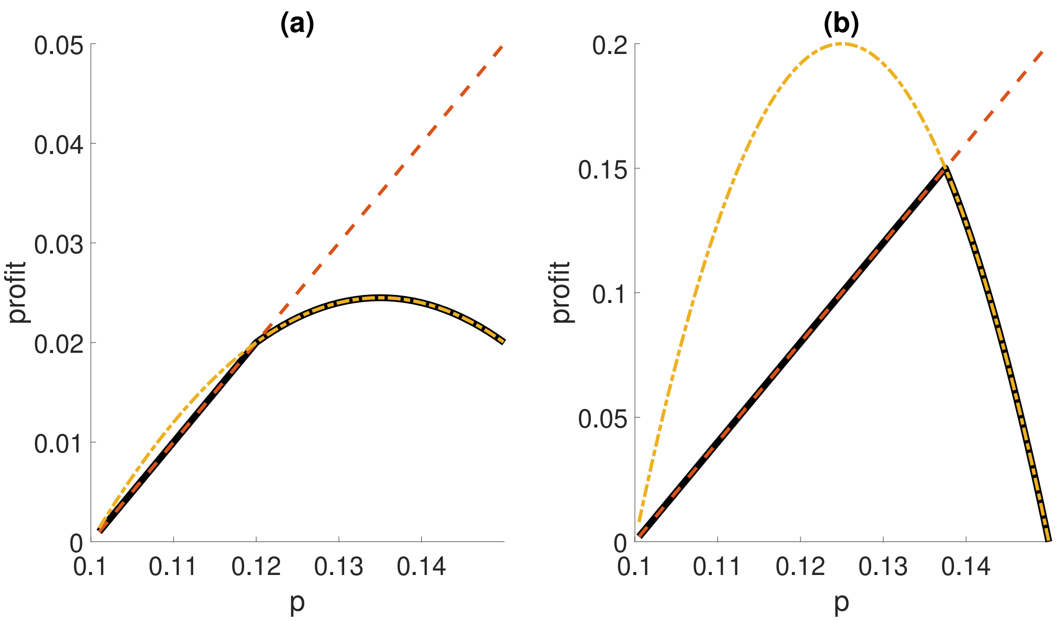

Figure 1 **The behavior of profit function Π for different values of p in the baseline setting, where $\Pi_F^B(p) = p - c$ and $\Pi_P^B(p) = (p-c)(\frac{2V-\delta\gamma-2p}{t_s})$**

Source: Authors' own work.

Next, we consider the optimization of θ.

Specifically, when $\theta < \frac{2c+2t_s-2V+\delta\gamma}{2\delta}$, the global maximum point $p^* = p_m = \frac{1}{4}(2c - \delta\gamma + 2\delta\theta + 2V)$, and

$$\Pi_P(\theta) = \Pi_P(p_m) = \frac{-4b\theta^2 t_s + 4c^2 - 4c(\delta(2\theta-\gamma) + 2V) + \delta^2(2\theta-\gamma)^2 + 4\delta V(2\theta-\gamma) + 4V^2}{8t_s} - F_k,$$

Under Assumption 1 (i.e., $b \ge \frac{\delta^2}{t_s}$), we obtain that $\Pi_P(\theta)$ is concave with the global maximum point $\theta_P^* = \frac{\delta(2V-2c-\delta\gamma)}{2(b\cdot t_s - \delta^2)}$.

When $\theta \ge \frac{2c+2t_s-2V+\delta\gamma}{2\delta}$, the global maximum point $p^* = p^\dagger = V - \frac{1}{2}t_s + \delta\frac{-\gamma+2\theta}{2}$, then we can get the profit function:

$$\Pi_F(\theta) = \Pi_F(p^\dagger) = V - \frac{1}{2}t_s + \delta\frac{-\gamma+2\theta}{2} - c - \frac{b\theta^2}{2} - F_k$$

$\Pi_F(\theta)$ is concave with θ. Then we can get the maximum point $\theta_F^* = \frac{\delta}{b}$.

In this case, the structural property of the profit function depends on the interior solution $\theta_P^* = \frac{\delta(2V-2c-\delta\gamma)}{2(b\cdot t_s-\delta^2)}$, $\theta_F^* = \frac{\delta}{b}$ and $\theta^\dagger = \frac{2c+2t_s-2V+\delta\gamma}{2\delta}$. We discuss as follows.

(a) when $V > c + t_s + \frac{\delta\gamma}{2}$, $\theta^\dagger < 0$, $\Pi(\theta) = \Pi_F(\theta)$ is concave for $\theta \in [0,1]$ with the global maximum point θ_F^*.

(b) when $c+t_s+\frac{\delta\gamma}{2}-\delta \le V \le c+t_s+\frac{\delta\gamma}{2}$,

(b_1) when $c+t_s+\frac{\delta\gamma}{2}-\frac{\delta^2}{b} \le V \le c+t_s+\frac{\delta\gamma}{2}$, $\theta^\dagger \le \theta_F^* \le \theta_P^*$, the profit function is quasi-concave with the global maximum point θ_F^*(see Figure fig:main$_profit(a))(\theta_F^*=\frac{\delta}{b}\in[0,1])$

(b_2) when $c+t_s+\frac{\delta\gamma}{2}-\delta \le V < c+t_s+\frac{\delta\gamma}{2}-\frac{\delta^2}{b}$, $\theta_P^* < \theta_F^* < \theta^\dagger$, the profit function is quasi-concave with the global maximum point θ_P^*(see Figure fig:main$_profit(b))(\theta_F^*=\frac{\delta}{b}\in[0,1])$

(c) when $V < c+t_s+\frac{\delta\gamma}{2}-\delta$, $\theta^\dagger > 1$, $\Pi(\theta)=\Pi_P(\theta)$ is concave for $\theta\in[0,1]$ with the global maximum point θ_P^*.

Therefore, we can conclude that:

(a) when $V \ge c+t_s+\frac{\delta\gamma}{2}-\frac{\delta^2}{b}$, the the profit function is quasi-concave with the global maximum point θ_F^*.

(b) when $V < c+t_s+\frac{\delta\gamma}{2}-\frac{\delta^2}{b}$, the the profit function is quasi-concave with the global maximum point θ_P^*.

□

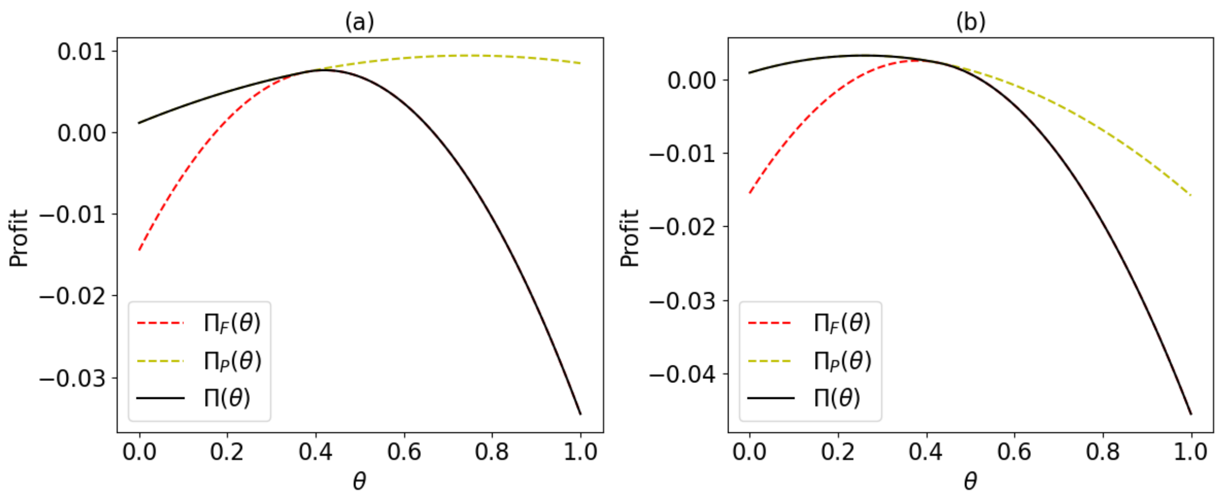

Figure 2 **The behavior of profit function Π for different values of θ in the main setting, where $\Pi_F(\theta)=V-\frac{1}{2}t_s+\delta\frac{-\gamma+2\theta}{2}-c-\frac{b\theta^2}{2}-F_k$ and $\Pi_P(\theta)=\frac{-4b\theta^2t_s+4c^2-4c(\delta(2\theta-\gamma)+2V)+\delta^2(2\theta-\gamma)^2+4\delta V(2\theta-\gamma)+4V^2}{8t_s}-F_k$**

Source: Authors' own work. Source: Authors' own work.

.4. Proof of Proposition 3

Part I: We can firstly get the customer surplus CS_P^B* and CS_F^B* of the baseline model.

From the result shown in Proposition 1, the discussion of customer surplus is as follows:

(a) when $V < c+t_s+\frac{\delta\gamma}{2}$, the maximum price $p^*=\frac{c+V}{2}-\frac{\delta\gamma}{4}$. In this case, customers located at $[0,x_1^0]$ will choose restaurant 1, and those located at $[x_2^0,1]$ will choose restaurant 2. We can get from the proof of Lemma 1, $x_1^0=\frac{V-p^*-\delta(1-\tau_1)}{t_s}, x_2^0=1-\frac{V-p^*-\delta(1-\tau_2)}{t_s}$. Therefore, the customer surplus:

$$\begin{aligned} CS_p &= \int_0^{x_1^0} U_1(x)\,dx + \int_{x_2^0}^1 U_2(x)\,dx \\ &= \frac{4c^2-4c(-\delta\gamma+2V)+\delta^2(5\tau_1^2+5\tau_2^2-4\tau_1-4\tau_2-6\tau_1\tau_2+4)+4V^2-4\delta V\gamma}{16t_s} \end{aligned}$$

(b) when $V \geq c + t_s + \frac{\delta\gamma}{2}$, the maximum price $p^* = \frac{1}{2}(-\delta\gamma - t_s + 2V)$. In this case, customers located at $[0, x_*]$ will choose restaurant 1, and those located at $[x_*, 1]$ will choose restaurant 2. We can get from the proof of Lemma 1, $x_* = \frac{1}{2} + \frac{\delta(\tau_1 - \tau_2)}{2t_s}$. Therefore, the customer surplus:

$$CS_f = \int_0^{x_*} U_1(\mathrm{x}) + \int_{x_*}^1 U_2(\mathrm{x}) = \frac{\delta^2(\tau_1 - \tau_2)^2 + t_s^2}{4t_s} \tag{4}$$

In order to compare CS_p and CS_f, we define $\Delta_{CS}(V) = CS_p - CS_f$:

$$\Delta_{CS}(V) = \frac{((V-c) - \frac{\delta\gamma}{2})^2 - (t_s)^2}{4t_s}$$

We find it when $V < c + t_s + \frac{\delta\gamma}{2}$, $CS = CS_p$, $\Delta_{CS} < 0$, which means $CS_p < CS_f$. When $V \geq c + t_s + \frac{\delta\gamma}{2}$, $CS = CS_f$, $\Delta_{CS} \geq 0$, which means $CS_p \geq CS_f$. □

Part II: Next we can get the customer surplus CS_P^M* and CS_F^M* of the main model.

(a) when $\delta \in [\frac{b\gamma - \sqrt{b^2\gamma^2 + 16b(c + t_s - V)}}{4}, \frac{b\gamma + \sqrt{b^2\gamma^2 + 16b(c + t_s - V)}}{4})$ will choose restaurant 1, and those located at $[x_2^0, 1]$ will choose restaurant 2. We can get from the proof of Lemma 1, $x_1^0 = \frac{V - p^* - \delta(1 - \tau_1 - \theta^*)}{t_s}, x_2^0 = 1 - \frac{V - p^* - \delta(1 - \tau_2 - \theta^*)}{t_s}$. Therefore, the customer surplus:

$$\begin{aligned} CS_P^M &= \int_0^{x_1} U_1(\mathrm{x}) + \int_{x_2}^1 U_2(\mathrm{x}) \\ &= \frac{b^2 t_s^2 (4c^2 + 4c\gamma + 2V) + \delta^2(-6\tau_1\tau_2 + 5(\tau_1^2 + \tau_2^2) - 4(\tau_1 + \tau_2) + 4) + 4V^2 - 4\delta V\gamma)}{16t_s(\delta^2 - bt_s)^2} \\ &\quad \frac{-8b\delta^4 t_s(\tau_1 - \tau_2)^2 + 4\delta^6(\tau_1 - \tau_2)^2}{16t_s(\delta^2 - bt_s)^2} \end{aligned}$$

(b) when $\delta \in [0, \frac{b\gamma - \sqrt{b^2\gamma^2 + 16b(c + t_s - V)}}{4}) \cup (\frac{b\gamma + \sqrt{b^2\gamma^2 + 16b(c + t_s - V)}}{4}, \infty)$ will choose restaurant 1, and those located at $[x_*, 1]$ will choose restaurant 2. We can get from the proof of Lemma 1, $x_* = \frac{1}{2} + \frac{\delta(\tau_1 - \tau_2)}{2t_s}$. Therefore, the customer surplus:

$$CS_F^M = \int_0^{x_*} U_1(\mathrm{x}) + \int_{x_*}^1 U_2(\mathrm{x}) = \frac{\delta^2(\tau_1 - \tau_2)^2 + t_s^2}{4t_s} \tag{5}$$

We start to discuss the central kitchen adoption choices as follows.

From the result shown in Proposition 1 and Proposition 2, the discussion of the comparison of the profits is as follows(we can prove $\frac{2(V-c-t_s)}{\gamma} \leq \frac{b\gamma - \sqrt{b^2\gamma^2 + 16b(c + t_s - V)}}{4}$ by demonstration):

(a) when $\delta \in [0, \frac{2(V-c-t_s)}{\gamma})$, the profits of the main model with a central kitchen π_F^M* and the baseline model without a central kitchen π_F^B* are, respectively:

$$\pi_F^B* = \frac{1}{2}(-\delta\gamma - t_s + 2V) - c$$

$$\pi_F^M* = \frac{1}{2}(-\delta\gamma - t_s + 2V) - c + \frac{\delta^2}{2b} - F_k$$

Therefore when $F_k \leq \frac{\delta^2}{2b}$, $\pi_F^B* \leq \pi_F^M*$. Otherwise, $\pi_F^B* > \pi_F^M*$.

Meanwhile, we can get that the consumer surplus of the main model with a central kitchen CS_F^M* and the baseline model without a central kitchen CS_F^B* are, respectively:

$$CS_F^B* = \frac{\delta^2(\tau_1-\tau_2)^2+t_s^2}{4t_s}$$

$$CS_F^M* = \frac{\delta^2(\tau_1-\tau_2)^2+t_s^2}{4t_s}$$

Therefore $CS_F^B* = CS_F^M*$.

(b) when $\delta \in [\frac{b\gamma-\sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4}, \frac{b\gamma+\sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4})$, the profits of the main model with a central kitchen π_P^M* and the baseline model without a central kitchen π_P^B* are, respectively:

$$\pi_P^B* = \frac{(-2c-\delta\gamma+2V)^2}{8t_s}$$

,

$$\pi_P^M* = \frac{b(-2c-\delta\gamma+2V)^2}{8bt_s-8\delta^2} - F_k = \frac{(-2c-\delta\gamma+2V)^2}{8t_s-8\delta^2/b} - F_k$$

Therefore when $F_k \le \frac{(-2c-\delta\gamma+2V)^2}{8t_s-8\delta^2/b} - \frac{(-2c-\delta\gamma+2V)^2}{8t_s}$, $\pi_P^B* \le \pi_P^M*$. Otherwise, $\pi_P^B* > \pi_P^M*$.

We can get that the consumer surplus of the main model with a central kitchen CS_P^M* and the baseline model without a central kitchen CS_P^B* are, respectively:

$$CS_P^B* = \frac{4c^2-4c(\delta\gamma+2V)+\delta^2(5\tau_1^2+5\tau_2^2-4\tau_1-4\tau_2-6\tau_1\tau_2+4)+4V^2-4\delta V\gamma}{16t_s}$$

$$CS_P^M* = \frac{b^2t_s^2\left(4c^2+4c\gamma+2V\right)+\delta^2\left(-6\tau_1\tau_2+5(\tau_1^2+\tau_2^2)-4(\tau_1+\tau_2)+4\right)+4V^2-4\delta V\gamma)}{16t_s\left(\delta^2-bt_s\right)^2}$$
$$\frac{-8b\delta^4t_s(\tau_1-\tau_2)^2+4\delta^6(\tau_1-\tau_2)^2}{16t_s\left(\delta^2-bt_s\right)^2}$$

,

We define $\Delta CS_b = CS_P^B* - CS_P^M*$,

$$\Delta CS_b = \frac{(4c\gamma\delta-8cV-4V\delta\gamma)(\delta^2-\mathrm{bt_s})^2+b^2{t_s}^2(-4c\gamma-2V+4V\delta\gamma)+\delta^2\left(\delta^2-2\mathrm{bt_s}\right)\left(\delta^2\gamma^2+4V^2+4c^2\right)}{16\mathrm{ts}(\delta^2-\mathrm{bt_s})^2}$$

From the expression of ΔCS_b, we can get that when $\delta^2 \le 2bt_s$ and $(4c\gamma\delta-8cV-4V\delta\gamma) \le 0$ and $(-4c\gamma-2V+4V\delta\gamma) \le 0$, $CS_P^B* \le CS_P^M*$. Otherwise, $CS_P^B* > CS_P^M*$.

(c) when $\delta \in [\frac{2(V-c-t_s)}{\gamma}, \frac{b\gamma+\sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4}) \cup [\frac{b\gamma+\sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4}, \infty]$, the profits of the main model with a central kitchen π_F^M* and the baseline model without a central kitchen π_P^B* are, respectively:

$$\pi_P^B* = \frac{(-2c-\delta\gamma+2V)^2}{8t_s}$$

,

$$\pi_F^M* = \frac{1}{2}\left(-\delta\gamma-t_s+2V\right)-c+\frac{\delta^2}{2b}-F_k$$

Therefore when $F_k \le \frac{1}{2}\left(-\delta\gamma-t_s+2V\right)-c+\frac{\delta^2}{2b}-\frac{(-2c-\delta\gamma+2V)^2}{8t_s}$, $\pi_P^B* \le \pi_F^M*$. Otherwise, $\pi_P^B* > \pi_F^M*$.

Meanwhile, we can get that the consumer surplus of the main model with a central kitchen CS_F^M* and the baseline model without a central kitchen CS_P^B* are, respectively:

$$CS_P^B* = \frac{4c^2 - 4c(-\delta\gamma + 2V) + \delta^2(5\tau_1^2 + 5\tau_2^2 - 4\tau_1 - 4\tau_2 - 6\tau_1\tau_2 + 4) + 4V^2 - 4\delta V\gamma}{16t_s}$$

,

$$CS_F^M* = \frac{\delta^2(\tau_1 - \tau_2)^2 + t_s^2}{4t_s}$$

We define $\Delta CS_c = CS_P^B * - CS_F^M *$,

$$\Delta CS_c = \frac{((V-c) - \frac{\delta\gamma}{2})^2 - (t_s)^2}{4t_s}$$

Therefore, we can conclude that, when $\delta \geq \frac{2(V-c-t_s)}{\gamma}$, $CS_P^B* \leq CS_F^M*$.

Next, we determine the continuity of the graph by substituting the boundary points of the piecewise function.

$$\kappa(\delta) = \begin{cases} \frac{\delta^2}{2b}(\kappa_a(\delta)), & \text{if } \delta \in [0, \frac{2(V-c-t_s)}{\gamma}), \\ \frac{(-2c-\delta\gamma+2V)^2}{8t_s - \frac{8\delta^2}{b}} - \frac{(-2c-\delta\gamma+2V)^2}{8t_s}(\kappa_b(\delta)), & \text{if } \delta \in \left[\frac{b\gamma - \sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4}, \frac{b\gamma + \sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4}\right), \\ \frac{1}{2}(-\delta\gamma - t_s + 2V) - c + \frac{\delta^2}{2b} - \frac{(-2c-\delta\gamma+2V)^2}{8t_s}(\kappa_c(\delta)), & \\ \text{if } \delta \in \left[\frac{2(V-c-t_s)}{\gamma}, \frac{b\gamma + \sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4}\right) & \\ \quad \cup \left[\frac{b\gamma + \sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4}, \infty\right). & \end{cases}$$

(a)We substitute $\delta_a = \frac{2(V-c-t_s)}{\gamma}$ into $\kappa_a(\delta)$ and $\kappa_c(\delta)$ and get:

$$\kappa_a(\delta_a) = \kappa_c(\delta_a) = \frac{2(c + \mathrm{ts} - V)^2}{b\gamma^2}$$

.

(b)We substitute $\delta_b = \frac{b\gamma - \sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4}$ into $\kappa_b(\delta)$ and $\kappa_c(\delta)$ and get:

$$\begin{aligned} &\kappa_b(\delta_b) = \kappa_c(\delta_b) \\ &= -\frac{\left(-\gamma\sqrt{b\left(b\gamma^2+16c+16t_s-16V\right)} + b\gamma^2 + 8c - 8V\right)\left(-\gamma\sqrt{b\left(b\gamma^2+16c+16t_s-16V\right)} + b\gamma^2 + 8c + 8t_s - 8V\right)}{128t_s} \end{aligned}$$

(c)We substitute $\delta_c = \frac{b\gamma + \sqrt{b^2\gamma^2+16b(c+t_s-V)}}{4}$ into $\kappa_b(\delta)$ and $\kappa_c(\delta)$ and get:

$$\begin{aligned} &\kappa_b(\delta_c) = \kappa_c(\delta_c) \\ &= -\frac{\left(\gamma\sqrt{b\left(b\gamma^2+16c+16t_s-16V\right)} + b\gamma^2 + 8c - 8V\right)\left(\gamma\sqrt{b\left(b\gamma^2+16c+16t_s-16V\right)} + b\gamma^2 + 8c + 8t_s - 8V\right)}{128t_s} \end{aligned}$$

□

.5. Proof of Lemma 2

Apparently, a customer would choose restaurant i, rather than restaurant $j \neq i$, only when his/her utility $U_i(x)$ is non-negative and no less than the utility $U_j(x)$ of choosing restaurant (i.e., $U_i(x) \geq 0, U_i(x) \geq U_j(x)$). By solving $U_1 = U_2$, we find a customer is indifferent between two restaurants at x_* as follows:

$$x_* = \frac{1}{2} + \frac{\delta(\lambda_1 - \lambda_2)}{2(\beta + t_s)}.$$

Moreover, by solving $U_i = 0$, we find the customer at x_i^0 has zero utility, where x_i^0 is as follows:

$$x_1^0 = \frac{V - p - \delta(1 - \lambda_1)}{\beta + t_s}, x_2^0 = 1 - \frac{V - p - \delta(1 - \lambda_2)}{\beta + t_s}. \tag{6}$$

Next, we discuss the customer demands for two restaurants as follows.

(a) When $x_1 \geq x_*$ and $x_2 \leq x_*$, which is equivalent to $p \leq V - \frac{\beta + t_s}{2} + \frac{\delta(-\gamma + 2\theta)}{2}$, the customers located at $[0, x_*]$ choose restaurant 1, and those located at $[x_*, 1]$ choose restaurant 2. In this case, the demand functions are given by:

$$D_1(p, \theta) = x_* = \frac{1}{2} + \frac{\delta(\tau_1 - \tau_2)}{2(\beta + t_s)}, D_2(p, \theta) = 1 - x_* = \frac{1}{2} - \frac{\delta(\tau_1 - \tau_2)}{2(\beta + t_s)}, \tag{7}$$

which satisfies $D_1(p) + D_2(p) = 1$.

(b) When $x_1^0 < x_* < x_2^0$, which is equivalent to $p > V - \frac{\beta + t_s}{2} + \frac{\delta(-\gamma + 2\theta)}{2}$, customers located at $[0, x_1^0]$ will choose restaurant 1, and those located at $[x_2^0, 1]$ will choose restaurant 2. In other words, the demand functions are given by:

$$D_1(p, \theta) = x_1^0 = \frac{V - p - \delta(1 - \tau_1 - \theta)}{\beta + t_s}, D_2(p, \theta) = 1 - x_2^0 = \frac{V - p - \delta(1 - \tau_2 - \theta)}{\beta + t_s}, \tag{8}$$

which satisfies $D_1(p) + D_2(p) < 1$.

□

.6. Proof of Proposition 4

We first fix θ and maximize the objective function by optimizing p. From the demand function shown in (30), we find that the profit function becomes $\Pi_F^E(p) = (p - c) - b\frac{\theta^2}{2} - F_K$, which is increasing in p, when $p \leq V - \frac{\beta + t_s}{2} + \delta\frac{-\gamma + 2\theta}{2}$, and the profit function becomes $\Pi_P^E(p) = (p - c)(\frac{\delta(-\gamma + 2\theta) - 2(p - V)}{(\beta + t_s)}) - b\frac{\theta^2}{2} - F_k$, which is concave in p, when $p > V - \frac{\beta + t_s}{2} + \delta\frac{-\gamma + 2\theta}{2}$.

In this case, the structural property of the profit function depends on the interior solution $p_e = \frac{1}{4}(2V + 2c - \delta\gamma + 2\delta\theta) = \arg\max_p \Pi_P^E(p)$, and $p^\dagger = V - \frac{\beta + t_s}{2} + \delta\frac{-\gamma + 2\theta}{2}$. We discuss as follows.

(a) when $p_e > p^\dagger$, which is equivalent to $V < \frac{1}{2}(2c + 2\beta + 2t_s + \delta\gamma - 2\delta\theta)$, the profit function is quasi-concave with the global maximum point p_e (see Figure $\mathrm{E}_{\mathbb{Q}}[fig: bs_p rofit](a)$).

(b) when $p_e \leq p^\dagger$, which is equivalent to $V \geq \frac{1}{2}(2c + 2\beta + 2t_s + \delta\gamma - 2\delta\theta)$, the profit function is concave with the global maximum price $p^\dagger$ (see Figure $\mathrm{E}_{\mathbb{Q}}[fig: bs_p rofit](b)$).

Next, we consider the optimization of θ.

Specifically, when $\theta < \frac{2c + 2t_s + 2\beta - 2V - \delta\gamma}{2\delta}$, the global maximum point $p^* = p_e = \frac{1}{4}(2c - \delta\gamma + 2\delta\theta + 2V)$, and

$$\Pi_P^E(\theta) = \Pi_P^E(p_e) = \frac{-4b\theta^2(\beta + t_s) + 4c^2 - 4c(\delta(-\gamma + 2\theta) + 2V) + \delta^2(-\gamma + 2\theta)^2 + 4\delta V(-\gamma + 2\theta) + 4V^2}{8(\beta + t_s)}$$

Under Assumption 2 (i.e., $b \geq \frac{\delta^2}{\beta+t_s}$), we obtain that $\Pi_P^E(\theta)$ is concave with the global maximum point $\theta_P^E* = \frac{\delta(2V-2c-\delta\gamma)}{2(b\cdot t_s+b\beta-\delta^2)}$.

When $\theta \geq \frac{2c+2t_s+2\beta-2V+\delta\gamma}{2\delta}$, the global maximum point $p^* = p^\dagger = V - \frac{1}{2}(\beta+t_s) + \delta\frac{-\gamma+2\theta}{2}$, then we can get the profit function:

$$\Pi_F^E(\theta) = \Pi_F^E(p^\dagger) = V - \frac{1}{2}(\beta+t_s) + \delta\frac{-\gamma+2\theta}{2} - c - \frac{b\theta^2}{2}$$

$\Pi_F^E(\theta)$ is concave with θ. Then we can get the maximum point $\theta_F^E* = \frac{\delta}{b}$.

In this case, the structural property of the profit function depends on the interior solution $\theta_P^E* = \frac{\delta(2V-2c-\delta\gamma)}{2(b\cdot t_s+b\beta-\delta^2)}$, $\theta_F^E* = \frac{\delta}{b}$ and $\theta^\dagger = \frac{2c+2t_s+2\beta-2V+\delta\gamma}{2\delta}$. We discuss as follows.

(a) when $V > c+t_s+\beta+\frac{\delta\gamma}{2}$, $\theta^\dagger < 0$, $\Pi^E(\theta) = \Pi_F^E(\theta)$ is concave for $\theta \in [0,1]$ with the global maximum point θ_F^*.

(b) when $c+t_s+\beta+\frac{\delta\gamma}{2} - \delta \leq V \leq c+t_s+\beta+\frac{\delta\gamma}{2}$,

(b_1) when $c+t_s+\beta+\frac{\delta\gamma}{2} - \frac{\delta^2}{b} \leq V \leq c+t_s+\beta+\frac{\delta\gamma}{2}$, $\theta^\dagger \leq \theta_F^* \leq \theta_P^*$, the profit function is quasi-concave with the global maximum point θ_F^*(see Figure $\mathrm{E}_{\mathbb{Q}}\,[fig:main_profit]\,(a)$)($\theta_F^* = \frac{\delta}{b} \in [0,1]$)

(b_2) when $c+t_s+\beta+\frac{\delta\gamma}{2} - \delta \leq V < c+t_s+\beta+\frac{\delta\gamma}{2} - \frac{\delta^2}{b}$, $\theta_P^* < \theta_F^* < \theta^\dagger$, the profit function is quasi-concave with the global maximum point θ_P^*(see Figure $\mathrm{E}_{\mathbb{Q}}\,[fig:main_profit]\,(b)$)($\theta_F^* = \frac{\delta}{b} \in [0,1]$)

(c) when $V < c+t_s+\beta+\frac{\delta\gamma}{2} - \delta$, $\theta^\dagger > 1$, $\Pi^E(\theta) = \Pi_P^E(\theta)$ is concave for $\theta \in [0,1]$ with the global maximum point θ_P^*.

Therefore, we can conclude that:

(a) when $V \geq c+t_s+\beta+\frac{\delta\gamma}{2} - \frac{\delta^2}{b}$, the the profit function is quasi-concave with the global maximum point θ_F^*.

(b) when $V < c+t_s+\beta+\frac{\delta\gamma}{2} - \frac{\delta^2}{b}$, the the profit function is quasi-concave with the global maximum point θ_P^*.

□

.7. Proof of Proposition 5

First of all, we talk about some results based on the baseline model with congestion effect:

(a) when $\delta > \frac{2(V-c-t_s-\beta)}{\gamma}$, $\Pi^B(p)$ is quasi-concave with maximum price:

$$p^* = \frac{c+V}{2} - \frac{\delta\gamma}{4}, \pi^* = \frac{(-2c-\delta\gamma+2V)^2}{8(\beta+t_s)}$$

$$\begin{aligned} CS_p &= \int_0^{x_1^0} U_1(x)\,dx + \int_{x_2^0}^1 U_2(x)\,dx \\ &= \frac{4c^2 - 4c(-\delta\gamma+2V) + \delta^2(5\tau_1^2+5\tau_2^2-4\tau_1-4\tau_2-6\tau_1\tau_2+4) + 4V^2 - 4\delta V\gamma}{16(\beta+t_s)} \end{aligned}$$

which leads to partial market coverage.

(b) when $\delta \leq \frac{2(V-c-t_s-\beta)}{\gamma}$, $\Pi^B(p)$ is concave with maximum price:

$$p^* = \frac{1}{2}\left(-\delta\gamma - \beta - t_s + 2V\right), \pi^* = \frac{1}{2}\left(-\delta\gamma - \beta - t_s + 2V\right) - c$$

$$CS_f = \int_0^{x^*} U_1(x)\,dx + \int_{x^*}^1 U_2(x)\,dx = \frac{(\beta+t_s)^2+\delta^2(\tau_1-\tau_2)^2}{4(\beta+t_s)}$$

which leads to full market coverage.

The proof of the results of the baseline model with consumer congestion is similar to supplementary material 2.

Then we talk about the results based on the main model with congestion effect:

(a) when $\delta \in [\frac{b\gamma-\sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4}, \frac{b\gamma+\sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4})$, the operator chooses a partial coverage strategy, and the optimal decisions and profit are as follows:

$$\begin{aligned}
\theta_P^E * &= \frac{\delta\,(2V-2c-\delta\gamma)}{2\,(b\cdot t_s+b\beta-\delta^2)},\\
p_P^E * &= \frac{b(\beta+t_s)(2c-\delta\gamma+2V)-4c\delta^2}{4b(\beta+t_s)-4\delta^2}\\
\pi_P^E * &= \frac{b(-2c-\delta\gamma+2V)^2}{8b(\beta+t_s)-8\delta^2}-F_k
\end{aligned} \tag{9}$$

$$\begin{aligned}
CS_p = &\frac{\left(b(\beta+t_s)(2c+\delta(\tau_1-3\tau_2+2)-2V)+2\delta^3(\tau_2-\tau_1)\right)^2}{32(\beta+t_s)\left(\delta^2-b(\beta+t_s)\right)^2}\\
&+\frac{\left(b(\beta+t_s)(2c+\delta(-3\tau_1+\tau_2+2)-2V)+2\delta^3(\tau_1-\tau_2)\right)^2}{32(\beta+t_s)\left(\delta^2-b(\beta+t_s)\right)^2}
\end{aligned}$$

(b) when $\delta \in [0, \frac{b\gamma-\sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4}) \cup (\frac{b\gamma+\sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4}, \infty)$, the operator chooses a full coverage strategy, and the optimal decisions and profit are as follows:

$$\begin{aligned}
\theta_F^E * &= \frac{\delta}{b},\\
p_F^E * &= \frac{1}{2}\left(-\delta\gamma+2\delta^2/b-\beta-t_s+2V\right),\\
\pi_F^E * &= \frac{1}{2}\left(-\delta\gamma+\delta^2/b-\beta-t_s+2V\right)-c-F_k
\end{aligned} \tag{10}$$

$$CS_f = \frac{\delta^2(\tau_1-\tau_2)^2+(\beta+t_s)^2}{4(t_s+\beta)}$$

The proof of the results of the main model with consumer congestion is similar to supplemantary material 3 and 4.

Then the discussion of the comparison of the profits is as follows(we can prove $\frac{2(V-c-t_s-\beta)}{\gamma} \leq \frac{b\gamma-\sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4}$ by demonstration):

(a) when $\delta \in [0, \frac{2(V-c-t_s-\beta)}{\gamma})$, the profits of the main model with a central kitchen $\pi_F^M *$ and the baseline model without a central kitchen $\pi_F^B *$ are, respectively:

$$\pi_F^B * = \frac{1}{2}\left(-\delta\gamma-\beta-t_s+2V\right)-c$$

$$\pi_F^M * = \frac{1}{2}\left(-\delta\gamma-\beta-t_s+2V\right)-c+\frac{\delta^2}{2b}-F_k$$

Therefore when $F_k \leq \frac{\delta^2}{2b}$, $\pi_F^B * \leq \pi_F^M *$. Otherwise, $\pi_F^B * > \pi_F^M *$.

Meanwhile, we can get that the consumer surplus of the main model with a central kitchen CS_F^M* and the baseline model without a central kitchen CS_F^B* are, respectively:

$$CS_F^B* = \frac{\delta^2(\tau_1-\tau_2)^2+t_s^2}{4t_s}$$

$$CS_F^M* = \frac{\delta^2(\tau_1-\tau_2)^2+t_s^2}{4t_s}$$

Therefore $CS_F^B* = CS_F^M*$.

(b) when $\delta \in [\frac{b\gamma-\sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4}, \frac{b\gamma+\sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4})$, the profits of the main model with a central kitchen π_P^M* and the baseline model without a central kitchen π_P^B* are, respectively:

$$\pi_P^B* = \frac{(-2c-\delta\gamma+2V)^2}{8(t_s+\beta)}$$

,

$$\pi_P^M* = \frac{b(-2c-\delta\gamma+2V)^2}{8b(t_s+\beta)-8\delta^2} - F_k = \frac{(-2c-\delta\gamma+2V)^2}{8(t_s+\beta)-8\delta^2/b} - F_k$$

Therefore when $F_k \leq \frac{(-2c-\delta\gamma+2V)^2}{8(\beta+t_s)-8\delta^2/b} - \frac{(-2c-\delta\gamma+2V)^2}{8(t_s+\beta)}$, $\pi_P^B* \leq \pi_P^M*$. Otherwise, $\pi_P^B* > \pi_P^M*$.

We can get that the consumer surplus of the main model with a central kitchen CS_P^M* and the baseline model without a central kitchen CS_P^B* are, respectively:

$$CS_P^B* = \frac{4c^2-4c(-\delta\gamma+2V)+\delta^2(5\tau_1^2+5\tau_2^2-4\tau_1-4\tau_2-6\tau_1\tau_2+4)+4V^2-4\delta V\gamma}{16(\beta+t_s)}$$

$$CS_p = \frac{(b(\beta+t_s)(2c+\delta(\tau_1-3\tau_2+2)-2V)+2\delta^3(\tau_2-\tau_1))^2}{32(\beta+t_s)\left(\delta^2-b(\beta+t_s)\right)^2} + \frac{(b(\beta+t_s)(2c+\delta(-3\tau_1+\tau_2+2)-2V)+2\delta^3(\tau_1-\tau_2))^2}{32(\beta+t_s)\left(\delta^2-b(\beta+t_s)\right)^2}$$

,

We define $\Delta CS_b = CS_P^B* - CS_P^M*$,

$$\Delta CS_b = \frac{(4c\gamma\delta-8cV-4V\delta\gamma)(\delta^2-\mathrm{bt_s}-b\beta)^2+b^2(t_s+\beta)^2(-4c\gamma-2V+4V\delta\gamma)+\delta^2\left(\delta^2-2b(t_s+\beta)\right)\right)\left(\delta^2\gamma^2+4V^2+4c^2\right)}{16(t_s+\beta))(\delta^2-\mathrm{bt_s}-b\beta)^2}$$

From the expression of ΔCS_b, we can get that when $\delta^2 \leq 2bt_s+2b\beta$ and $(4c\gamma\delta-8cV-4V\delta\gamma) \leq 0$ and $(-4c\gamma-2V+4V\delta\gamma) \leq 0$, $CS_P^B* \leq CS_P^M*$. Otherwise, $CS_P^B* > CS_P^M*$.

(c) when $\delta \in [\frac{2(V-c-t_s-\beta)}{\gamma}, \frac{b\gamma+\sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4}) \cup [\frac{b\gamma+\sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4}, \infty]$, the profits of the main model with a central kitchen π_F^M* and the baseline model without a central kitchen π_P^B* are, respectively:

$$\pi_P^B* = \frac{(-2c-\delta\gamma+2V)^2}{8(t_s+\beta)}$$

,

$$\pi_F^M* = \frac{1}{2}\left(-\delta\gamma-t_s-\beta+2V\right)-c+\frac{\delta^2}{2b}-F_k$$

Therefore when $F_k \leq \frac{1}{2}\left(-\delta\gamma-t_s-\beta+2V\right)-c+\frac{\delta^2}{2b}-\frac{(-2c-\delta\gamma+2V)^2}{8(t_s+\beta)}$, $\pi_P^B* \leq \pi_F^M*$. Otherwise, $\pi_P^B* > \pi_F^M*$.

Meanwhile, we can get that the consumer surplus of the main model with a central kitchen CS_F^M* and the baseline model without a central kitchen CS_P^B* are, respectively:

$$CS_P^B* = \frac{4c^2 - 4c(-\delta\gamma + 2V) + \delta^2(5\tau_1^2 + 5\tau_2^2 - 4\tau_1 - 4\tau_2 - 6\tau_1\tau_2 + 4) + 4V^2 - 4\delta V\gamma}{16(\beta + t_s)}$$

,

$$CS_F^M* = \frac{\delta^2(\tau_1 - \tau_2)^2 + (\beta + t_s)^2}{4(\beta + t_s)}$$

We define $\Delta CS_c = CS_P^B * - CS_F^M *$,

$$\Delta CS_c = \frac{((V-c) - \frac{\delta\gamma}{2})^2 - (t_s + \beta)^2}{4(t_s + \beta)}$$

Therefore, we can conclude that, when $\delta \geq \frac{2(V-c-t_s-\beta)}{\gamma}$, $CS_P^B* \leq CS_F^M*$.

Next, we determine the continuity of the graph by substituting the boundary points of the piecewise function.

$$\kappa(\delta) = \begin{cases} \frac{\delta^2}{2b}(\kappa_a(\delta)), & \text{if } \delta \in [0, \frac{2(V-c-t_s-\beta)}{\gamma}), \\ \frac{(-2c-\delta\gamma+2V)^2}{8(\beta+t_s)-\frac{8\delta^2}{b}} - \frac{(-2c-\delta\gamma+2V)^2}{8(\beta+t_s)}(\kappa_b(\delta)), & \text{if } \delta \in \left[\frac{b\gamma - \sqrt{b^2\gamma^2 + 16b(c+t_s+\beta-V)}}{4}, \frac{b\gamma + \sqrt{b^2\gamma^2 + 16b(c+t_s+\beta-V}}{4}\right. \\ \frac{1}{2}(-\delta\gamma - t_s - \beta + 2V) - c + \frac{\delta^2}{2b} - \frac{(-2c-\delta\gamma+2V)^2}{8(\beta+t_s)}(\kappa_c(\delta)), & \\ \text{if } \delta \in \left[\frac{2(V-c-t_s-\beta)}{\gamma}, \frac{b\gamma + \sqrt{b^2\gamma^2 + 16b(c+t_s+\beta-V)}}{4}\right) & \\ \quad \cup \left[\frac{b\gamma + \sqrt{b^2\gamma^2 + 16b(c+t_s+\beta-V)}}{4}, \infty\right). & \end{cases}$$

(a)We substitute $\delta_a = \frac{2(V-c-t_s-\beta)}{\gamma}$ into $\kappa_a(\delta)$ and $\kappa_c(\delta)$ and get:

$$\kappa_a(\delta_a) = \kappa_c(\delta_a) = \frac{2(c + t_s + \beta - V)^2}{b\gamma^2}$$

.

(b)We substitute $\delta_b = \frac{b\gamma - \sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4}$ into $\kappa_b(\delta)$ and $\kappa_c(\delta)$ and get:

$$\kappa_b(\delta_b) = \kappa_c(\delta_b)$$
$$= -\frac{\left(\gamma\sqrt{b\left(b\gamma^2 + 16(c+t_s+\beta-V)\right)} + b\gamma^2 + 8c - 8V\right)\left(-\gamma\sqrt{b\left(b\gamma^2 + 16(c+t_s+\beta-V)\right)}) + b\gamma^2 + 8c + 8t_s + 8\beta - 8V\right)}{128(\beta + t_s)}$$

(c)We substitute $\delta_c = \frac{b\gamma + \sqrt{b^2\gamma^2+16b(c+t_s+\beta-V)}}{4}$ into $\kappa_b(\delta)$ and $\kappa_c(\delta)$ and get:

$$\kappa_b(\delta_c) = \kappa_c(\delta_c)$$
$$= -\frac{\left(\gamma\sqrt{b\left(b\gamma^2 + 16(c+t_s+\beta-V)\right)} + b\gamma^2 + 8c - 8V\right)\left(\gamma\sqrt{b\left(b\gamma^2 + 16(c+t_s+\beta-V)\right)} + b\gamma^2 + 8c + 8t_s + 8\beta - 8V\right)}{128(\beta + t_s)}$$

□

.8. Proof of Proposition 6

For baseline model with consumer congestion:

(a)when $\delta \leq \frac{2(V-c-t_s-\beta)}{\gamma}$, the profits of the extension model with the effect of customer congestion π_P^E* is:

$$\pi_P^E* = \frac{b(-2c-\delta\gamma+2V)^2}{8b(\beta+t_s)}$$

$$\frac{\partial\pi^*}{\partial\beta} = -\frac{8(2V-2c+\delta\gamma)^2}{(8\beta+8t_s)^2}.$$

$$\frac{\partial^2\pi^*}{\partial\beta\partial\delta} = \frac{16\gamma(2V-2c-\delta\gamma)}{(8\beta+8t_s)^2}.$$

Therefore we can get $\frac{\partial^2\pi^*}{\partial\beta\partial\delta} \geq 0$, which means congestion effect can circumvent the negative impact of food quality inconsistency.

(b)when $\delta > \frac{2(V-c-t_s-\beta)}{\gamma}$, the profits of the extension model with the effect of customer congestion π_F^E* is:

$$\pi_F^E* = \frac{1}{2}(-\delta\gamma-\beta-t_s+2V)-c$$

$$\frac{\partial\pi_F^E*}{\partial\beta} = -1$$

$$\frac{\partial^2\pi_P^E*}{\partial\beta\partial\delta} = 0$$

Therefore we can get $\frac{\partial^2\Pi^*}{\partial\beta\partial\delta} = 0$, which means the congestion effect and the negative impact of food quality inconsistency are independent.

For main model with consumer congestion: From the result shown in proposition 4, the discussion of the profits of the extension model is as follows:

(a) when $V \leq c+t_s+\beta+\frac{\delta\gamma}{2}-\frac{\delta^2}{b}$, the profits of the extension model with the effect of customer congestion π_P^E* is:

$$\pi_P^E* = \frac{b(-2c-\delta\gamma+2V)^2}{8b(\beta+t_s)-8\delta^2} - F_k$$

$$\frac{\partial\pi_P^E*}{\partial\beta} = -\frac{8b^2(-2c-\delta\gamma+2V)^2}{(8b(\beta+t_s)-8\delta^2)^2} \leq 0$$

$$\frac{\partial^2\pi_P^E*}{\partial\beta\partial\delta} = -\frac{256b^2\delta(2V-2c-\delta\gamma)^2}{(8b(\beta+t_s)-8\delta^2)^3} - \frac{16b^2\gamma(2V-2c-\delta\gamma)}{(8b(\beta+t_s)-8\delta^2)^2}$$

We find that when $\gamma \leq \frac{16\delta(2V-2c)}{8b(\beta+t_s+8\delta^2)}$, $\frac{\partial^2\pi_P^E*}{\partial\beta\partial\delta} \geq 0$, which means ongestion effect can circumvent the negative impact of food quality inconsistency, when $\gamma > \frac{16\delta(2V-2c)}{8b(\beta+t_s+8\delta^2)}$, $\frac{\partial^2\pi_P^E*}{\partial\beta\partial\delta} < 0$, which means congestion effect can intensify the negative impact of food quality inconsistency.

(b) when $V > c+t_s+\beta+\frac{\delta\gamma}{2}-\frac{\delta^2}{b}$, the profits of the extension model with the effect of customer congestion π_F^E* is:

$$\pi_F^E * = \frac{1}{2}\left(-\delta\gamma + \delta^2/b - \beta - t_s + 2V\right) - c - F_K$$

$$\frac{\partial \pi_F^E *}{\partial \beta} = -1$$

$$\frac{\partial^2 \pi_P^E *}{\partial \beta \partial \delta} = 0$$

Therefore, the larger β becomes, the more pronounced the negative impact. Moreover, we get $\frac{\partial^2 \pi_P^E *}{\partial \beta \partial \delta} = 0$, which means the congestion effect and the negative impact of food quality inconsistency are independent.

□

.9. Proof of Proposition 7

Part I:

We first fix θ and maximize the objective function by optimizing p. From the demand function shown in (30), we find that the profit function becomes $\Pi_F^E(p) = (p - c_0 + \alpha\theta) - b\frac{\theta^2}{2} - F_K$, which is increasing in p, when $p \leq V - \frac{t_s}{2} + \delta\frac{-\gamma+2\theta}{2}$, and the profit function becomes $\Pi_P^E(p) = (p - c_0 + \alpha\theta)(\frac{\delta(-\gamma+2\theta)-2(p-V)}{t_s}) - b\frac{\theta^2}{2} - F_k$, which is concave in p, when $p > V - \frac{t_s}{2} + \delta\frac{-\gamma+2\theta}{2}$.

In this case, the structural property of the profit function depends on the interior solution $p_e = \frac{1}{4}(-2\alpha\theta - \gamma\delta + 2c_0 + 2\delta\theta + 2V)$, and $p^\dagger = V - \frac{t_s}{2} + \delta\frac{-\gamma+2\theta}{2}$. We discuss as follows.

(a) when $p_e > p^\dagger$, which is equivalent to $V < \frac{1}{2}(2c + 2\beta + 2t_s + \delta\gamma - 2\delta\theta)$, the profit function is quasi-concave with the global maximum point p_e (see Figure 1 (a)).

(b) when $p_e \leq p^\dagger$, which is equivalent to $V \geq \frac{1}{2}(2c + 2\beta + 2t_s + \delta\gamma - 2\delta\theta)$, the profit function is concave with the global maximum price $p^\dagger$ (see Figure 1 (b)).

Next, we consider the optimization of θ.

Specifically, when $\theta < \frac{2c_0+2t_s-2V+\delta\gamma}{2\delta+2\alpha}$, the global maximum point $p^* = p_e = \frac{1}{4}(2c - \delta\gamma + 2\delta\theta + 2V)$, and

$$\Pi_P^E(\theta) = \Pi_P^E(p_e) = \frac{1}{8}\left(\frac{(2\alpha\theta - \gamma\delta - 2c_0 + 2\delta\theta + 2V)^2}{t_s} - 4b\theta^2\right) - F_k$$

Under Assumption 3 (i.e., $b \geq \frac{(\delta+\alpha)^2}{t_s}$), we obtain that $\Pi_P^E(\theta)$ is concave with the global maximum point $\theta_P^E * = \frac{(\delta+\alpha)(2V-2c_0-\delta\gamma)}{2bt_s-2(\delta+\alpha)^2}$.

When $\theta \geq \frac{2c_0+2t_s-2V+\delta\gamma}{2\delta+2\alpha}$, the global maximum point $p^* = p^\dagger = V - \frac{1}{2}t_s + \delta\frac{-\gamma+2\theta}{2}$, then we can get the profit function:

$$\Pi_F^E(\theta) = \Pi_F^E(p^\dagger) = V - \frac{1}{2}t_s + \delta\frac{-\gamma+2\theta}{2} - c_0 + \alpha\theta - \frac{b\theta^2}{2} - F_k$$

$\Pi_F^E(\theta)$ is concave with θ. Then we can get the maximum point $\theta_F^E * = \frac{\delta+\alpha}{b}$.

In this case, the structural property of the profit function depends on the interior solution $\theta_P^E * = \frac{(\delta+\alpha)(2V-2c_0-\delta\gamma)}{2bt_s-2(\delta+\alpha)^2}$, $\theta_F^E * = \frac{\delta+\alpha}{b}$ and $\theta^\dagger = \frac{2c_0+2t_s-2V+\delta\gamma}{2\delta+2\alpha}$. We discuss as follows.

(a) when $V > c + t_s + \frac{\delta\gamma}{2}$, $\theta^\dagger < 0$, $\Pi^E(\theta) = \Pi_F^E(\theta)$ is concave for $\theta \in [0,1]$ with the global maximum point θ_F^*.

(b) when $c + t_s + \frac{\delta\gamma}{2} - \delta - \alpha \leq V \leq c + t_s + \frac{\delta\gamma}{2}$,

(b_1) when $c+t_s+\frac{\delta\gamma}{2}-\frac{(\delta+\alpha)^2}{b}\le V\le c+t_s+\frac{\delta\gamma}{2}$, $\theta^\dagger\le\theta_F^*\le\theta_P^*$, the profit function is quasi-concave with the global maximum point θ_F^*(see Figure 2 (a))($\theta_F^*=\frac{\delta}{b}\in[0,1]$)

(b_2) when $c+t_s+\frac{\delta\gamma}{2}-\delta-\alpha\le V< c+t_s+\frac{\delta\gamma}{2}-\frac{(\delta+\alpha)^2}{b}$, $\theta_P^*<\theta_F^*<\theta^\dagger$, the profit function is quasi-concave with the global maximum point θ_P^*(see Figure 2 (b))($\theta_F^*=\frac{\delta}{b}\in[0,1]$)

(c) when $V<c+t_s+\frac{\delta\gamma}{2}-\delta-\alpha$, $\theta^\dagger>1$, $\Pi^E(\theta)=\Pi_P^E(\theta)$ is concave for $\theta\in[0,1]$ with the global maximum point θ_P^*.

Therefore, we can conclude that:

(a) when $V\ge c+t_s+\frac{\delta\gamma}{2}-\frac{(\delta+\alpha)^2}{b}$, the the profit function is quasi-concave with the global maximum point θ_F^*.

(b) when $V< c+t_s+\frac{\delta\gamma}{2}-\frac{(\delta+\alpha)^2}{b}$, the the profit function is quasi-concave with the global maximum point θ_P^*.

Part II: After considering the effect of η, from the proof of Proposition Part I , we can get that:

(a)when $\eta\ge\frac{c+t_s+\frac{\delta\gamma}{2}-\frac{(\delta+\alpha)^2}{b}}{V}$

$$\Pi_F(\eta)=\frac{1}{2}\left(-\delta\gamma+(\delta+\alpha)^2/b-t_s+2V\eta\right)-c-F_k(\eta)$$

$\Pi_F(\eta)$ is concave with η. Then we can get the maximum point $\eta_F^*=\frac{V}{k}$.

(b)when $\eta<\frac{c+t_s+\frac{\delta\gamma}{2}-\frac{(\delta+\alpha)^2}{b}}{V}$

$$\Pi_P(\eta)=\frac{b(-2c-\delta\gamma+2V\eta)^2}{8bt_s-8(\delta+\alpha)^2}-F_k(\eta)$$

$\Pi_P(\eta)$ is quasi-concave with η. Then we can get the maximum point $\eta_P^*=\frac{V(2c+\delta\gamma)}{2\left(V^2-kt_s+\frac{k(\delta+\alpha)^2}{b}\right)}$.

In this case, the structural property of the profit function depends on the interior solution $\eta_P^*=\frac{V(2c+\delta\gamma)}{2\left(V^2-kt_s+\frac{k(\delta+\alpha)^2}{b}\right)}$, $\eta_F^*=\frac{V}{k}$ and $\eta^\dagger=\frac{c+t_s+\frac{\delta\gamma}{2}-\frac{(\delta+\alpha)^2}{b}}{V}$. We discuss as follows.

(a) when $k\le\frac{V^2}{c+t_s-\frac{(\delta+\alpha)^2}{b}+\frac{\delta\gamma}{2}}$, $\eta^\dagger\le\eta_P^*\le\eta_F^*$, the profit function is quasi-concave with the global maximum point η_F^*.

(b) when $k>\frac{V^2}{c+t_s-\frac{(\delta+\alpha)^2}{b}+\frac{\delta\gamma}{2}}$, $\eta_F^*<\eta_P^*<\eta^\dagger$, the profit function is quasi-concave with the global maximum point η_P^*.

□