Book review: The evolution of research on teaching mathematics: international perspectives in the digital era

Book Title: The Evolution of Research on Teaching Mathematics
Book Subtitle: International Perspectives in the Digital Era
Editors: *Agida Manizade*, *Nils Buchholtz*, *Kim Beswick*
Series Title: Mathematics Education in the Digital Era
Publisher: Springer Cham,
DOI: https://doi.org/10.1007/978-3-031-31193-2
Copyright Information: The Editor(s) (if applicable) and The Author(s), 2023
ISBN and Publication Details Hardcover ISBN: 978-3-031-31192-5 (Published: 11 August 2023)
Softcover ISBN: 978-3-031-31195-6 (Published: 11 August 2023)
eBook ISBN: 978-3-031-31193-2 (Published: 10 August 2023)
Review DOI 10.1108/QRJ-02-2026-380

Considering digital and epistemological context on teaching mathematics

The introduction part of *The Evolution of Research on Teaching Mathematics* by Manizade, Buchholtz and Beswick, the editors, explains the background, objectives and structure of the book. The lack of a systematic scientific overview of the complete chain of effects between teacher characteristics, activities and students' learning processes served as the impetus for writing this book. This book examined the present state of research on teaching mathematics and explored the likely direction of future research development. This book adapts Medley's (1987) framework process-product research, which has been updated to presage process-product research (see Figure 1) by taking into account additional variables that emerged from recent advances in technology and research methodology in the digital era.

Although this book focuses only on Western culture, it also mentioned other countries from other continents (e.g. Africa and Asia), especially Japan, which is quite often mentioned. From the reviewers' perspective (i.e. Indonesia), some of the theories are or have been developed. One of them is RME, wherein some Scopus-indexed journals in Indonesia explicitly call for original research articles on the subject (e.g. *Journal on Mathematics Education* and *Infinity Journal*). The Indonesian readers can certainly consider other theories by taking socio-cultural aspects into account and, if required, making adjustments. According to the curriculum document, Indonesian teachers should, in theory, determine learning goals before considering how to teach them when developing lesson plans. Additionally, Indonesia's most recent curriculum, known as "*Kurikulum Merdeka*" or the Independent Curriculum, has already integrated technology, including introducing programming as a specific discipline. However, like in England, which is described in this book, it only regulates the content without providing adequate pedagogical guidance for teachers.

Past, present and future of research on teaching mathematics within cultural, epistemological and digital context

This book is divided into two parts. The first part consists of six chapters that explain each online variable, which are units of analysis of research that can be under the control of

Qualitative Research Journal
Vol. 26 No. 1, 2026
pp. 145-154

e-ISSN: 1448-0980
p-ISSN: 1443-9883

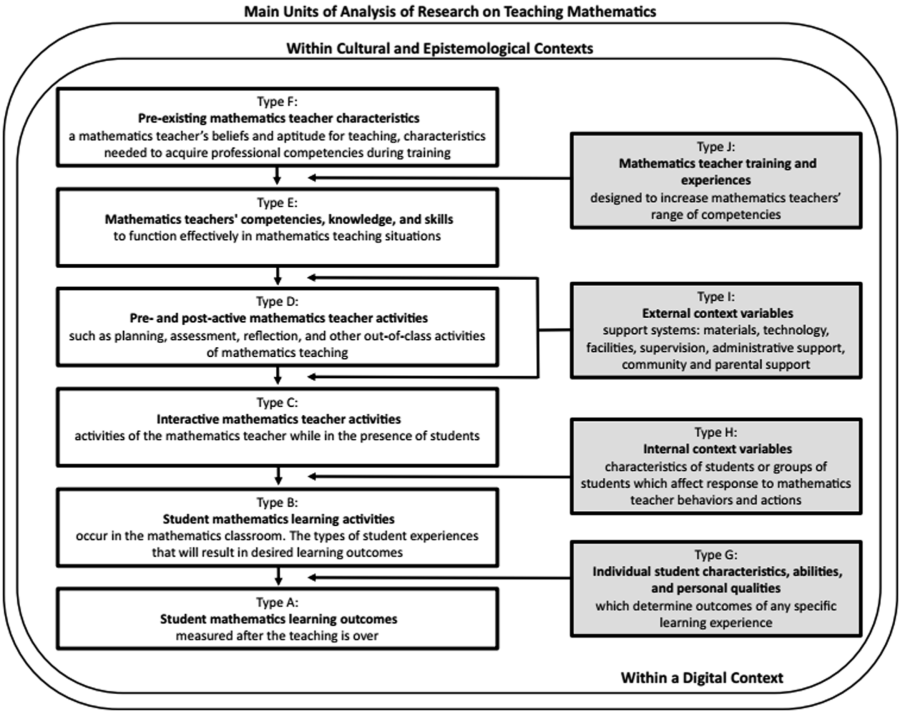

Figure 1. Updated framework of research on teaching mathematics

mathematics teachers, including Type F to Type A. The second part consists of four chapters that explain each offline variable, which are units of analysis that are not under the direct control of teachers, including Type J to Type G. A final chapter in this book outlines the direction of the evolution of mathematics education research in the future.

Chapter 1, "Pre-existing mathematics teacher characteristics," by Olive Chapman discusses the Type F unit. Pre-existing mathematics teacher characteristics (PTMCs) include prospective teachers of mathematics' (PTs') knowledge and skills, pedagogical knowledge and abilities and beliefs at the beginning of their mathematics teacher education (MTE), which reflect the nature of their background knowledge or ability related to their school experiences with mathematics. PTs' pre-existing mathematical knowledge of various content areas and problem-posing skills was hampered by a low conceptual understanding. PTs' pedagogical abilities have many shortcomings related to the ability to observe and draw appropriate conclusions or offer suggestions regarding instruction, to notice and provide solid evidence to support students' reasoning, to recognize the potential of tasks or difficulties students may encounter and to classify a high level of complexity problem. PTs' beliefs are dominated by the Platonist and absolutist perspective of mathematics, the traditional perspective or "teacher-centered" perspective of teaching and learning mathematics, narrow mathematical conceptions, PS and multiple representations and a lack of understanding of the use of technology to support students' learning and to develop concepts.

Future research should focus on PTMC for PTs at the point of entry into MTE; the types of PTMC within and beyond the previously mentioned categories; addressing affective factors such as PTs' attitudes and what they value and PTs' ability to reflect on types of beliefs. Furthermore, research should focus on the impact of technology and culture on the PTs' PTMC at the point of entry into MTE; conceptualizing PTMC in relation to teacher education, technology and culture and exploring PTMC at the point of entry to MTE, which requires different or better instruments and analysis and uses a more rigorous mixed-methods research design with more rigorous statistical analysis and use of technology.

Chapter 2, "The evolution of research on mathematics teachers' competencies, knowledge, and skills," by Buchholtz, Kaiser and Schwarz discusses the Type E unit. The narrative reviews in this chapter present how Type E variables encountered paradigm shifts and how these paradigms are critiqued. Medley's process-product paradigm took over in the 1960s as a result of the deficiency of the personality or traits paradigm, which did not explain how to measure personality traits. Since the mid-1980s, the focus of the research based on this paradigm has been on the more complex structures of instructional quality or the combination of multiple variables and on teachers' cognition, which underlies Shulman's work. However, Shulman had a static understanding of knowledge, and the idea of "transforming" or "translating" subject matter into pedagogical forms amounted to a routine, mechanistic transmission. Thus, the expert paradigm sparks and holds that teachers are considered experts because of their ability to successfully manage a highly specialized, complex task such as school teaching due to the students' behavior.

Future research should address how suitable instruments can be used to survey content knowledge (CK), pedagogical content knowledge (PCK) and general pedagogical knowledge in connection to teaching practice, particularly leading to situation-specific teacher competencies observation and measurement based on the expert paradigm; expanding studies that used Medley's paradigm that was strongly influenced by cognitivism by including several cognitive perspectives and advancing methodology by critically analyzing the validity of measuring instruments.

Chapter 3, "The research on mathematics teaching and planning: theoretical perspectives and implications of teachers' pre-post classroom activities," by Manizade, Moore and Beswick discusses the Type D unit. Observable Type D actions produced by the decision-making process that reflect how teachers' thinking – based on the Type E elements they possess – determines the effectiveness of the Type C implementation and the degree to which the teaching purpose is met. In designing lesson plans, teachers might choose to focus on the process of student learning – then it is less likely that they will think of students' conceptual development, and lessons can become more prescribed and rigid – and discovery of concepts or on specific content outcomes – would drive teachers to likely consider common student challenges, typical questions or difficulties students might experience based on their developmental levels, as well as strategies to deal with those.

The most important part is the literature review of theoretical perspectives for teaching mathematics that are present in Type D, which includes eight epistemological perspectives: situated learning theory (SLT), behaviorism, cognitive learning theory, social constructivism (SC), structuralism, problem solving (PS), culturally relevant pedagogy and project and problem-based learning (PBL). Definitions, teaching goals, examples from the literature and pros and cons of each perspective are explained. This chapter also explains their respective cultural contexts and the implications of Type D for lesson planning.

Future research should address the theory-practice gap that can be seen as an attempt to link Type E variables directly to Type C variables with insufficient attention to Type D variables. When this gap is successfully overcome, it should be possible to trace a coherent theoretical perspective along the chain of Type E-C-D variables, influenced and constrained by Type I-J variables. Future research should also focus on some theoretical perspectives that have not been researched well (e.g. SLT and PBL); investigate the potential of digitalization for improving teachers' practice of lesson planning, assessment and reflection and investigate the theoretical perspective through which the teacher views the teaching goals.

Chapter 4, "Interactive mathematics teacher activities," by Beswick, Rawlings-Sanaei and Tuohilampi discusses the Type C unit. Large-scale international surveys, particularly TIMSS and PISA, provide information including teaching mathematics resources, instructional practices and teachers' use of technology. Regarding teaching resources, lack of time constrains the kind of students' activities and impedes the adoption of innovative practices. Regarding instructional practices, there was increased practice of computational skills and decreased practice of PS, contrary to current trends that emphasize reasoning and PS as the teaching goals. Additionally, time limits resulted in structured teaching practices – explicitly stating learning goals, allowing students to practice until they understand and providing summaries – as the most frequently used – and the use of project work was less frequent. Regarding the use of technology, the majority of students reported that they never or almost never do computer activities to support learning that resulted in their low achievement. Teachers also face challenges to use new technologies and engage in higher-order pedagogical tasks.

Those findings are similar in atypical teaching practices. Student-centered approaches are not widely used, and on the other hand, it was hard to find an innovative perspective on behaviorist approaches. Teachers seemed to struggle to shift from procedural knowledge-developing pedagogies to ones that facilitate robust understanding (Schoenfeld, 2018; Schoenfeld *et al.*, 2020).

During an intervention from the researcher, teachers may adopt the new practice; however, future research should document what happened before and after these interventions. Researchers' beliefs constrain their study, resulting in a mismatch between the teacher behaviors that researchers advocate and the pedagogies that students most commonly report experiencing. This raises questions about how a teacher's practice can be influenced. Future research should also document how teachers have reacted to the recent opportunities due to the growing digital world and fully identify the COVID-19 pandemic's effects.

Chapter 5, "Student mathematics learning activities," by Timmerman discusses the Type B unit. Over the last 3 decades, there has been a shift in focus toward student thinking needed to develop a mathematical conceptual understanding and identify how students should experience solving mathematical problems. However, teaching practices (Type C) that support student learning with understanding were missing. Historical reviews also address the dilemma of balancing between the needs of mathematics and the learner; if either side dominates too much, then the entire curriculum is disrupted. Thus, curriculum frameworks should examine student engagement in learning activities (Type B), including technological environments, that have evolved in both cognitive and affective aspects as students become mathematics knowers and doers.

This chapter provides three theoretical perspectives that explain student engagement in learning activities to develop mathematical CK with understanding. The first theory is Hackenberg's (2010) scheme theory, which defined mathematical learning as a process in which people make accommodations in schemes in continuous interaction with their experiential world. According to Simon, perturbations in scheme theory do not demonstrate how learning happens, and scheme theory does not explain what a learner "attends to" in order to achieve a learning goal. Thus, learning through activity (LTA), which is a research model, examines how learners engage in learning activities to develop mathematical concepts (Simon *et al*., 2016, 2018). Reflective abstraction is now understood as a construction of higher-level concepts based on lower-level actions, rather than as a chronological sequence of actions for developing a new concept. A concept in LTA consists of a goal and an action that develop in two stages. The first stage is participatory or initial reflective abstraction; a learner engages in activity and uses existing concepts to start developing new knowledge. The second stage is anticipatory, only when a learner can call upon an earlier abstraction (concept) in different contexts. PS activities may provide an opportunity to examine LTA's theory in terms of providing a more detailed explanation of how teachers can promote a transition between the two stages beyond individual students to small- and whole-group work methods in the classroom.

Liljedahl's (2016) AHA! Experience is the theory in which encouraged changes in students' behaviors and dispositions; that is, engagement in learning activities of PS and perseverance. This chapter explains three kinds of perseverance based on Liljedahl's theory:

(1) productive disposition – viewing mathematics as sensible, useful and worthwhile, coupled with belief in diligence and one's own efficacy; (2) productive struggle – intellectual effort students do to understand difficult concepts that are within their reasoning capabilities and (3) productive failure – students' initial individual PS attempts were unsuccessful in finding correct solutions and became productive when supported with appropriate classroom instruction.

Future research should attribute to transitioning from past theories and methods of measuring procedural student performance goals to a vision of measuring conceptual student learning goals; provide evidence of the effect of students' engagement in learning activities while building CK and doing mathematics; connect student struggles to some of Polya's PS phases with a new lens for analyzing students' sense-making and examine students' productive struggles (Type B) and teacher–student interactions (Type BC research) using an appropriate framework. Additionally, new areas of research are needed with a focus on the complexity of the learning and teaching process – the interrelationships between teachers, students, mathematical activities, curriculum content and the added effect of technology.

Chapter 6, "Student mathematics learning outcomes," by Radišic discusses the Type A unit. From the 1930s to the 1980s, there was a shift in focus with mathematical learning outcomes defined, from knowledge and understanding of content and a clear set of associated procedural skills to PS and beyond computational ability. Many theoretical frameworks have been used to define mathematical competence, and the idea shifts away from using a strict cognitive lens. The KOM framework (Niss, 2003) divides eight competencies into two groups: aspects of involvement with and in mathematics and dealing with and managing mathematical language and tools. However, the KOM framework did not consider affective and dispositional factors. Proficiency strands (Kilpatrick *et al.*, 2001) see proficiency as a multidimensional trait – a combination of five strands that are mutually intertwined and codependent. Unlike the KOM, proficiency strands include motivational action tendencies. The last frameworks discussed in this chapter are TIMSS and PISA, coming from the ILSA domain. TIMSS organizes competence around two dimensions: the content dimension, which is the subject matter to be assessed, and the cognitive dimension, which is the thinking processes to be assessed. On the other hand, mathematical literacy as a mathematics framework from PISA entailed situations and/or contexts in which problems were situated. The mathematical content categories and the processes were employed to solve them.

The challenge in the assessment is to produce measures that allow for an understanding of how students come to use mathematics (Type A) in different social settings and how to produce instruction that helps them even better (Type C). However, one size of assessment does not fit all purposes. Formative assessment aims to improve teaching and learning processes, while summative assessment aims to determine whether an individual has reached a certain level of competence. Large-scale assessments, such as ILSAs, describe and inform about a particular system rather than an individual. Technology set a new understanding of students' competencies, affected curricular goals and initiated the need for computer-based assessments (CBAs).

Future research should grasp what student outcomes mean and require to master mathematics, understand the role of dispositional factors in the conceptualizations of competence, especially given their fleeting position across existing frameworks, and discover a joint criterion or framework in large-scale assessment and well-founded models and theories in the use of CBAs.

Chapter 7, "Abilities and personal qualities and the teacher's role in improving mathematics learning outcomes," by Faragher discusses the Type G unit. As inclusive education spreads across the world, general mathematics teachers are now responsible for educating learners with intellectual disabilities. These are three categories of student characteristics. First, mathematics learning disabilities are inherent in the student, with some caused by neurobiological factors (e.g. developmental dyscalculia and dyslexia, which impact, respectively, the understanding of quantity and retrieving language-based arithmetic aspects).

Computer interventions reported could develop new neural pathways through repeated tasks. However, transfer to the mathematics learning area is still not available. Second, mathematics learning difficulties, which are not inherent in the student but caused by poor teaching, environmental factors, affective factors, minority status, previous academic attainment, gender, age, health, family socio-economic characteristics or school characteristics. Low-attaining students required training and task repetition, but this could cause low motivation due to monotonous and tedious tasks. Third, mathematics learning difficulties, which are related to their school environment engagement. To prevent learners from experiencing mathematics anxiety, it should be prevented from developing in the first place.

Teachers should focus on learning trajectories that can be developed using universal design for learning (UDL), which emphasizes meeting as many learning support needs as possible in a single lesson plan. Planning (presage) always provides multiple ways of presenting information, engaging with content (e.g. use rich PS tasks that can engage and challenge all learners with a single task) and demonstrating accomplishment. Diversity is expected, planned for and valued for adding richness and alternative ways of thinking; therefore, teachers can make adjustments once the requirements are known. UDL or other planning methods might be used to design year-level adjusted curriculum (YLAC). YLAC's purpose is to begin with the curriculum being planned for the class and then meet specific learning needs through planning adjustments. Digital tools might also help. For example, students with writing difficulties can be recorded demonstrating techniques or presenting their work, and students with limited expressive language can be observed through making choices. However, additional study into the use of digital tools to assess the learning of students with intellectual disabilities is required.

Future research should learn much more about Type G variables and their effect on Type A, particularly in inclusive mathematics classrooms. Furthermore, there is much to learn about how teachers develop professionally by reflecting on the interplay between learner variables, student activities and student outcomes.

Chapter 8, "Individual student internal contexts and considerations for mathematics teaching and learning," by Che and Baker discusses the Type H unit. Mathematics identity is a dynamic construct that dialectically shapes and is shaped by social context. Internal context includes students' construction of themselves and of mathematics that is ever incomplete and is continually ongoing. Students' cognitive processes to understand mathematics in the 1970s were influenced by behaviorism. However, for the past several decades, constructivism has expanded. Knowing is not a process of discovering external, independent and pre-existing realities; thus, mathematics does not exist outside of the mind of the knower. Students' views about the nature of mathematics are informed by their experiences as students that are categorized as Type C and Type B. If their experience comes from a traditional perspective where mathematics is seen as a static discipline, they will suspect that mathematics is about memorization, is dry and boring and is a waste of time. Thus, teachers should support productive disposition. Students' identity constructions to understand themselves as learners are connected with affective processes including mathematics self-concept, self-efficacy, identity and disposition. Self-concept is important to become persistent. Self-efficacy relates to students' belief in success at given tasks, which connects to motivation, emotional well-being and performance. Identity provides ways to understand the complexity of students' decision-making. Motivation can fuel higher performance, which can fuel further increases in performance and strengthened motivation; the reverse can also hold.

Future research should increase the presence of postmodern research perspectives on student internal context; consider critical theoretical approaches in consideration of student and educational context; (re)envision notions of student identity and student performance in ways that are open to irregularity and spontaneity while maintaining rigor in research; develop deeper understandings of the systemic nature of concealed, asymmetric relationships of power and the ways those of social contexts of inequity are revealed in children's educational lived experiences and identities; understand students' identities and internal social contexts in a variety of technological learning environments and understand patterns and asymmetry in student access to important online learning communities.

Chapter 9, "External context-related research: digital resources as transformers of the mathematics teachers' context," by Gueudet and Pepin discusses the Type I unit. This chapter focuses solely on Type I variables in the digital resources' context, including both the materials themselves and various aspects of teachers' external context (e.g. community support). This connects Type I variables to Type C-D-E variables. Research on educational policies has shifted from the technologies' use in the classroom to how educational authorities use digital curriculum resources (DCRs) to support teacher design and classroom practices. Early studies showed that policy translation to practice varies due to economic situations, and unequal technology access has become an obstacle in all countries. Recent studies have shown that, though curriculum specifies content, it offers little pedagogical guidance. Furthermore, teachers are at a loss on how to assess the quality of freely available DCRs and how to design or modify DCRs. Research has produced various tools for assessing digital resource quality, revealing the need to re-conceptualize quality, to consider new possibilities for connectivity and to view teachers as designers of their own curriculum (Type C-D variables, because design occurs both out-of-class and in-class). Research on teachers' integration has shifted from a single educational technology to complex sets of digital resources available. This was linked with the development of theoretical frameworks and new conceptualizations of digital resource integration. In a digitalized context, students develop as self-directed learners with peer support, while teachers serve as knowledge development scaffolders. Teachers need new requirements, including adjusting their perspective on mathematics (e.g. seeing programming as an integral part of mathematics). Digital resources have impacted both the potential and actual collective dimensions of teachers' work.

Future research should focus on educational policies relevant to the offering of digital curriculum and the tensions between supporting teacher creativity and efforts of the national agencies supplying resources to help teachers align with education reforms; provision and quality of specific DCRs (e.g. for particular mathematical topic areas, including programming); digital assessment procedures, developing from simple tests to complex digital environments where students can collaborate; distant and hybrid teaching at all educational levels, and its relationship to equity concerns and educational digital resources for teacher professional development (PD) (Type J variable).

Chapter 10, "Competency framework for the qualification of facilitators of mathematics," by Peters-Dasdemir, Holzäpfel, Barzel and Leuders discusses the Type J unit. Mathematics teacher training includes pre-service activities at university and PD programs for in-service teachers. PD programs are a crucial intervention that significantly improves teaching quality. In mathematics, focusing on teachers' CK and PK development has a stronger effect than concentrating only on CK. The professionalization process during PD programs involving facilitators leads teachers to recognize new ideas and innovative approaches that they could use in the classroom. Thus, the competencies of facilitators are a new presage variable in the presage-process-product framework. This chapter aims to provide a complete description of the competencies required by facilitators to conduct effective PD through a Delphi study that involved researchers, stakeholders and teachers with experience in continuous PD, carried out along with the three-tetrahedron model (3 TM) (Prediger *et al.*, 2019, p. 410). The result leads to the German Centre for Mathematics Teacher Education framework covering four areas of the competencies necessary for the qualification of facilitators from the perspective of mathematics education: professional values and beliefs, professional self-monitoring, competencies at the PD level and competencies at the classroom level.

Future research should identify the success of PD from the facilitator to the student outcomes; develop instruments needed to measure competencies in the framework presented, which could be researched in more detail regarding implementation, taking into account the interplay of competencies and develop quality standards for facilitators and qualification programs in different countries with differing structures, as well as implement them to guarantee the nationwide success of PD programs.

This book closes with chapter “Continuing evolution of research on teaching and learning: exploring emerging methods for unpacking research on teachers, teaching and learning” by Orrill, Gearty and Wang. The discussion includes methodologies that have become more widespread since Medley introduced his framework, some examples of research methods that offer new ways of thinking about research questions and presage process-product research in the 21st century.

In 1987, qualitative research was rarely used in education. However, quantitative research methodologies are particularly appropriate for a teaching and learning paradigm that depends on knowledge transfer from the expert to the novice. The emergence of cognitive, socio-cultural and critical theories has made qualitative research a critical research tool. Since qualitative methodologies became a norm within the field, some researchers use qualitative and quantitative methods together, called mixed methods approaches, to better understand the interactions inherent in the learning environment. On the other hand, technological advancements can make quantitative research analysis more robust and accessible. The numerous statistical and psychometric models that have emerged in the last several decades, including item response theory, mixture Rasch models, diagnostic classification models and topic models, are particularly important for presage-process-product research. This chapter then introduces four methods that provide new perspectives on the interconnected nature of the variables for teaching and learning: teacher experiments, design-based research, cultural-historical activity theory and quantitative ethnography. The definition, benefits and limitations and examples of each method are presented. Lastly, this chapter provides three examples of how technology has changed the kind of data researchers may collect and the questions that they can pose: eye tracking, dynamic geometry software, 360° video and other full-room video capture. Again, definitions, benefits and limitations and examples of each are presented.

Recommendations

This book is at least suitable to be read by educators, policymakers and, especially, researchers. For educators, this book provides a general idea for them to develop lesson plans, implement them in the classroom and design assessments. This involves Type A-B-C-D variables. For policymakers, this book provides an overview of how Western countries have established and executed curricula, how technology might be incorporated into them and – mostly importantly – explains all of the offline variables that cannot be directly controlled by teachers but by them.

For researchers, they might greatly benefit from this book in writing a research proposal. For the proposal’s introduction section, the researcher can consider which variable(s) are most intriguing to them. For example, if they are interested in Type A and Type B variables, this book provides a sufficient literature review about the lack of PTs’ pre-existing mathematical knowledge and abilities as presented in Chapter 1. Related to this, researchers might draw attention to the study findings presented in Chapter 2, which indicate that teachers’ PCK is closely connected to their CK. However, when it came to predicting student performance, teachers’ PK was more explanatory than their CK. In this instance, Type J may also be taken into account by researchers as a mediating variable for Type A and Type B variables. After examining the past and current research development, researchers can look at the section on the suggestions made by the book’s authors for further study. This would be an appropriate research gap.

For the proposal’s theoretical framework section, this book offers and thoroughly explains a wide range of theories and frameworks. For instance, Chapter 3 presents various theoretical perspectives for teaching mathematics, while Chapter 6 presents different frameworks for students’ mathematics learning outcomes. In order to eliminate any ambiguity and gray area between theories and frameworks, the author carefully and clearly delineates their respective bounds. Thus, researchers are able to select the best suitable one. For example, as explained above, the KOM framework and the proficiency strands differ from one another. Regarding the differences in theoretical perspectives for teaching mathematics, the author explains that SLT is different from SC in that it does not assume that knowledge is possessed by an individual. Rather than having a determined ontology, knowledge is experienced and engaged in. Furthermore, the authors also explain that the structuralist perspective focused on discovering the existing mathematical structure, whereas the constructivist perspective focused on exploring to construct students' own concepts from scratch. As a result, while structuralists are able to talk about misconceptions and misunderstandings, constructivists do not use that term because the student's concept is its own referent. However, the existence of the term "alternate conceptions" in constructivism is not further explained in this book (Fujii, 2020; Hennessey *et al*., 2012). Lastly, for the proposal's methodology section, researchers just need to open the last chapter to obtain everything they need so they can choose the most appropriate method aligned with their research question(s).

Conclusion

An Indonesian proverb says, "*Tak ada gading yang tak retak*," which means nothing is perfect. This book contains several technical errors. For example, on page 74, it said that the framework for teacher knowledge developed by COACTIV identifies three different facets of subject-specific knowledge. However, then it only mentions two different facets. After checking the reference source, there are indeed only two facets. Additionally, Chapter 10 mentions CPD several times without ever giving the abbreviation. After checking the reference source, CPD stands for continuous PD. However, would you pass up this outstanding book just because of that? We do not think so.

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