THEMES AND STRATEGIES FOR TRANSFORMATIVE ONLINE INSTRUCTION
A Review of Literature and Practice

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INTRODUCTION

Institutions and instructors who provide distance education are faced with a rapidly expanding literature base for teaching courses online. As new programs and initiatives develop distance learning components, each is faced with the daunting task of sifting the rapidly expanding literature base in search of successful learning environments, instructional task formats, and communication strategies. Some of the existing frameworks and recommendations are based on research, while many others are built simply on the experiences of online educators.

In this article, we address the challenge of providing high-quality online instruction and review the current literature as it relates to six themes that have structured our approaches since 2005. In particular, we focus on the common context of our combined efforts—providing effective instruction in mathematics and professional development for mathematics educators at the secondary and collegiate levels.

Over the past 10 years, four graduate-level education programs have developed an extensive literature base on best practices in computer-mediated distance education in general and for mathematics education in particular. These projects include two National Science Foundation-funded Centers for Learning and Teaching, a mathematics and science project, and a successful master of science program for secondary mathematics teachers. Their combined review of the existing literature regarding online instruction (Mayes,

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Luebeck, Mays, & Niemiec, 2006) resulted in identification of six themes that informed the projects' efforts to provide quality online mathematics and mathematics education coursework. This foundation was recently expanded through another extended literature review in 2010. The process of mining the literature has been informed and enhanced by our own growing knowledge base gained through experimenting with course designs and delivery modes, interviewing successful online faculty, and conducting qualitative analysis of course materials and transcripts.

THEMES FROM THE ONLINE LEARNING LITERATURE

We first present the six themes and their associated indicators in matrix form (Table 1). This is followed by a summary of the literature and its embedded recommendations for each individual theme. We close with a discussion of how these themes and the literature supporting them interact with our own online experiences, which span 10 years of development and implementation and include audiences ranging from middle and high school teachers to doctoral-level teacher educators.

Learners and Instructors

Learners are drawn to online courses for their convenience and flexibility (Sullivan, 2001). Learner expectations for online courses include flexibility of time, place, and pace; affordability; relevance and applicability; competence; reliability and ongoing support; personal choice and personalization; and rapid feedback (Choy, McNickle, & Clayton, 2002; DiPaolo, 2002). In the early days of online instruction, courses were specialized, technology was limited, and students fit a narrow profile. The online student demographic has changed dramatically in recent years, as universities and education centers compete to provide online undergraduate and graduate courses for their students. In particular, the online setting provides a powerful and previously unavailable professional development alternative for teachers who are place-bound due to work, family, finances, or geography. Although the audience for our programs is comprised of adult learners—typically inservice secondary mathematics teachers—these themes and recommendations apply to all online learners.

Parker (2003) identified self-motivation and self-direction as learner characteristics that are essential to online success. Students need to develop "sophisticated abilities in problem-solving, making judgments, searching, analyzing, thinking critically, and collaborating with others" (Cowan, 2006, p. 3). They must move from the traditional classroom role of passive learning into new roles as active online inquirers. They must accept greater responsibility for their learning through synthesizing and applying new ideas or concepts, by stimulating their own curiosity, and by participating in the learning environment (Yang & Cornelious, 2005). For some adult learners, the self-discipline and self-pacing inherent in distance learning can be a drawback. Thus a role adjustment for learners should be clearly communicated by the instructor to ensure success in an online environment.

Learner characteristics can be intensified in an online environment, creating unexpected obstacles to teaching and learning. For example, a normally extrinsic, active, and socially engaged learner may find that an online course lacks the support and rewards he or she derives from interacting with people in the face to face setting. Diversity in ethnicity, gender, professional experience, and location can be another potential barrier, but in many situations differences among students add depth and perspective to the online learning experience (Luebeck & Bice, 2005). Faculty should be prepared to capitalize on the diverse experiences and knowledge bases represented within an online audience.

Instructor roles and titles identified by researchers and practitioners include content facilitator, technologist, designer, manager/

TABLE 1
Themes and Indicators for Assessing the Quality of Online Instruction

Themes	*Indicators*	
Learners and Instructors • How do learner and instructor characteristics influence online learning?	• Learner characteristics o Demographics o Diversity o Motivation o Technical fluency	• Instructor roles o Manager o Technical advisor o Facilitator o Social director o Educationalist
Medium • What cognitive and technological tools effectively support online learning?	• Transparent versus transformative use of technology • Synchronous versus asynchronous delivery • Influence of technology-based factors o Embedded cognitive tools (e.g., whiteboards) o Access to print and electronic resources	
Community and Discourse • What social and interactive tools effectively support online learning?	• Reducing isolation and creating connections • Role of community in effective online learning o Building community through course structure o Building community through discussion • Varied levels of interaction (e.g., learner-instructor; learner-learner)	
Pedagogy • What instructional strategies effectively support online learning?	• Active, problem-based, learner-centered instruction • Multiple modes of interaction and collaboration • Use of modeling and reflective tools • Workload and awareness of student needs	
Assessment • How do formative and summative assessment practices influence online learning?	• Prompt formative feedback • Quality, honesty, and security in assessment • Effective measures of student understanding	
Content • What unique opportunities and challenges arise in content-specific learning?	• Symbolic nature of mathematics • Iconic/visual nature of mathematics • Abstract nature of mathematics • Mathematical technology (e.g., software and calculators)	

administrator, process facilitator, adviser/counselor, assessor, and researcher (Goodyear, Salmon, Spector, Steeples, & Tickner, 2001; Lim & Lee, 2008); coach, counselor, and mentor (Yang & Cornelious, 2005); moderator and tutor (Cowan, 2006); and facilitator, expert, planner, and tutor (Craig, Coldwell, & Goold, 2009).

Many of the findings and frameworks for instructor roles presented in the literature can be synthesized into five major roles. The *manager* develops the online course structure, designs and implements teaching and learning processes, and manages course timelines. The *technical advisor* resolves technical issues quickly so students can remain focused on their learning. Ideally, faculty should also have technical skills in the use of computer communications and computer applications (Huff, 2002). The *facilitator* nurtures student engagement, interest, and motivation, while the *social director* provides affective support through interpersonal communication and keeps discourse flowing. Finally, the *educationalist* fosters collaborative learning through discussions, role plays and debates; ask questions and provides feedback; stimulates reflective thinking; and synthesizes the comments and views of the learners.

It is in the educationalist role that an online instructor has the opportunity and responsibility to create and facilitate collaborative, inquiry-oriented, and student-centered learning environments (Carey, Kleiman, Russell, Venable, & Louie, 2008; Chinnappan, 2006; Johnson & Green, 2007; Maor, 2003; Sendag & Odabasi, 2009; Zhou & Stahl, 2007). This has been accomplished in online mathematics courses through synchronous activity rooms (Maor, 2003) and online discussion boards designed to enhance mathematical discourse and foster a community of learners (Johnson & Green, 2007). Chinnappan (2006) believes that online instructors need to adopt a socioconstructivist approach in facilitating teacher reflections, learning, and mentoring by their peers.

The multiple roles of an online course instructor are synthesized in the construct of "teaching presence"—the design, facilitation, and direction of cognitive and social processes for the purpose of realizing personally meaningful and educationally worthwhile learning outcomes (Anderson, Rourke, Garrison, & Archer, 2001). Developing teaching presence and engaging learners are challenging tasks, but embedded in the literature are useful recommendations for preparing instructors for this challenge:

1. Using surveys or personal statements, make an early effort to identify the unique characteristics and circumstances of the students including learning preferences, prior experiences with online instruction, and background in the content area.
2. Provide tutorials and other support to familiarize students with the technical skills needed to use course management and/or online conferencing systems as well as any software or technological tools that will be used in the course.
3. Provide professional development for course instructors regarding how to nurture an online learning community, facilitate discourse in the online environment, and develop a strong teaching presence through careful course design and instruction.
4. Similarly, provide resources for students to understand the unique demands and expectations of the online environment such as increased cognitive discipline, greater independence, and personal responsibility for engagement and learning.
5. Structure courses around an array of high-quality, interactive learning materials and activities that encourage student-centered collaboration and investigation.

Medium

Beyond discussion boards and other text-based tools, instructional resources in the online environment include two-way audio and video communication, electronic white-boards, interactive formative assessment tools, Java-based applets, blogs and wikis, and shared access to software. Extensive access to Web-based resources allows teachers to engage in active learning through accessing reserved materials, researching electronic resources, exploring interactive applets, and dialoguing with guest experts. But despite the easy access, students in an online course may underuse the provided resources, ignoring active Web links and printing hard copies of materials (Weems, 2002).

The delivery of courses via online instruction can be viewed as either *transparent* or *transformative* (Bernard et al., 2004). The transparent perspective views online instruction as not significantly different from face-to-face, with effectiveness more dependent on learner characteristics, quality of instruction, and learning tasks. The transformative viewpoint values innovative strategies and the variety of resources that become possible in an online environment. A transformative approach to online learning may result in increased reflection due to writing and peer interaction (Hawkes, 2001) and improved problem solving and critical thinking due to peer modeling and mentoring (Garrison, Anderson, & Archer, 2001; Lou, 2004; Lou, Dedic, & Rosenfield, 2003; Lou &

MacGregor, 2004; McKnight, 2001). However, Bernard et al. (2004) warn that the online environment's potential to emulate face-to-face classroom interaction poses a risk that the transformative process could be undermined.

Asynchronous course management tools like Blackboard and synchronous online conferencing tools such as Elluminate can be used separately or in combination to enhance the online experience. Asynchronous interactions provide participants with anytime, anywhere freedom to access course components and allow time to construct deeper understanding. Rovai and Jordan (2004) argued that asynchronous formats can enhance group communication and cooperative learning and promote a level of reflective interaction often lacking in a face-to-face, teacher-centered classroom.

Synchronous approaches enable community building and support authentic interactive learning experiences, which contribute to increased retention (Bernard et al., 2004; Kubala, 2000). To enhance the effectiveness of synchronous course sessions, Hofmann (2004) suggests assigning *prework*—activities or assignments that students accomplish before the real-time class session begins. Instructors can then focus on essential content and address questions and issues that arise from the prework. Instructors may create a student guide, often in the form of a slide presentation that includes the key points of a presentation or lesson (Ludlow, Collins, & Menlove, 2006). Similarly, a leader guide can help instructors stay on time and on topic during a live class session.

The following recommendations offer strategies to support transformative online instruction, regardless of the mode of delivery or electronic supports used:

1. Provide a permanent course site where students can engage in asynchronous discussions, exchange messages, submit assignments, access assessment data, and obtain course materials such as syllabi, unit overviews, instructor notes, and readings. Use PDF format whenever possible to reduce accessibility problems.
2. Incorporate audiovisual media, but sparingly and within reasonable limits (20- to 30-minute modules). Consider using video clips to introduce and conclude lessons or to provide expertise on a topic.
3. Early in the course, develop norms for discussion and synchronous interaction. Be consistent in implementing user interfaces, electronic resources, and course tools.
4. Take full advantage of the available technology to provide innovative learning opportunities that are not readily implemented in the face-to-face environment.

Community and Discourse

Learners in online courses may feel isolated and disconnected both from the instructor and from other learners in the course. Structuring a course to address this isolation can increase student success and reduce retention concerns. To promote a sense of connectedness, Sunderland (2002) recommends immediacy in response to student inquiries by determining a minimal length of time in which the instructor will respond.

Lack of student-teacher interaction in online courses is another major concern. Ramasamy (2009) posits that weak interaction negatively influences course satisfaction. O'Dwyer, Carey, and Kleiman (2007) found that students in an Algebra I online program, while satisfied overall, felt that interaction with the online instructor was not adequate. The researchers attributed this at least in part to course structures that did not promote discourse.

Interactions between learners are also essential to the development of community. Ramasamy (2009) believes control of learning needs to shift from the teacher to the student in online courses. Zemel, Xhafa, and Stahl (2005) found that secondary students participating in small online groups tended to take either an expository approach (students sharing their individual solutions) or an exploratory approach that led to conjoint production of a solution. Learners can also be encouraged to take on the role of peer mentors when discussing problems or reporting solutions.

According to Greelan and Taylor (2001), discourse should be open and content-centered; it may be facilitated through threaded discussions, chat rooms, or synchronous class sessions that include audio/video exchange, e-mail, and instant messaging. Effective organization of discussion boards is an important means of structuring courses to promote discourse. Dogan-Dunlap (2004) increased interaction in matrix algebra and research methods courses through a discussion board requiring students to comment on others' reflections on assigned group projects and class assignments. Carey et al. (2008) investigated the impact of using a facilitated cohort design versus an online self-paced design on middle school algebra teachers' mathematical understanding, pedagogical beliefs, and instructional practices. They found the facilitated cohort design fostered interactions and maximized the role of the facilitator, while the self-paced course design minimized interactions, reduced the facilitator's role, and impeded the use of discussion boards.

Creating a sense of community in online courses ultimately enhances collaborative learning (King & Puntambekar, 2003). There are a number of strategies for building this into the course structure: incorporating a social discussion thread into asynchronous sites (Geelan & Taylor, 2001); grading online discussions to improve engagement and learning (Rovai, 2003); posting personal introductions (Testone, 2003); and displaying participant pictures online (Sullivan, 2001). Face-to-face introductory sessions are an effective means of building community and double as an opportunity for technology training and setting course expectations (Cooper, 2000; Huff, 2002).

The following recommendations summarize specific examples from the literature:

1. Stimulate instructor-student interaction by providing opportunities for discourse that are content centered and skillfully facilitated.
2. Stimulate student-student interaction through structured discussions, chat rooms, and synchronous class sessions. Engage students in facilitation, peer review, group discussion, and exploratory learning.
3. Build social tools into the course Web site, including a "coffee shop" or "teacher's lounge" where learners can talk about noncourse issues, a forum for frequently asked questions, and a location where students can post photos and brief autobiographies.
4. Vary discussions to include instructor-led whole-class discussions, small groups facilitated by student leaders, critiquing of individual assignments, and open forums.
5. Build collaboration through group projects and a facilitated cohort design.

Pedagogy

Online pedagogy research typically focuses on collaborative learning, peer collaboration, and reflective discussions (Love, Keinert, & Shelley, 2006; O'Dwyer et al., 2007; Sendag & Odabasi, 2009). Bernard et al. (2004) believe that online courses should use constructivist approaches that provide for interpersonal interaction and create learner-centered environments. Along with Sendag and Odbase (2009), they also espouse the use of problem-based learning to induce high levels of cooperation and collaboration among participants and to encourage deep learning and critical thinking skills. These pedagogies shift the role of instructor to that of content facilitator (G. G. Smith, Ferguson, & Caris, 2001). Evens et al. (2008) found that creating a collaborative problem-solving environment had a positive impact on teachers' opinions about their classroom practices and that small group interaction increased their problem-solving skills. These factors helped build a community of teachers who shared their reflections and expanded their knowledge.

Studies of online mathematics courses in particular also emphasize collaborative learning (Chinappan, 2006; Evans et al., 2008; Johnson & Green, 2007; Zemel, Xhafa, & Stahl, 2005; Zhou & Stahl, 2007). Johnson and Green (2007) successfully used online discussion boards to enhance collaboration among learners. Chinnappan (2006) added an asynchronous learning forum to a face-to-face course focused on K-6 mathematical pedagogy and found that learning in a collaborative online environment improved the pedagogical content knowledge of teachers. According to Chinnappan (2006), collaborative learning allows beginning teachers to construct their pedagogical knowledge "through social interactions, meaning negotiation and shared understandings" (p. 357).

However, students' ability to effectively work together should not be assumed. It is important that online instructors model problem solving and use strategies to engage learners in collaborative problem-solving situations. Offenholley (2006) used a technique called "round robin" to get all students involved in solving an equivalent form of problem. She had students work on a problem individually, then post their answers. If one student found the correct answer, he or she would generate equivalent forms of the question by changing the numbers to get other students to solve the problem.

Such structured and guided approaches to online discussion are an important pedagogical component of distance courses. Hovermill and Crites (2008) reported on the effectiveness of newly developed mathematics education courses that emphasized connections between mathematics content, effective pedagogy, and reflective practice. Student teachers participated in online discussions about journal articles that addressed issues such as student misconceptions, multiple models of learning trajectories, and assessment strategies. The courses resulted in an increase in student satisfaction and student teachers found the program effective. However, Kramarski and Mizrachi (2006) warn that unstructured online discussions might not lead to an impact on knowledge acquisition unless the instructors scaffold mathematical discussion and practice and reinforce mutual reasoning. They suggest using *metacognitive guidance* to structure online discussions. This involves developing and asking sets of questions that are organized into four groups: comprehension, connection, strategy, and reflection.

Reflective thinking is an essential pedagogical strategy for online courses in teacher education. Owston, Sinclair, and Wideman (2008) believe that teachers need to share with colleagues both student work and difficulties with new instructional models. Their approach led to increased teacher confidence in implementing new models. Teachers more often shared teaching concerns with peers, and rural teachers expressed reduced feelings of isolation. To support reflection, various online mathematics education courses have incorporated a focus on children's conceptual understanding (Cady, 2007), conducted hypothetical lessons (Chinnappan, 2006), and used case studies (Alexander, Lignugaris/Kraft, & Forbush, 2007). The case studies involved narratives and PowerPoint slides meant to develop preservice teachers' instructional skills. Each case study included three components: My turn, Our turn, and Your turn. In "My turn," an instructor models how the first case study should be analyzed through narration. "Our turn" provides a second case study for students to complete through class discussion. Finally, "Your turn" presents a third case study to be completed individually by the students. This case study approach improved teachers' lesson design skills and students' outcomes.

How does online pedagogy differ from face-to-face approaches? Ramasamy (2009) compared the pedagogy in online mathematics courses to face-to-face courses and found that teachers in an online environment make assumptions about students' prerequisite knowledge, often failing to explain in detail mathematical theorems based on student questions. To address this, the researcher recommended that online instructors provide more examples and offer hyperlinks to connect missing skills and information to subskills. After working with freshman students in an asynchronous matrix algebra course, Hodges (2009) identified seven self-regulation strategies that instructors can promote in their students: working practice quizzes, taking notes, reviewing notes, seeking a friend's assistance, watching week-to-week grade progress, setting a goal of getting a good grade, and rewarding completed work with leisure activities.

It is a paradox of distance education that although participants are geographically isolated from the instructor, they may receive significantly more one-on-one interaction than do site-based participants (Geelan & Taylor, 2001). Interaction and learning are enhanced by the pedagogical strategies summarized below.

1. Create a learner-centered environment by reducing time spent on lecturing and by raising expectations for student facilitation, presentation, and discussion.
2. Emphasize constructivist approaches and problem-based learning. For a teacher audience, motivate lessons through application to the practice of teaching.
3. Use cooperative and collaborative learning techniques to engage learners in meaningful problem solving, discussion, reflection, and analysis.
4. Incorporate hypothetical lessons, case studies, reflective discussions, and analysis of student work as instructional tools.
5. Explicitly model problem solving, collaboration strategies, metacognitive behaviors, and self-regulation strategies through instruction and discussions.

Assessment

Along with convenience, flexibility, and relevance, today's online learners expect immediate feedback. Vonderwall, Liang, and Alderman (2007) found that instructor feedback was important for student learning and was sought as an essential element of assessment by students. Riccomini (2002) and Kashy, Albertelli, Bauer, Kashy, and Thoennessen (2003) determined that ongoing assessment of student performance linked to immediate feedback and individualized instruction supported online learning. Projects, portfolios, self-assessments, peer evaluations, threaded discussions, chats, timed quizzes, weekly assignments with immediate feedback, and the use of rubrics have been identified as effective online assessment techniques (Gaytan & McEwen, 2007).

Arend (2007) showed that formative assessment based on nongraded assignments can motivate more practice and improve instructor feedback. Implementing the understanding by design framework (Wiggins & McTighe, 2005) can address the need for formative feedback through incorporation of performance tasks that allow for multiple submittals. Vonderwall et al. (2007) found that "asynchronous online discussions facilitate a multidimensional process of assessment demonstrated in the aspects of discussion structure, self-regulatory cognitions and activities, learner autonomy, learning community and student writing skills" (p. 321). Students in their study valued having assessment criteria and rubrics to assess learning, and suggested that rubrics be detailed and topic-specific.

Summative assessments meant to test individual mastery can be a challenge in online environments, since they commonly require either open-note tests or a system of proctoring tests offsite (Cooper, 2000; Serwatka, 2002). In any case, it is essential to align summative assessments with course objectives that are performance-based and competency-based (Thompson, 2004). Karr, Weck, Sunal, and Cook (2003) found that student performance on online course assessments is often worse than in face-to-face courses, perhaps because more inadvertent hints about assessment are supplied when lecturing. In her study of 60 online courses offered by community colleges, Arend (2007) found that the most frequently used summative assessment methods are discussion, exam, and written assignment, with other methods including final/midterm, quiz, journal, peer review, group project, and presentation. Although nearly every course used discussion as a form of assessment, the average weight of discussion in course grades was only 17%.

Researchers have explored the validity and usefulness of peer assessment in online courses. Wen and Tsai (2008) identified peer assessment activity as valuable in online research methods courses for science and mathematics teachers. Their findings showed positive participant attitudes toward online peer assessment. However, the scores assigned by peers and instructors were inconsistent, raising concerns about the validity of this method. Differences were attributed to participants' lack of knowledge regarding research methods and to the subjectivity of their opinions. Bouzidi and Jaillet (2009) did obtain consistent peer assessment results in three mathematics courses. The authors concluded that online peer assessment was reliable for assessments involving calculations, mathematical reasoning, and short algorithms. Comparing assessments done by at least four peers increased reliability, and comparing self-assessment with peer assessment increased validity. The authors noted areas of concern including "the quality of the marking schemes, the clarity of questions, the grading of the assessments [sic] quality, the technical assistance, and the assessment of examples" (p. 266).

Research suggests that formative, summative, rubric-based, and peer assessments all have a place in online teaching and learning. Recommendations include the following:

1. Incorporate frequent and immediate formative feedback about engagement in discussion and other course activities as well as for course assignments and tasks.
2. Create summative assessments that are aligned with performance- and competency-based objectives. Build in strategies for equitable administration and scoring routines such as detailed, topic-specific scoring rubrics for assignments prior to their due date.
3. Clearly define expectations for discussion participation using rubrics and examples. Experiment with peer assessment of discussions as well as assignments and projects.

Content

Although many recommendations for online learning are universal, subject matter does impact online teaching in specific ways. Teaching mathematics content online is hampered by challenges to exchanging ideas, exploring concepts, and presenting work that may not exist in other content areas. This final theme focuses on considerations for teaching mathematics and mathematics education online.

Engelbrecht and Harding (2005) praise the flexibility and power of teaching on the Internet and the resultant paradigm shift to distributed learning; however, the authors warn that by its nature mathematics provides particular challenges. The symbolic nature of mathematics leads to distinct problems in reproducing mathematical symbols in an online environment. Mathematics is also conceptual by nature, and concepts may be difficult to develop given the isolation inherent in online learning. In addition assessment of mathematics online is difficult due to its iconic, symbolic, and abstract components.

Smith, Torres-Ayala, and Heindel (2008) found that mathematics instructors in an online program expressed a variety of content-relevant concerns: modeling problem solving due to the sequential nature of mathematics; students' ability to learn abstract concepts; sharing visuals; and difficulties in communication due to the symbolic nature of mathematics as a language. Communicating abstract mathematical concepts in symbol form, sharing graphics and visual representations, and discussing mathematics problems with group members are not easy tasks online. In addition, Lawless (2000) found workload, a primary factor in retention, to be amplified in online mathematics courses due to the problem-solving nature of mathematical tasks.

However, there are positive aspects of teaching mathematics online that allow for a dynamic learning environment. Experience has shown that online discussion can sustain meaningful interaction as students explore challenging mathematics. In many ways mathematics is less verbal and subjective than other subjects, so debate and interpretation may play a lesser role, making the subject more amenable to online teaching. New technologies such as mathematical markup language, Java applets, and video conferencing tools such as writing tablets and whiteboards are overcoming some of the barriers identified by Smith (2008). Finally, depending on the course content, students may download or purchase software to produce and share their work with ease.

In spite of challenges that arise from the symbolic and abstract nature of mathematics, researchers have found online mathematics courses to be effective overall. Weems (2002) found that students performed as well in online mathematics courses as they did in face-to-face versions of the courses. Karr et al. (2003) further conjectured that students had to become more independent as learners and dig deeper to understand material if they took a mathematics course online. Hughes, McLeod, Brown, Maeda, and Choi (2007) determined that high school students' scores on an algebra understanding achievement test were higher in online courses. Love, Keinert, and Shelley (2006) found that students in a Web-based discrete mathematics course performed better than the students in a traditional classroom, and similar results were found by O'Dwyer, Carey, and Kleiman (2007) in their study of the effectiveness of an Algebra I online model.

A number of studies conducted in a variety of online mathematics courses have found positive impacts on student learning if appropriate pedagogical strategies are incorporated. Cope and Suppes (2002) provided for varying ability in a computer-based calculus and linear algebra course by allowing students to review archived presentations and to progress at their own pace. In an experimental study with 40 undergraduate primary school mathematics teachers, Sendag and Odabasi (2009) explored how online problem-based learning sessions positively influenced learners' critical thinking skills. Taylor and Mohr (2001) used a range of student-centered strategies, including relevant contextual materials, informal language, and reflective practice using diaries and essays to positively impact students' confidence and anxiety about learning mathematics online. Testone (2003) supported the need for student-centered strategies in teaching developmental mathematics online, recommending that communication be clear and concise, highly visible through frequent contact, and empathic.

Mathematics education courses can also succeed in the online environment. Signer (2008) studied the effect of an online course on teachers' views of inservice education, their own learning, and the quality of interactions, finding that inservice teachers had positive attitudes about all three. Owston et al. (2008) studied a blended learning model with goals of improving middle school mathematics and science/technology teachers' pedagogical knowledge, classroom practice, and attitudes. Pre- and postsurvey results showed that teachers' pedagogical beliefs changed significantly, with teachers embracing the program's open-ended, constructivist approach.

Perhaps more than for other subjects, teaching mathematics online calls for a transformative approach. Traditional mathematics instruction can be modified with new approaches to representing information, carrying out procedures, and exploring concepts. Technological tools can be introduced to enhance mathematical communication and to enable a richer and deeper exploration of meaningful mathematics. The following recommendations support such a transformation.

1. Use student-centered and exploratory approaches such as problem-based learning to actively involve learners in the construction of mathematical knowledge.
2. Provide tools and establish conventions that simplify mathematical communication.
3. Take full advantage of available software, technology, and Internet-based resources for exploring and presenting mathematical ideas.
4. Provide sufficient detail and solicit student feedback when explaining concepts, theorems, or examples in both synchronous and asynchronous online lessons.
5. Be mindful of barriers to teaching mathematics online, including the symbolic, iconic, and conceptual nature of mathematics and workload issues related to the time required to complete mathematics assignments.

LESSONS LEARNED FROM EXPERIENCE

The recommendations and research findings reported so far are supported by our own experiences administrating a variety of online programs, both synchronous and asynchronous in design, for teachers and learners of mathematics. In this final section we share additional insights and practices that did not specifically appear in the literature but have proven helpful in creating positive and fruitful online learning experiences.

Technology

The instructional value of any technology is only as good as the quality of its implementation and the skill and comfort levels of its users. The technology backgrounds of online learners are often highly inconsistent. Students should be asked to provide background regarding prior experience with relevant Web-based communication platforms, computer software, calculators, and online tools. The following advance measures are also recommended:

Introduce opportunities for students to gain familiarity with the course site and required technology through electronic scavenger hunts and other precourse exercises. When synchronous instruction will be used, an informal opening activity is vital for the identification and resolution of technical problems before instruction begins. Create activities that allow testing of all hardware technology—Web access, software, two-way video and audio components, and other course-related devices. As the course proceeds, maintain a discussion site where students can report problems and resolve technical issues. These are often successfully self-managed as students are very willing to share shortcuts, quick fixes, and expertise.

In situations where students will enroll in multiple courses (e.g., a master's program) and/or use advanced communication technology (e.g., Elluminate), an initial face-to-face orientation and technology training session for all instructors and participants will prepare students for the rigor of online learning and reduce time spent later on technology issues.

Community

Although we fully embrace the transformative value of online instruction, we also value the importance of human interaction in creating a creative, motivational, and challenging learning environment where students are comfortable presenting their own theories and challenging the ideas of others. In an ideal situation, online learners will have an opportunity to meet face-to-face before commencing their virtual classroom experience. Such opportunities can be built into programs where students will engage over time in a cohort model, but may not be possible in single-course situations.

Relationships of trust among instructors and students are slower to develop in the absence of face-to-face interaction. Camaraderie can be nurtured through humor, through the social discussion boards described earlier, and through instructor initiative by being the first to share background information and post photos or video clips. Instructors may also consider providing "virtual office hours" each week via telephone, Web-based chat rooms, or video-supported platforms to promote learner-instructor interaction. Regardless of the medium used, instructors should always provide constructive and timely feedback.

Course activities can be used to build a sense of community into the learning process. Communication structures (whole group, small group, pairs, and individual reflection) should change often to maximize interactions and to cross-pollinate ideas and perspectives. Students may be assigned to facilitate small group discussions based on readings or activities. Coursework can be presented in forums or working groups. Pairs or groups of students can be assigned to work on projects or research specific aspects of a problem. Collaboration can be a challenge in the online environment, so group work should be accompanied by clear timelines and tasks. Instructors may want to further direct students to meet at specific times in synchronous study groups.

In general, discussion groups and other community structures should have access to explicit instructions for how to initiate presentations, support or extend ideas, and critique, challenge, or defend thinking. Prompts may be provided for student leaders to initiate a discussion, and groups should be monitored to determine when further instructor input is needed. Finally, students themselves should be encouraged to critique the community-building strategies implemented in the course and to brainstorm ideas for the best approach to communicate with their fellow learners effectively.

Content

Although the nature of mathematics creates some challenges in terms of communicating ideas in symbolic and graphical form, our experience demonstrates that both procedural and conceptual understanding of mathematics can be fostered in the online environment. We have successfully taught mathematics content courses in geometry, number and algebra, discrete mathematics, and calculus/analysis. The online environment is fertile ground for the mathematical processes that support learning such as problem solving and reasoning, representation, and mathematical communication. The technologies used in mathematics and the technologies of online instruction interact in a complementary manner to advance students' understanding.

Mathematics education courses for teachers (e.g., learning theory, standards-based pedagogy, curriculum, and assessment) are enriched by the varied perspectives and experiences of their participants. In some classes, teachers share personal vignettes, compare and contrast instructional strategies, and examine their own students' work. In others, teachers use their classrooms as real-time "living laboratories" to experiment with mathematics teaching, learning, and assessment strategies introduced in online courses.

CONCLUSION

This article provides a compendium of recommendations for online instruction gleaned from research and expository literature based on the experiences of those who design, deliver, and study such instruction. It is hoped that the strategies, structures, and perspectives reported here will advance the work of online educators who strive for a transformative approach to online learning.

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