Detecting and analyzing explosive bubbles and their relationship with volatility: evidence from Tunisia

Sirine Ben Yaala and Jamel Eddine Henchiri
RED Laboratory, Higher Institute of Management, University of Gabes, Gabes, Tunisia

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Abstract

Purpose – This study aims to identify and analyze speculative bubbles in the Tunisian stock market from 2004 to 2023 and examine the evolution of return volatility during these periods.

Design/methodology/approach – The research uses the Supremum Augmented Dickey-Fuller (SADF) and Generalized Supremum Augmented Dickey-Fuller (GSADF) tests, alongside Monte Carlo and bootstrap simulations (Sieve-bootstrap and Wild-bootstrap), to detect speculative bubbles. The Markov-Switching Generalized Autoregressive Conditional Heteroskedasticity model is used to analyze volatility regimes.

Findings – The study identifies multiple speculative bubbles with varying timing, duration and response to external events. The GSADF test proves more effective than the SADF test for detecting longer, more frequent bubbles. Despite methodological differences, strong correlations among bootstrap techniques improve bubble identification. Bubble periods align with a high-volatility regime (regime 2), emphasizing volatility's role in bubble formation.

Research limitations/implications – This study enhances the understanding of speculative bubble formation in emerging markets, highlighting the importance of considering national financial market specifics in bubble analysis.

Practical implications – The findings offer valuable insights for investors, regulators and policymakers, helping inform decisions and improve financial regulation to foster market stability.

Social implications – By identifying speculative bubbles, the research helps mitigate economic uncertainty, protects savings and supports financial stability, aiding policymakers in curbing excessive speculation and promoting sustainable economic growth.

Originality/value – This research contributes to the understanding of speculative bubbles in the underexplored Tunisian stock market, using innovative methodologies for a comprehensive analysis of bubbles and volatility dynamics.

Keywords Speculative bubbles, SADF, GSADF, Volatility regimes, MSGARCH

Paper type Research paper

1. Introduction

Popularly known as the *Random Walk Theory*, the efficient market hypothesis assumes that market prices incorporate all available information. The first use of the efficiency concept in

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the context of asset markets is attributed to Fama (1965), who posited that an efficient market is one that reacts quickly to information available to any investor at a relatively low cost.

The market efficiency assumption is central to debates in financial theory, as it links information to the behavior of stock prices. Stock prices evolve based on new information, and under the assumption of rationality, this information is integrated almost instantaneously into stock prices.

However, traders and trend followers can violate investor's rationality. Some investors are not perfectly rational and can act as noise traders. Their behavior leads to deviations in prices from fundamental values. In addition, some market participants act correlatedly, and this herd behavior can be the origin of speculative bubbles.

Speculative bubbles are complex phenomena defined in various ways across the literature. One key perspective describes bubbles as prolonged periods of rising prices lasting 15–40 months, often followed by sharp declines and significant depreciation, as noted by Kindleberger, Aliber and Solow (2005) and Goetzmann (2015). Another perspective emphasizes the divergence between market prices and intrinsic values, with Coudert and Verhille (2001) defining bubbles as instances when asset prices exceed their fundamental values, reflecting mispricing driven by speculation. Levine *et al.* (2014) further highlight a lag between market prices and intrinsic values caused by speculative behavior. In addition, the distinction between rational and irrational bubbles is significant; Blanchard (1979) introduces rational bubbles, where investors expect to profit before a collapse, while Bailey (2005) discusses irrational bubbles driven by psychological factors and herd behavior.

The detection of speculative bubbles has led to various econometric approaches. Early methods, like the variance bounds test (Shiller, 1981) and the unit root test (Campbell & Shiller, 1987), struggled to distinguish collapsing bubbles from stationary processes. Phillips and Yu (2011) and Phillips, Shi and Yu (2015) addressed this with the Supremum Augmented Dickey-Fuller (SADF) and Generalized Supremum Augmented Dickey-Fuller (GSADF) tests, enabling real-time identification of bubble origins and endings. These tests, validated by studies such as Zeren and Yilanc (2019) and Korkmaz, Bari and Adalı ((2021), are effective early warning systems for financial crises.

The explosive nature of speculative bubbles results in market instability and output contraction (Bean, 2004). Such crises can manifest in two forms: a localized disruption limited to the affected market (relative crash) or a global crisis impacting all financial markets, potentially extending to the entire financial system (systemic crisis).

These findings emphasize the need to better understand the mechanisms of bubble formation and their impact on price volatility, which is crucial for preventing crises and ensuring financial market stability.

Studies examine the relationship between speculative bubbles and price volatility, with volatility as a key indicator of bubbles. Shi and Song (2015) identify increased volatility as a sign of speculative bubbles, while Blasques, Koopman and Lucas (2014) use volatility regime shift models to predict turbulent periods. These findings emphasize the need to understand bubble formation mechanisms and their impact on volatility to improve financial market stability and efficiency.

Despite extensive research on speculative bubbles in global markets, there is a notable lack of literature regarding the Tunisian stock market. This study aims to fill three key gaps: First, it will compare the effectiveness of the SADF and GSADF tests in detecting bubbles in Tunisia. Second, it seeks to enhance detection precision by incorporating various simulation methods, including Monte Carlo and bootstrap techniques. Finally, the research will investigate the relationship between speculative bubbles and volatility regimes using the Markov-Switching

Generalized Autoregressive Conditional Heteroskedasticity (MSGARCH) model, providing insights for effective mitigation strategies in the Tunisian context.

By focusing on the Tunisian stock market, this study contributes to the literature on speculative bubbles and financial market volatility. It uses an innovative methodological approach that combines the GSADF and SADF tests, simulation methods and the MSGARCH model for a detailed analysis of speculative bubbles and market volatility. The study also provides practical implications for investors, regulators and policymakers to mitigate the harmful effects of speculative bubbles, enhancing financial market stability and efficiency.

The article is structured as follows: Section 2 reviews previous studies, Section 3 outlines the methodology, Section 4 describes the data set, Section 5 discusses results, and Section 6 concludes with key findings and suggestions for future research.

2. Literature review

The detection of speculative bubbles and their link to volatility has long intrigued researchers and policymakers. These bubbles, marked by rapid price increases followed by sharp declines, pose substantial economic risks. Early identification is crucial for mitigating their impact on financial stability. In addition, understanding the variance dynamics within these bubbles improves our understanding of market volatility.

2.1 Speculative bubbles detection

Detecting explosive bubbles in financial markets is crucial due to their significant economic consequences, marked by rapid price surges followed by sharp declines. The variance bounds test, introduced by Shiller (1981) and LeRoy and Porter (1981), posits that when a rational bubble exists, the variance of observed asset prices should exceed that of fundamental values. This foundational approach led to more advanced detection techniques.

Campbell and Shiller (1987) proposed a unit root test, identifying two scenarios for bubble detection: one where the asset price is nonstationary while the fundamental value is stationary, and another where both are nonstationary, requiring co-integration tests to confirm bubble presence. Diba and Grossman (1988) emphasized that the gap between asset and fundamental prices aids bubble detection, using unit root and co-integration tests.

However, these tests have limitations. Evans (1991) noted that they often misclassify time series with periodically collapsing bubbles as stationary. To address this, Phillips and Yu (2011) and Phillips et al. (2015) developed the SADF and GSADF tests for identifying bubbles at their formation and distortion, allowing real-time identification.

Bago, Souratié, Ouédraogo, Ouédraogo and Dembélé (2019) used GSADF on South African stock data, identifying three bubbles related to the 1979 oil crisis and the 2008 global financial crisis, showing exogenous shocks' influence on price variations. Wang, Chang and Min ((2022) applied SADF and GSADF tests in Taiwan and Mainland China's tourism stock markets, finding evidence of bubbles during COVID-19.

Zeren and Yilanc (2019) reported multiple bubbles across 15 countries, highlighting the GSADF test as an early warning tool for financial crises. Escobari, Garcia and Mellado (2017) identified explosive bubbles in six Latin American markets, concluding that these periods started earlier and lasted longer than in the USA during the 2008 crisis. Korkmaz et al. (2021) used Recursive Augmented Dickey-Fuller (RADF), SADF and GSADF tests to identify bubbles in various stock markets, with the GSADF proving effective except for the S&P 500.

Li, Xiao, Yang, Guo and Yang (2021) examined periodically failing bubbles in the Chinese stock market, identifying eight bubbles in the Main-Board Market, six in small and medium-sized enterprises and four in the growth enterprise market, primarily attributed to liquidity factors.

2.2 Variance and speculative bubble dynamics

Variance is crucial in identifying speculative bubbles, with elevated levels recognized as indicators of speculative growth or impending price collapse. Recent studies support this perspective. Fang (2001) analyzed daily closing prices on the Taiwan Stock Exchange (1995–1998) using the Autoregressive Conditional Heteroskedasticity (3)-M model, finding that return variance significantly increased during the 1997–1998 Asian crisis, indicating heightened volatility. Taherian & Minouei (2016) studied the Tehran Stock Exchange with the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, revealing a significant link between conditional return variance and bubble presence, suggesting that volatility fluctuations are tied to speculative behavior.

Omoruyi, Hassan and Evbaziegbere (2017) noted that high, mean-reverting variance serves as a key indicator of speculative bubbles, emphasizing the need for regulatory oversight to manage market volatility. Diniz, Prince and Maciel (2023) further highlighted the role of variance in volatile assets like cryptocurrencies, finding that shifts in variance signal unstable market conditions and potential bubble formations. Finally, Ethereum exhibited higher variance during high-volatility periods, indicating greater susceptibility to speculative dynamics than Bitcoin.

2.3 Association between speculative bubbles and volatility

Several studies have analyzed the relationship between bubble periods and volatility regimes in financial markets. Blasques et al. (2014) analyzed the American stock market (1990–2010) using volatility regime shift models to anticipate turbulent periods, improving forecasts by capturing abrupt volatility changes. Sornette, Cauwels and Smilyanov (2018), in their analysis of 40 historical bubbles, showed that asset price volatility increases significantly during bubbles, indicating that volatility can serve as an early warning signal for unsustainable price rises and crashes.

Ardia, Bluteau and Rüede (2019) tested Bitcoin's volatility patterns using MSGARCH models, finding significant regime changes in its GARCH process. Ciaian, Rajcaniova and Kancs (2016) identified a positive association between bubble phases and increased volatility driven by intense speculative activity, noting that increased investor demand and trading volume contribute to rapid price swings.

Similarly, Hafner (2020) explored speculative bubbles in cryptocurrencies, adapting traditional unit root tests to account for time-varying volatility. His model combines a deterministic long-run component – anticipating reduced volatility as cryptocurrencies mature – with a stochastic short-run component, capturing volatility clustering typical of speculative assets.

Balcombe and Fraser (2017) adopted Bayesian Markov switching models to investigate bubble dynamics across nine data series, finding significant regime switching, with each asset reflecting distinct bubble behaviors.

3. Methodology

To identify speculative bubbles in the Tunisian stock market and analyze associated volatility regimes, we used the SADF and GSADF tests alongside the MSGARCH model.

3.1 Bubble detection methods: Supremum Augmented Dickey-Fuller and Generalized Supremum Augmented Dickey-Fuller tests

To accurately identify the phases of explosive price behavior in Tunisia, we use right-tailed unit root tests, specifically the SADF and GSADF tests.

3.1.1 Supremum Augmented Dickey-Fuller test. The SADF test, developed by Phillips and Yu (2011), is based on the recursive calculation of the Augmented Dickey-Fuller (ADF) statistic, starting from a fixed point and progressively increasing the observation window. Initially, the starting point of the window is set to the first observation ($r_1 = 0$). The end point of the first window (r^2) is determined by the chosen minimum window size (r_0), resulting in an initial window size of $r_w = r^2$.

The SADF statistic is defined as the maximum value in the sequence ADF_{r2}:

$$\mathbf{SADF}(\mathbf{r}_0) = \underbrace{\mathbf{sup}}_{\mathbf{r}_0 \leq \mathbf{r}_2 \leq 1} \mathbf{ADF}_0^{\mathbf{r}_2} \tag{1}$$

where:

SADF(r_0) = The SADF statistic calculated at the minimum window size r_0;
sup = The supremum operator;
r_0 = The minimum size of the window used for the initial observation;
r^2 = The endpoint of the observation window, varying between r_0 and 1; and
$ADF_0^{r_2}$ = The ADF statistic calculated at window size r^2 and at the initial time point.

To date the explosive behavior in the price series, the supreme value of the ADF statistic obtained is compared to the critical values from the right of its limit distribution. The following equation represents this:

$$\underbrace{\mathbf{sup}}_{\mathbf{r}_0 \leq \mathbf{r}_2 \leq 1} \mathbf{ADF}_0^{\mathbf{r}_2} \rightarrow \underbrace{\mathbf{sup}}_{\mathbf{r}_0 \leq \mathbf{r}_2 \leq 1} \frac{\int_0^{\mathbf{r}_2} \mathbf{W}_\mathbf{d}^2 \mathbf{W}}{\mathbf{W}_\mathbf{d}^2 \mathbf{W}} \tag{2}$$

where:

W_d = A stochastic process (e.g. Brownian motion);
W = A standard Weiner process that implies convergence in distribution;
$\int_0^{r_2} W_d W$ = Captures the accumulated value of the process up to r^2; and
$W_d^2 W$ = Represents a scaling factor involving the square of the stochastic process.

The principle of timestamping in the context of the SADF test involves a comparative analysis of each element within the statistical sequence $ADF_0^{r_2}$ against the critical value at the extreme right of the standard ADF test. The origin date of a bubble, denoted as T_{r_e}, corresponds to the first chronological observation in which the statistic $ADF_0^{r_2}$ exceeds the corresponding critical value. Conversely, the estimated date of the bubble's bursting, denoted as T_{r_f}, is defined as the first chronological observation in which the statistic $ADF_0^{r_2}$ falls below the applicable critical value.

Formally, the prediction of the beginning ($\hat{r}_e$) and end ($\hat{r}_f$) dates of an explosive bubble based on the SADF test is defined as follows:

$$\hat{\mathbf{r}}_\mathbf{e} = \underbrace{\mathbf{inf}}_{\mathbf{r}_0 \leq \mathbf{r}_2 \leq 1} \left\{ \mathbf{r}_2; \mathbf{ADF}_0^{\mathbf{r}_2} > \mathbf{scu}_{\mathbf{r}_2}^{a_t} \right\} \tag{3}$$

$$\hat{\mathbf{r}}_\mathbf{f} = \underbrace{\mathbf{inf}}_{\hat{\mathbf{r}}_\mathbf{e} \leq \mathbf{r}_2 \leq 1} \left\{ \mathbf{r}_2; \mathbf{ADF}_0^{\mathbf{r}_2} > \mathbf{scu}_{\mathbf{r}_2}^{a_t} \right\} \tag{4}$$

where:

$\widehat{r}_e$ = The beginning date of the bubble;

$\widehat{r}_f$ = The end date of the bubble;

$ADF_0^{r_2}$ = The ADF statistic calculated at window size r^2 and at the initial time point; and

$scu_{r_2}^{a_t}$ α is the 100 (1 − α)% critical value of the standard ADF statistic based on T_{r_2} observations.

3.1.2 Generalized Supremum Augmented Dickey-Fuller test. The GSADF test, proposed by Phillips et al. (2015), is an extension of the SADF test that provides greater flexibility in estimation windows. Unlike the SADF test, which uses a fixed starting point r_1 across all estimates, the GSADF test allows r_1 to vary within the interval [0, r^2-r_0], enabling a more comprehensive detection of potential bubble episodes.

The GSADF statistic is defined as follows:

$$GSADF(r_0) = \underbrace{\sup}_{\substack{r_0 \leq r_2 \leq 1 \\ 0 \leq r_1 \leq r_2 - r_0}} ADF_{r_1}^{r_2} \quad (5)$$

where:

$GSADF(r_0)$ = The GSADF statistic, which depends on an initial window size r_0.

r_1 = Represents the starting point of each sample window, which can vary from 0 up to $r_2 - r_0$.

$ADF_{r_1}^{r_2}$ = The ADF statistic calculated for each subsample with a starting point r_1 and an endpoint r_2.

Phillips et al. (2015) proposed a dating strategy based on the Backward Supremum Augmented Dickey-Fuller (BSADF) statistic, defined as follows:

$$BSADF_{r_2}(r_0) = \underbrace{\sup}_{0 \leq r_1 \leq r_2 - r_0} ADF_{r_1}^{r_2} \quad (6)$$

The start date of the bubble burst is identified as the first chronological observation where the BSADF statistic exceeds the critical value:

$$\widehat{\mathbf{r}}_{\mathbf{e}} = \underbrace{\mathbf{inf}}_{\mathbf{r}_0 \leq \mathbf{r}_2 \leq 1} \left\{\mathbf{r}_2; \mathbf{BSADF}_0^{\mathbf{r}_2} > \mathbf{scu}_{\mathbf{r}_2}^{a_t}\right\} \quad (7)$$

The estimated end date of the exuberance period is defined as the first observation where the BSADF statistic surpasses the corresponding critical value:

$$\widehat{\mathbf{r}}_{\mathbf{f}} = \underbrace{\mathbf{inf}}_{\widehat{\mathbf{r}}_{\mathbf{e}} \leq \mathbf{r}_2 \leq 1} \left\{\mathbf{r}_2; \mathbf{BSADF}_0^{\mathbf{r}_2} > \mathbf{scu}_{\mathbf{r}_2}^{a_t}\right\} \quad (8)$$

where:

$BSADF_0^{r_2}$ = BSADF statistic calculated from the starting point 0 to $r^2$$scu_{r_2}^{a_t}$ α is the 100 (1−α)% critical value of the standard ADF statistic based on T_{r_2} observations.

Three methods are used to simulate the critical values in the SADF and GSADF tests: Monte Carlo, Sieve-bootstrap (Gutierrez, 2011) and Wild-bootstrap (Harvey et al., 2016). The Monte Carlo method is valued for its simplicity and flexibility across various models but is computationally intensive and can produce variable results due to its stochastic nature (Robert, 1999). The Sieve-bootstrap method, proposed by Gutierrez (2011), is particularly effective for time series data, as it captures dependence structures like autocorrelation, though it requires precise specification of the autoregressive model, which increases its complexity (Kreiss & Paparoditis, 2011). The Wild-bootstrap method, introduced by Harvey, Leybourne, Sollis and Taylor (2016), is beneficial for addressing heteroscedastic errors and providing improved size control and critical value estimation. However, it necessitates careful modeling of the error distribution, making its implementation more complex than the Monte Carlo method (Davidson & Flachaire, 2008).

3.1.2.1 Monte Carlo method description. The Monte Carlo procedure involves generating synthetic data through numerous simulations and follows these steps:

Step 1: Generate a sequence of random walk series with an asymptotically negligible drift over a sample size of *T*;

Step 2: For each simulated time series, apply the ADF test. This test determines whether the simulated data exhibit a unit root (indicating nonstationarity) or are stationary;

Step 3: Calculate the SADF and GSADF statistics from the simulated series;

Step 4: Compare the test statistics obtained from the simulated data with the critical values;

Step 5: Repeat the simulation process multiple times (e.g. 2,000 replications) to obtain a distribution of test statistics; and

Step 6: The 100 $(1 - \alpha)$% critical value of each test statistic, derived from Monte Carlo simulation, is the 100 $(1 - \alpha)$ percentile of the corresponding distributions obtained in step 5.

3.1.2.2 Sieve-bootstrap method description. Gutierrez (2011) simulates the critical values of the SADF and GSADF tests using the Sieve-bootstrap method. The basic steps of this method are outlined as follows:

Step 1: Fit the following autoregressive process using ordinary least squares (OLS) for the complete sample period t = 1, …, T, to obtain the estimated residuals:

$$\Delta y_t = \mu + p y_{t-i} - \varepsilon_{i=1}^{T} \varnothing_i \Delta y_{t-i} + \varepsilon_t \tag{9}$$

Where:

Δy_t = Change in the variable y at time t;

μ = Constant term;

p = Autoregressive coefficient;

T = Total number of observations in the sample; and

εt = Error term at time t.

Step 2: Resample $\varepsilon_{\mathbf{t}}^{*}$ based on the centered residuals $\widehat{\boldsymbol{\varepsilon}}_{\mathbf{t}} - \overline{\boldsymbol{\varepsilon}}_{\mathbf{t}}$

Where $\overline{\varepsilon}_t = \sum_{t=1}^{T} \frac{\widehat{\varepsilon}_t}{T}$.

Step 3: Generate the bootstrapped series y_t* using the OLS estimates $\widehat{\phi}_k$ and the residuals $\varepsilon_{\mathbf{t}}^{*}$.

Step 4: Use the bootstrapped series and repeat steps 2–4 B times to obtain the critical values of the SADF and GSADF tests.

3.1.2.3 Wild-bootstrap method description. Harvey et al. (2016) introduced an alternative bootstrap method for estimating critical values in the context of the SADF and GSADF tests. Like Gutierrez's method, this approach involves resampling the residuals obtained from fitting an autoregressive model to the data. However, only Step 2 differs from Gutierrez's procedure, while Steps 1, 3 and 4 remain the same.

In step 2, Harvey et al. (2016) resample ε_t^* based on weighted residuals $w_t \dot{A}_t$, where w_t is randomly drawn from a standard normal distribution. These weighted residuals are then used to generate ε_t^* by replacing the estimated residuals $\hat{\varepsilon}_t$.

3.2 Volatility *regime* analysis method: Markov-Switching Generalized Autoregressive Conditional Heteroskedasticity model

The MSGARCH model, developed by Haas, Mittnik and Paolella (2004), extends the conventional GARCH model by incorporating multiple volatility regimes. It assumes that returns follow a Markov process, with transitions between volatility states depending solely on the current regime. This approach improves the model's ability to capture time-varying volatility and regime shifts in financial data.

Formally, the conditional volatility at time *t* in the MSGARCH model is expressed as:

$$y_t = \mu_t(S_t) + \sigma_t(S_t)\,\eta_t \tag{10}$$

where:

$y_t = \ln(P_t) - \ln(P_{t-1})$ = the log-returns of stock price index (Tunisian stock market index [TUNINDEX]) at time *t*, $t = 1, \dots, T$.

$\mu_t(S_{1:t})$ and $\sigma_t(S_{1:t})$ denote measurable functions concerning a tribe generated by the random vector $(y_{1:t-1}, S_{1:t})$, where the processes $\{\eta_t\}$ and $\{S_t\}$ are independent. Therefore, the conditional mean of the return at time *t* is $E[y_t \mid y_{1:t-1}, S_{1:t}] = \mu_t(S_{1:t})$ and the conditional variance is $Var[y_t|y_{1:t-1}, S_{1:t}] = \sigma_t^2\,(S_{1:t})$. Since both the conditional mean and variance now depend on the unobserved Markov chain, they are no longer deterministic, given the observable market information.

The MSGARCH model is distinguished by the fact that the conditional distribution of y_t depends on the entire regime path from 1 to *t*, which makes calculating the log-likelihood practically impossible and estimating the parameters very difficult. To simplify the estimation process, we consider a subclass of regime-switching models:

$$y_t = \mu_t(S_t) + \sigma_t(S_t)\,\eta_t \tag{11}$$

where $\mu_t(S_t)$ and $\sigma_t(S_t)$ denote functions that are measurable concerning a tribe generated by the random vector $(y_{1:t-1}, S_t)$. In this model, the conditional probability density of y_t depends only on the regime at time *t*, rather than on the entire history of regimes:

$$f(y_t \mid y_{1:t-1}, S_{1:t}, \theta) = f(y_t \mid y_{1:t-1}, S_t, \theta) \tag{12}$$

where $S(\cdot)$ represents the family of GARCH conditional heteroskedasticity function (GARCH and EGARCH), which defines the conditional variance process.

4. Data description

We use the daily closing price series of the TUNINDEX extracted from the Datastream database. The decision to focus on the Tunisian Stock Market is based on the assumption that, in an era of financial globalization and increasing interdependence among stock markets, the Tunisian market can be influenced by global crises. The study period spans from January 5, 2004, to December 29, 2023, comprising 4,989 daily observations.

5. Results and discussion

5.1 Speculative bubbles detection results and correlation analysis of simulation methods

5.1.1 Speculative bubble detection results. By testing for the existence of speculative bubbles in the Tunisian Stock Market using right-tailed unit root tests, we obtained the SADF and GSADF test statistics through Monte Carlo simulations, as well as the Sieve-bootstrap and Wild-bootstrap methods, as detailed in Table 1.

The results show that the SADF and GSADF test statistics, using Monte Carlo, wild-bootstrap and sieve-bootstrap procedures on the TUNINDEX, exceed their critical values at the 1%, 5% and 10% significance levels. This confirms the presence of rational bubbles on the Tunis Stock Exchange from January 2004 to December 2023.

The results of identifying explosive bubbles are shown in Table 2.

The analysis of speculative bubbles in the Tunisian Stock Market, based on the SADF and GSADF tests, reveals that bubble durations vary in response to significant economic, financial and political events. Bago et al. (2019) and Li et al. (2021) highlight the importance of exogenous shocks in bubble formation. The GSADF test is more effective than the SADF in detecting longer and more frequent bubbles, as shown by Korkmaz et al. (2021). In addition, the choice of simulation method significantly impacts results: Monte Carlo simulations identify longer bubbles, while Sieve-bootstrap and Wild-bootstrap methods capture shorter, fragmented bubbles, indicating greater sensitivity to market corrections.

The Tunisian Stock Market was significantly affected by the subprime crisis, primarily due to economic ties with Europe, which led to reduced trade, production and heightened speculation in the TUNINDEX. Moreover, political instability following the Arab Spring and terrorist attacks in 2015 further intensified market volatility and eroded investor confidence. In addition, the European sovereign debt crisis in 2012 compounded these effects, as foreign investors' reactions pressured the local market.

5.1.2 Correlation analysis of simulation methods for identifying speculative bubbles in the Tunisian Stock Market using the Supremum Augmented Dickey-Fuller and Generalized Supremum Augmented Dickey-Fuller tests. The correlation analysis of three simulation methods for detecting speculative bubbles in the Tunisian Stock Market using the SADF and GSADF tests reveals notable differences. The SADF test shows a strong correlation between the Monte Carlo and Sieve-bootstrap methods (0.951833) and a moderate correlation with the Wild-bootstrap method (0.815121). The Sieve-bootstrap and Wild-bootstrap methods have a lower correlation (0.796147), indicating some divergence. In contrast, the GSADF test shows

Table 1. Results of the SADF and GSADF test statistics applied to the TUNINDEX Stock Index from January 2004 to December 2023

Procedure	t-calculated	t-statistics 1%	5%	10%	*p*-value
SADF					
Monte Carlo	7.189721	2.063540	1.603520	1.394152	0.0000
Sieve-bootstrap	7.189721	2.361277	1.694200	1.405327	0.0000
Wild-bootstrap	7.085482	6.434227	4.736883	3.936739	0.0000
GSADF					
Monte Carlo	9.814678	8.402781	7.179462	7.037146	0.0000
Sieve-bootstrap	7.943072	6.872400	6.138024	5.916405	0.0000
Wild-bootstrap	8.746001	7.803691	6.880064	6.570910	0.0000

Source(s): Authors' own creation

Table 2. Speculative bubbles identified by the SADF and GSADF tests in the Tunisian Stock Market

	SADF test			GSADF test		
Procedure	Beginning date	End date	Duration	Beginning date	End date	Duration
Monte Carlo	06/04/2005	02/12/2008	3 years, 7 months and 21 days	05/04/2005	29/09/2008	3 years, 5 months and 28 days
	30/03/2009	28/03/2011	1 year and 11 months	12/08/2009	16/12/2010	1 year, 4 months and 6 days
	05/04/2012	20/06/2012	2 months and 15 days	20/04/2015	03/08/2015	3 months and 15 days
	12/07/2012	14/09/2012	2 months and 2 days	19/07/2017	31/08/2017	1 month and 13 days
	08/03/2018	04/02/2019	10 months and 20 days	21/12/2017	10/09/2018	8 months and 23 days
	01/06/2023	14/09/2023	3 months and 13 days	30/08/2022	30/09/2022	1 month and 1 day
	–	–	–	26/05/2023	25/08/2023	3 months and 1 day
Sieve-bootstrap	06/04/2005	03/12/2008	3 years, 7 months and 27 days	05/04/2005	14/10/2008	3 years, 6 months and 9 days
	11/02/2009	29/03/2011	2 years, 1 month and 18 days	04/01/2009	31/12/2010	1 year, 11 months and 27 days
	30/03/2012	13/09/2012	5 months and 14 days	13/04/2015	31/07/2015	3 months and 18 days
	26/03/2018	14/12/2018	8 months and 18 days	01/08/2017	31/08/2017	30 days
	01/06/2023	11/09/2023	3 months and 10 days	11/01/2018	24/10/2018	9 months and 13 days
	–	–	–	01/09/2022	30/09/2022	29 days
	–	–	–	23/05/2023	23/08/2023	3 months
Wild-bootstrap	06/04/2005	20/07/2007	2 years, 3 months and 14 days	05/04/2008	21/10/2008	6 months and 16 days
	15/11/2007	03/10/2008	10 months and 18 days	31/03/2009	03/01/2010	9 months and 3 days
	05/06/2009	10/01/2011	1 year, 7 months and 5 days	07/04/2015	31/07/2015	3 months and 24 days
	21/03/2018	19/10/2018	6 months and 28 days	28/07/2017	31/08/2017	1 month and 3 days
	–	–	–	12/01/2018	27/11/2018	10 months and 15 days
	–	–	–	01/09/2022	30/09/2022	29 days
	–	–	–	11/05/2023	25/08/2023	3 months and 14 days

Source(s): Authors' own creation

stronger correlations, particularly between Sieve-bootstrap and Wild-bootstrap (0.972267), with the Monte Carlo method also showing strong relationships (0.903883 with Sieve-bootstrap and 0.897326 with Wild-bootstrap). These findings suggest that while all methods are consistent, the GSADF test exhibits greater agreement among the simulation methods, particularly between Sieve-bootstrap and Wild-bootstrap, enhancing their reliability in identifying speculative bubbles.

Table 3 presents the correlation matrices for these simulation methods.

5.2 Volatility *regime* analysis during speculative bubbles

The MSGARCH model estimation results for TUNINDEX returns (Table 4) show varying conditional variance, with high volatility (regime 2: 146.02% annualized volatility) and low volatility (regime 1:37.48%). This reinforces Fang's (2001) finding that heightened volatility

Table 3. Correlation matrix between three simulation methods for identifying speculative bubbles in the Tunisian Stock Market using the SADF and GSADF tests

Simulation methods	Monte Carlo	Sieve-bootstrap	Wild-bootstrap
Correlation matrix of three simulation methods for identifying speculative bubbles in the Tunisian stock market using the SADF test			
Monte Carlo	1	0.951833	0.815121
Sieve-bootstrap	0.951833	1	0.796147
Wild-bootstrap	0.815121	0.796147	1
Correlation matrix of three simulation methods for identifying speculative bubbles in the Tunisian stock market using the GSADF test			
Monte Carlo	1	0.903883	0.897326
Sieve-bootstrap	0.903883	1	0.972267
Wild-bootstrap	0.897326	0.972267	1

Source(s): Authors' own creation

Table 4. Parameter estimation results of the MSGARCH model for TUNINDEX return volatility

Parameter	Estimate	Standard error	*t*-statistic	*p*-value
$\alpha_{0.1}$	0.06742	0.0001	51.21716	0.0000***
$\alpha_{1.1}$	0.08625	0.0009	298.4291	0.0000***
$\alpha_{2.1}$	0.35175	0.0000	478.2159	0.0000***
β_1	0.81402	0.0000	3,524.027	0.0000***
$\dot{A}_1$	2.91176	0.0010	530.0439	0.0000***
Inc. vol		37.48%		
$\alpha_{0.2}$	0.08492	0.0002	135.413	0.0000***
$\alpha_{1.2}$	0.14678	0.0009	466.426	0.0000***
$\alpha_{2.2}$	0.29072	0.0000	359.009	0.0000***
β_2	0.7626	0.0000	40,184.12	0.0000***
$\dot{A}_2$	25.8264	0.6042	37.08391	0.0000***
Inc. vol		146.02%		
$\square_{1.1}$	0.7120	0.0075	329.516	0.0000***
$\square_{2.2}$	0.1869	0.0097	195.007	0.0000***

Note(s): ***Indicate significance at the 1% level
Source(s): Authors' own creation

signals market instability during crises, supported by Taherian and Minouei (2016) and Omoruyi et al. (2017), who linked increased return variance with speculative bubbles during economic strain.

Both regimes' positive and significant volatility asymmetry indicates that negative shocks increase volatility, while positive shocks reduce it. These findings align with Ardia et al. (2019), who observed similar shifts in Bitcoin's volatility dynamics, and Ciaian et al. (2016), who linked speculative bubbles to heightened volatility due to increased investor speculation. Hafner (2020) also highlighted the importance of time-varying volatility in bubble detection.

The smoothed transition probabilities for the TUNINDEX from 2004 to 2023 are illustrated in Figure 1. When the transition probability for regime 1 exceeds 0.5, it indicates low volatility, while a probability below 0.5 signifies high volatility in regime 2. The figure clearly distinguishes between the two regimes, with the low-volatility regime (regime 1) predominating. High-volatility periods are notably associated with speculative bubbles.

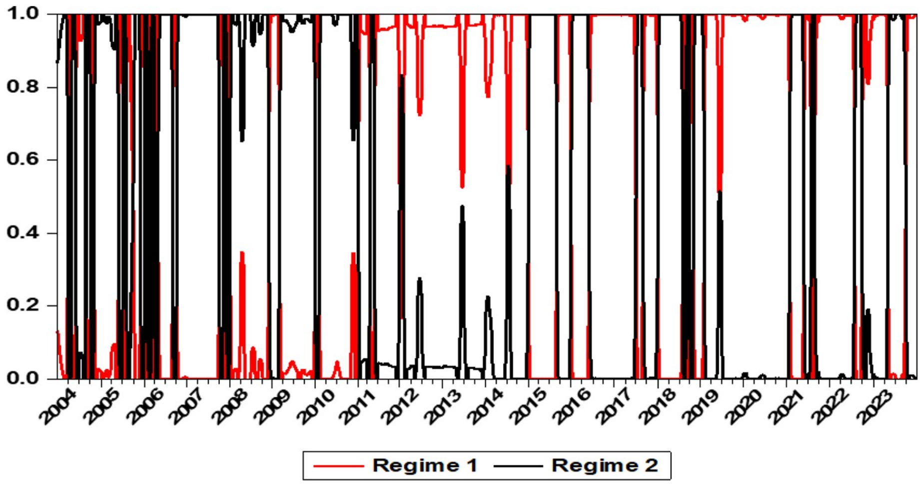

Figure 1. Evolution of smoothed transition probabilities between low (regime 1) and high (regime 2) volatility regimes for TUNINDEX using MSGARCH models
Source(s): Authors' own creation

Table 5. Association of TUNINDEX bubble periods identified by the GSADF test with volatility regimes based on MSGARCH transition probabilities

Bubble–volatility regime association	Monte Carlo	Sieve-bootstrap	Wild-bootstrap
Bubbles	2,309	2,560	1,078
Regime 1	728	598	132
Regime 2	1,581	1,962	946
Regime1%	31.52	23.36	12.24
Regime 2%	68.48	76.64	87.76

Source(s): Authors' own creation

Based on various methods used to identify bubble periods, we analyzed their association with volatility regimes: low volatility (regime 1) and high volatility (regime 2). Table 5 shows that bubbles detected by the GSADF test are predominantly linked to the high-volatility regime, regardless of whether critical values were determined via Monte Carlo simulation or bootstrap methods. High volatility increases market uncertainty, leads to greater information asymmetry and triggers different behaviors from investors. Informed investors reduce demand and prices by adjusting their portfolios, while uninformed, risk-averse investors extrapolate rising volatility, fueling speculative bubbles. These findings are consistent with Frankel (2008), who suggests that uninformed traders make adaptive volatility predictions, contributing to market crashes. Similarly, Antonakakis and Scharler (2012) demonstrated that volatility typically rises before a crash and remains elevated afterward.

6. Conclusion

This study examined speculative bubbles in the Tunisian stock market from 2004 to 2023 using the SADF and GSADF tests, along with Monte Carlo, Sieve-bootstrap and Wild-bootstrap simulations. It also analyzed return volatility during bubble periods using the MSGARCH model.

The GSADF test proved more effective than the SADF test, detecting longer and more frequent bubbles, consistent with Korkmaz et al. (2021). The simulation methods influenced the results: Monte Carlo simulations identified more sustained bubble durations, while Sieve-bootstrap and Wild-bootstrap methods captured shorter, more responsive bubbles, highlighting the market's sensitivity to corrections.

The identified bubbles coincided with major economic, financial and political events, such as the 2008 subprime crisis, the Arab Spring, the 2015 terrorist attacks and the 2012 European debt crisis. These external shocks intensified market volatility, with Tunisia's economic ties to Europe and political instability, particularly in tourism, further contributing to instability. These findings emphasize the importance of considering political and economic risks in investment strategies (Bago et al., 2019; Li et al., 2021).

Analysis of TUNINDEX returns during bubbles revealed two volatility regimes: low (37.48%) and high (146.02%) volatility. Regime 1, characterized by low volatility, persisted longer while regime 2 was shorter. These results align with Fang (2001), Taherian and Minouei (2016) and Omoruyi et al. (2017), linking volatility and bubbles during economic strain. The study also showed that negative shocks increase volatility, supporting Ardia et al. (2019). The correlation between bubble periods and high volatility highlights how elevated volatility increases uncertainty, influencing investor behavior, as Frankel (2008) and Antonakakis and Scharler (2012) observed.

This study enhances the understanding of speculative bubbles in Tunisia, contributing to the financial economics literature on emerging markets. Methodologically, it highlights the GSADF test's effectiveness, the accuracy of the Wild and Sieve-bootstrap methods and the MSGARCH model's ability to distinguish volatility regimes during bubbles. Societally, it helps mitigate economic uncertainty, safeguards savings and fosters financial stability, boosting public confidence. Practically, it provides insights for managing bubbles and associated risks, enabling investors to make informed decisions, diversifying portfolios, guiding regulators in ensuring market stability and supporting policymakers in addressing instabilities to build a resilient stock market with robust regulation and crisis management.

Limitations include the study's focus on the Tunisian Stock Market, limiting generalizability. Future research could explore rational and irrational bubbles in emerging markets, assess market factors like liquidity and investor behavior, evaluate policy interventions and use machine learning to improve bubble detection and crisis prevention.

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Authors' contribution: Sirine Ben Yaala – Conceptualization (Equal), Data curation (Equal), Formal analysis (Equal), Investigation (Equal), Methodology (Equal), Project administration (Equal), Resources (Equal), Software (Equal), Supervision (Equal), Validation (Equal), Visualization (Equal), Writing – original draft (Equal), Writing – review & editing (Equal); Jamel Eddine Henchiri – Formal analysis (Equal), Investigation (Equal), Methodology (Equal), Project administration (Equal), Supervision (Equal), Validation (Equal), Visualization (Equal), Writing – review & editing (Equal).

Corresponding author
Sirine Ben Yaala can be contacted at: sirine17@live.fr

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