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A Self-enforcing International Environmental Agreement on Matching Rates: Can It Bring About an Efficient and Equitable Outcome?*

Toshiyuki Fujita

Faculty of Economics, Kyushu University, Japan; tfujita@econ.kyushu-u.ac.jp

ABSTRACT

We incorporate matching schemes into a model of transboundary environmental agreements and investigate their effectiveness using three-stage game models. In the first stage, each country decides whether to accede to the agreement. In the second stage, the signatories collectively choose a common matching rate. Finally, in the third stage, each signatory and non-signatory determines its unconditional flat abatement noncooperatively, taking the value of the matching rate as given. An additional abatement is imposed upon each signatory, which is obtained by multiplying the total of all the other countries' flat abatements by the matching rate. The analysis of a matching agreement game with symmetric countries as players suggests the existence of a self-enforcing agreement leading to an efficient and equitable outcome, which shows that matching schemes are effective.

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Introduction

One remarkable feature of transboundary environmental problems is that no organization has the supranational power to control anthropogenic pollutants. Hence, to develop measures to protect the environment, it is essential to conclude an international agreement. An agreement is said to be *self-enforcing* if no country has an incentive to change decisions on its accedence. The design of any agreement should aim to prevent free riding and thus realize the large self-enforcing agreement.

Earlier theoretical analyses of international environmental agreements, such as Carraro and Siniscalco (1993), Barrett (1994), have shown that the size of self-enforcing agreement is generally small.[1] They typically describe the agreements as two-stage games with sovereign countries as players. In the first stage, each country decides simultaneously whether to accede to the agreement. In the second stage, signatories collectively choose the abatements to maximize their total payoffs, whereas each non-signatory behaves noncooperatively. The payoff is defined as a function of the abatements of all countries. Each country considers its final payoff and decides whether to be a signatory.

In this paper, we apply the *matching* concept proposed by Guttman (1978) in the context of noncooperative voluntary provisions of public goods to international environmental agreements, and investigate the effectiveness of *matching agreements* on environmental protection.[2] Matching agreements do not fix the pollution abatement but rather determine the so-called "*matching rate*" for each country. Subsequently, each country fixes an unconditional *flat abatement* noncooperatively. Consequently, a country has — additionally to its unconditional flat abatement — to conditionally provide

1 More recent studies that examine various factors include Na and Shin (1998), Barrett (2001), and Lange and Vogt (2003).

2 Matching schemes have also been studied in Guttman (1987), Danziger and Schnytzer (1991), Guttman and Schnytzer (1992), Althammer and Buchholz (1993), Varian (1994a, 1994b), Boadway *et al.* (2007), and Buchholz *et al.* (2009, 2011, 2012).

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a matching component consisting of the product of its announced matching rate(s) and other countries' flat abatement levels. Thus, the matching component is conditional on other countries' contributions.

Rübbelke (2006) is the first study that applies matching schemes to environmental problems. He extends Guttman (1978, 1987) and focuses on the ancillary benefits of the global environmental policies. In conclusion, Rübbelke (2006) emphasizes that matching agreements are effective because generally there is no need for changes in the matching rate, as each country can update its flat abatement rate whenever a new ancillary benefit is discovered. Boadway *et al.* (2011) study matching schemes in the framework of a noncooperative pollution abatement game with asymmetric countries, and show that such schemes lead to efficient subgame perfect equilibria. Furthermore, they extend their model to various cases, such as the case when countries conduct emissions quota trading and the case of a dynamic two-period setting, and get the result that the efficiency of matching schemes holds in these cases.

In the models of Rübbelke (2006) and Boadway *et al.* (2011), each country chooses its matching rate noncooperatively like in all other studies of matching schemes, although Rübbelke (2006) uses the term "matching agreement." We analyze matching schemes within the framework of self-enforcing agreements in the present study because a number of international negotiations concern environmental issues and thus it is more realistic to consider the matching behaviors of countries as the outcomes of their commitments made in an agreement, rather than as purely voluntary actions. We formulate an agreement on the matching rate based on the simple pollution abatement models by Barrett (1994) and Na and Shin (1998) to show that there exists an equilibrium in which all countries participate in the agreement and that an efficient and equitable outcome is realized.

As noted earlier, Carraro and Siniscalco (1993) and Barrett (1994), among other earlier studies of international environmental agreements, show that the size of self-enforcing agreements would be small unless considerable restrictions are imposed on the payoff structures and behavioral patterns of each country in the setting or assumptions of the model. The concept of a matching agreement in our model, by contrast, is noteworthy because it does not force the countries to undertake any commitment other than following the simple rules of matching.

The contributions of our study to the literature are as follows: it defines a matching agreement through the application of matching theory to a

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framework of an international environmental agreement by Barrett (1994) and Na and Shin (1998); and it clearly shows the self-enforcing agreement scheme by matching that has not been achieved before. In addition, we have extended previous studies by allowing the existence of countries that do not commit to the matching, and analyzed the interdependencies between the actions of signatories and those of non-signatories.

The remainder of this paper is organized as follows. In section "Model for Matching Agreements," after formulating a model of international pollution abatement and obtaining the first-best solution, we define a game that incorporates matching agreements into the basic model and explain the game's solution concepts. In section "Solution of the Matching Agreement Game," we prove the existence of efficient and self-enforcing agreements and show the effectiveness of matching agreements. Finally, we conclude in section "Conclusion."

Model of Matching Agreements

International Environmental Agreements

Let us start by describing the game of standard international environmental agreements. Players of the game are the governments of n symmetric countries that share the environment. Let $N = \{1, \ldots, n\}$ denote the set of countries. We focus on certain transboundary pollutants. The benefit derived from a single country's pollution abatement is proportional to its amount of abatement, which affects all countries. The abatement cost is incurred entirely by the abating country. Let $\boldsymbol{x} = (x_1, \ldots, x_n)$ denote the vector of abatement of countries, and the payoff to country $i(\in N)$ is

$$\pi_i = \sum_{j \in N} x_j - C(x_i), \tag{1}$$

where $C(\cdot)$ is a three-times differentiable cost function that satisfies $C(0) = C'(0) = 0$, $\lim_{x\to\infty} C(x) = \lim_{x\to\infty} C'(x) = \infty$ and $C' > 0, C'' > 0, C''' \geq 0$ for all $x > 0$. As can be seen from (1), the technological structure and damage due to pollution for each country are identical.

The first-best abatement that maximizes the total payoff to all countries or, in other words, satisfies the Samuelson condition is $x_i = \Omega(n) \equiv C'^{-1}(n)$ for all $i \in N$, obtained as a solution of $\frac{\partial}{\partial x_i} \sum_{j \in N} \pi_j = 0$.[3] This condition can

[3] Since $C'(\cdot)$ is monotone, it has an inverse function.

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also be written as $\sum_{j\in N}\frac{1}{C'(x_j)}=1$. Put differently, the sum of all countries' ratios of marginal benefits from abatement to marginal costs should be equal to unity, which is the individual country's abatement necessary to increase the total abatement of n countries by one unit. The properties $\Omega'>0$ and $\Omega''\leq 0$ are derived from the assumptions on cost function C.[4] Let us assume that for each country, the emissions exceed $\Omega(n)$ in the current state. When each country decides an abatement noncooperatively, country i selects $x_i=\Omega(1)$ $(<\Omega(n))$, which maximizes its own payoff by solving $\frac{\partial\pi_i}{\partial x_i}=0$, that is, $\frac{1}{C'(x_i)}=1$; thus, letting each country act independently does not lead to an efficient outcome.

Let us examine the possibility of solving the problem by implementing an agreement on abatement x_i using the model of international environmental agreements. The game is divided into two stages. In the first stage, each country individually decides whether to participate in the agreement. In the second stage, all signatories and each non-signatory determine the amount of abatement noncooperatively. In this framework, when the number of countries is large and the cost function satisfies the properties described above, the agreement is shown to be not self-enforcing.

Proposition 1 *For the pollution-abatement agreement game with n symmetric countries, an efficient agreement among all countries is not self-enforcing when the number of countries is four or greater.*

Proof: The final payoff to a country is determined by the number of signatories and by whether the country accedes to the agreement. Let $s(=0,1,\ldots,n)$ denote the number of signatories. If the payoffs to signatories and non-signatories are written as $\pi^S(s)$ and $\pi^F(s)$, respectively, the condition for an agreement with all countries to be self-enforcing is

$$\pi^S(n)\geq\pi^F(n-1). \tag{2}$$

Equation (2) indicates that participating in an agreement would be rational for a country if all other countries participate in the agreement. When the number of signatories is s, the abatement of signatories is $\Omega(s)$. A simple

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[4] Let $x=\Omega(s)$. Although s is a nonnegative integer, we suppose that Ω can be defined for any real number between 0 and n. From the definition of Ω, we have $s=C'(x)$, and it follows that $\Omega'=dx/ds=(ds/dx)^{-1}=1/C''(x)>0$, and $\Omega''=d/ds(dx/ds)=(dx/ds)d/dx(1/C''(x))=-C'''(x)/\{C''(x)\}^3\leq 0$.

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calculation yields $\pi^S(n) = n\Omega(n) - C(\Omega(n))$ and $\pi^F(n-1) = (n-1)\Omega(n-1) + \Omega(1) - C(\Omega(1))$. The first proof in Appendix A shows that $\pi^S(n) < \pi^F(n-1)$ when $n \geq 4$, and thus that (2) does not hold. ■

Proposition 1 is considered to be a generalized version of Proposition 2 of Barrett (1994, p. 888), which uses specific functional forms to show that when n is sufficiently large, an efficient agreement by all countries is not self-enforcing. As presented in the proof above, country i will not join the agreement if all other countries do so, because noncooperative behavior outside the agreement would be more beneficial than entering the agreement. Since this kind of incentive works for all countries, an agreement by all countries is not self-enforcing when the number of countries is large.

Matching Agreement Game and the Solution Concept

We next introduce matching rules in the framework of section "International Environmental Agreements" and define the matching agreement. The game is divided into three stages. In the first stage, the matching rules are announced and each country individually decides whether to accede to the agreement. If there exist countries that accede to the agreement, signatories collectively determine a common matching rate by negotiation in the second stage.[5] In the third stage, each country determines its flat abatement noncooperatively, taking the matching rate as given. If no countries accede to the agreement in the first stage, the second stage will be skipped and each country will proceed directly to the third stage. In this case, the situation is the same as the standard noncooperative game.

Under the matching agreement, an additional abatement is imposed upon each signatory, which is the amount calculated by multiplying the matching rate fixed in the second stage by the total flat abatement determined by all other countries including non-signatories. We assume that the matching agreement has a certain binding authority and that signatories are committed to the matching rate and comply with the abatement settled in the agreement after the third stage.[6] As non-signatories do not commit to the

[5] It is possible to examine an alternative case where the matching rate between signatories is different from that between signatories and non-signatories. However, we find that the consideration of such a case does not alter the characteristics of the solution. Therefore, we only describe the case of a common matching rate.

[6] This assumption means that no signatory refuses the additional abatement derived from the matching agreement.

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matching rules, there is no matching component, and total abatements are the same as their flat abatements.[7] After the actual abatement, the final payoff to each country is determined.

Let $a_i(\geq 0)$ denote the flat abatement of country $i \in N$ and $b(\geq 0)$ the matching rate of the agreement. In addition, let S denote the set of signatories, while $s \equiv |S|$. If country i is a signatory, that is, $i \in S$, the total abatement imposed on i is $x_i = a_i + b\sum_{j \neq i} a_j$ according to the matching rules. If it is a non-signatory, that is, $i \notin S$, $x_i = a_i$. Therefore, from (1),

$$\pi_i = \begin{cases} \displaystyle\sum_{j \in S} a_j(1 + b(s-1)) + \sum_{j \notin S} a_j(1 + bs) - C\left(a_i + b\sum_{j \neq i} a_j\right) & (i \in S), \\ \displaystyle\sum_{j \in S} a_j(1 + b(s-1)) + \sum_{j \notin S} a_j(1 + bs) - C(a_i) & (i \notin S). \end{cases} \tag{3}$$

As the payoff depends on the flat abatement vector $\boldsymbol{a} \equiv (a_1, \ldots, a_n)$, matching rate b, and size of the agreement (i.e., number of signatories) s, we subsequently denote the right-hand side of the first and second lines of (3) by $\pi_i^S(\boldsymbol{a}, b, s)$ and $\pi_i^F(\boldsymbol{a}, b, s)$, respectively.

Let us define the solution of the matching agreement game. We solve the game backwards. In the third stage, given the size of the agreement and matching rate determined in the first and second stages, country i determines its flat abatement a_i as the optimal response to the set of flat abatements of other countries $\boldsymbol{a}_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$. The Kuhn–Tucker condition for maximizing country i's benefit is thus

$$\frac{\partial \pi_i}{\partial a_i} \leq 0, \quad \frac{\partial \pi_i}{\partial a_i} \cdot a_i = 0. \tag{4}$$

From (3), we obtain

$$\frac{\partial \pi_i}{\partial a_i} = \begin{cases} 1 + b(s-1) - C'\left(a_i + b\displaystyle\sum_{j \neq i} a_j\right) & (i \in S), \\ 1 + bs - C'(a_i) & (i \notin S). \end{cases}$$

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[7] Put differently, non-signatories commit to a matching rate of zero.

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When the optimal response $a_i^*(b, s, \boldsymbol{a}_{-i})$ takes a positive value as an interior solution, $\frac{\partial \pi_i}{\partial a_i} = 0$ leads to

$$a_i^*(b, s, \boldsymbol{a}_{-i}) = \begin{cases} \Omega(1 + b(s-1)) - b\sum_{j \neq i} a_j & (i \in S), \\ \Omega(1 + bs) & (i \notin S). \end{cases} \tag{5}$$

If the right-hand side of (5) is nonpositive for some i, then $a_i^*(b, s, \boldsymbol{a}_{-i}) = 0$ from (4). When all countries make optimal responses, that is, when the simultaneous equations $a_i^* = a_i^*(b, s, \boldsymbol{a}_{-i}^*) \forall\, i \in N$ are satisfied, the combination of flat abatement $(a_1^*, \ldots, a_n^*)$ is the equilibrium of the third stage. Below, we express this as $\boldsymbol{a}^*(b, s)$.

The solution of the second stage is b^* which satisfies

$$\sum_{j \in S} \pi_j^S(\boldsymbol{a}^*(b^*, s), b^*, s) \geq \sum_{j \in S} \pi_j^S(\boldsymbol{a}^*(b, s), b, s) \quad \forall\, b \geq 0. \tag{6}$$

Inequality (6) shows that the matching rate determined in the second stage is b^*, which maximizes the total payoff to signatories. As signatories make decisions given the size of the agreement, b^* is dependent on s. Therefore, this solution is expressed as $b^*(s)$.

Finally, the solution of the first stage is S^*, which satisfies the following equations simultaneously:

$$\begin{aligned} \forall\, i \in S^*, \quad & \pi_i^S(\boldsymbol{a}^*(b^*(s^*), s^*), b^*(s^*), s^*) \\ & \geq \pi_i^F(\boldsymbol{a}^*(b^*(s^*-1), s^*-1), b^*(s^*-1), s^*-1), \end{aligned} \tag{7}$$

$$\begin{aligned} \forall\, j \notin S^*, \quad & \pi_j^F(\boldsymbol{a}^*(b^*(s^*), s^*), b^*(s^*), s^*) \\ & > \pi_j^S(\boldsymbol{a}^*(b^*(s^*+1), s^*+1), b^*(s^*+1), s^*+1), \end{aligned} \tag{8}$$

where $s^* \equiv |S^*|$. Note that when $S^* = \emptyset (s^* = 0)$, the right-hand side of (7) is not defined and, therefore (8) is the only condition for $S^* = \emptyset$ to be the solution. Similarly, (7) is the only condition for $S^* = N(s^* = n)$ to be the solution. Inequality (7) shows that even if one country withdraws from the agreement of size s^*, the final payoff to that country does not increase. Inequality (8) shows that if one country outside the agreement enters into the agreement of size s^*, the final payoff to that country decreases. Inequalities (7) and (8) are the characteristics known as the *internal stability* and *external stability* of the agreement, and an agreement that satisfies these two

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characteristics is self-enforcing. If a certain s^* satisfies (7) and (8) in the first stage, no country has an incentive to change the decision pertaining to entering into the agreement, and an agreement of size s^* is realized as an outcome led by the equilibrium.

Solution of the Matching Agreement Game

In this section, we examine the solution of matching agreement games. We refer to the agreement in which all countries participate as the *full agreement*, and Lemma 1 shows that the matching rules realize an efficient outcome when the full agreement is formed.

Lemma 1 *For the pollution-abatement matching agreement game with n symmetric countries, the first-best abatement is led by the solutions of the second and third stages when the full agreement is formed in the first stage.*

Proof: Let b denote the matching rate determined in the second stage. We assume a symmetric equilibrium as the solution of the third stage, that is, $a_i^* = a_j^*$ for all $i, j \in S(=N)$. Subsequently, from (5), we have $a_i^* = \Omega(1 + b(n-1)) - b(n-1)a_i^*$ for all $i \in N$, and it follows that $a_i^* = \frac{\Omega(1+b(n-1))}{1+b(n-1)}$. The total abatement of country i becomes $x_i^* = a_i^* + b(n-1)a_i^* = \Omega(1+b(n-1))$. As $\Omega(\cdot)$ is a monotonically increasing function, x_i^* is equal to the first-best level $\Omega(n)$ if and only if $b = 1$. Therefore, the matching rate $b^* = 1$ is determined in the second stage, and an efficient outcome is realized. ∎

When $s = n$, (3) becomes $\pi_i = \sum_{j\in N} a_j(1 + b(n-1)) - C(x_i)$, and $\frac{\partial \pi_i}{\partial a_i} = 1 + b(n-1) - C'(x_i) = 0$ leads to

$$\frac{1}{C'(x_i)} = \frac{1}{1 + b(n-1)} \tag{9}$$

for all i. This abatement level satisfies the Samuelson condition when $b = 1$. Again, the right-hand side of (9) represents the abatement needed for country i to increase the total abatement of n countries by one unit. This can be considered as a Lindahl price, or the effective cost of abatement for country i.

Next, Lemma 2 shows that the full agreement is self-enforcing. This means that all countries accede to the agreement as a result of noncooperative decisions and that no single country has an incentive not to accede.

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Lemma 2 *In the first stage of the pollution-abatement matching agreement game with n symmetric countries, the full agreement is self-enforcing.*

Proof: As there are no countries outside the agreement, we only need to check the internal stability. We show that in the case where country i alone does not accede to the agreement ($S = N\backslash\{i\}$), it cannot increase its payoff compared with the case when it accedes to form the full agreement.

When $S = N\backslash\{i\}$ and the matching rate is b, from (5), the flat abatement determined by each country is calculated as follows:

$$\begin{cases} a_i^* = \Omega(1 + b(n-1)), \\ a_j^* = \max\left(\Omega(1 + b(n-2)) - b\displaystyle\sum_{k\neq j} a_k^*, 0\right) \quad (j \in S). \end{cases}$$

Again, we assume a symmetric equilibrium in the agreement, that is, $a_j^* = a_k^*$ for any $j, k \in S$. Assuming $a_j^* > 0$ for all $j \in S$, we obtain $a_j^* = \Omega(1 + b(n-2)) - b(a_i^* + (n-2)a_j^*)$, and it follows that $a_j^* = \frac{\Omega(1+b(n-2)) - b\Omega(1+b(n-1))}{1+b(n-2)}$. Consequently, the flat abatement determined by each signatory in the third stage is

$$a_j^* = \begin{cases} \dfrac{\Omega(1 + b(n-2)) - b\Omega(1 + b(n-1))}{1 + b(n-2)} & (b \in B_1), \\ 0 & (b \in B_2), \end{cases}$$

where $B_1 \equiv \{b \geq 0 \,|\, \Omega(1 + b(n-2)) \geq b\Omega(1 + b(n-1))\}$ and $B_2 \equiv \{b \geq 0 \,|\, \Omega(1 + b(n-2)) < b\Omega(1 + b(n-1))\}$. We have $B_1 \subset [0, 1)$ because $\Omega(1 + b(n-2)) < b\Omega(1 + b(n-1))$ holds when $b \geq 1$.

Let us derive the matching rate that maximizes the total payoff to signatories. When $b \in B_1$, the total abatement of each country in equilibrium is $x_i^* = \Omega(1 + b(n-1))$ and $x_j^* = \Omega(1 + b(n-2))(j \in S)$. Therefore, for all $j \in S$, the payoff in equilibrium is

$$\pi_j^S = \Omega(1 + b(n-1)) + (n-1)\Omega(1 + b(n-2)) - C(\Omega(1 + b(n-2))).$$

Considering this as a function of b and differentiating with respect to b, we have

$$\frac{d\pi_j^S(b)}{db} = (n-1)\Omega'(1 + b(n-1)) + (n-2)^2(1-b)\Omega'(1 + b(n-2)) > 0;$$

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thus, π_j^S is a monotonically increasing function in the range of $b \in B_1$. Hence, if we denote by $\bar{b}$ the maximum value of b that satisfies $\Omega(1 + b(n-2)) = b\Omega(1 + b(n-1))$, it follows that $\bar{b} \in (0, 1)$ and

$$\pi_j^S(\bar{b}) \geq \pi_j^S(b) \quad \forall\, b \in B_1. \tag{10}$$

Next, when $b \in B_2$, from $x_i^* = \Omega(1 + b(n-1))$ and $x_j^* = b\Omega(1 + b(n-1))$ $(j \in S)$, the payoff to country j in equilibrium is

$$\pi_j^S = \{1 + b(n-1)\}\Omega(1 + b(n-1)) - C(b\Omega(1 + b(n-1))).$$

Differentiating this with respect to b again, we obtain

$$\begin{aligned}\frac{d\pi_j^S(b)}{db} &= (n-1)\Omega(1 + b(n-1)) + \{1 + b(n-1)\}(n-1)\Omega'(1 + b(n-1)) \\ &\quad - \{\Omega(1 + b(n-1)) + b(n-1)\Omega'(1 + b(n-1))\} \\ &\quad \times C'(b\Omega(1 + b(n-1))).\end{aligned}$$

Using the second and third proofs in Appendix A, we can show the following inequalities:

$$\lim_{b \to \bar{b}+0} \frac{d\pi_j^S(b)}{db} > 0, \tag{11}$$

$$\frac{d\pi_j^S(b)}{db} < 0 \quad \forall\, b \geq 1. \tag{12}$$

From (10), (11), and (12), we have

$$\exists\, b' \in B_2 \cap (0, 1) \quad \text{s.t. } \pi_j^S(b') \geq \pi_j^S(b) \quad \forall\, b \geq 0,$$

which means that all signatories select flat abatements of zero, and that b', which maximizes π_j^S, exists in the range of $(0, 1)$.

The matching rate determined by the agreement is less than unity; thus, we obtain $x_i^* > x_j^*$, that is, the payoff to country i is smaller than the average payoff to all countries. Moreover, the total payoff is maximized when the full agreement is formed. Therefore, we have

$$\pi_i^F(n-1) < \frac{1}{n}\left\{\pi_i^F(n-1) + \sum_{j \in S} \pi_j^S(n-1)\right\} \leq \frac{1}{n}\sum_{j \in N} \pi_j^S(n) = \pi_i^S(n).$$

From the above, the payoff to country i is smaller than that when it accedes to the full agreement. Thus, there is no incentive for country i to refuse accedence to the agreement. ■

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Now, we can obtain the following proposition directly from Lemmas 1 and 2.

Proposition 2 *In the first stage of the pollution-abatement matching agreement game with n symmetric countries, there exists a self-enforcing solution that leads to an efficient outcome.*

Proof: Lemmas 1 and 2 show that the full agreement, which leads to an efficient outcome, is self-enforcing. ■

When the full agreement is formed, the matching rate is unity; thus, the total abatement of each country equals the sum of the flat abatements of all countries. Even if an individual country decides not to accede to the agreement, that country will become the sole contributor of a positive flat abatement, and therefore the matching rate determined by the agreement among the remaining countries will be less than unity. Given the commitment of a zero matching rate by the non-signatory, the marginal decrease in the matching rate from unity leads to less abatement for both signatories and the non-signatory. The benefit to signatories from less abatement of themselves exceeds the cost of less abatement of the non-signatory, and this brings about an incentive for signatories to collectively select a lower matching rate. Consequently, the payoff to the non-signatory decreases and the full agreement is thus self-enforcing. For the outcome led by the equilibrium, the equality of all countries' abatements means that the payoffs are also equal. This is a desirable outcome from the perspective of equity.

Conclusions

We have considered international agreements on matching rates as a model of transboundary environmental agreements and investigated their effectiveness.

In section ‘‘Model of Matching Agreements,’’ we first present Proposition 1, which states that efficient agreements are not normally self-enforcing in the game of international environmental agreement on pollution abatement. This proposition is regarded to be a generalization of the proposition provided by Barrett (1994). Subsequently, we present a model that includes matching rules. The players in the proposed model are symmetric countries whose benefits are specified as linear functions. In the first stage, the rules of

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matching are announced and each country individually decides whether to accede to the agreement. In the second stage, signatories collectively determine a common matching rate through negotiations. In the third stage, each country determines its flat abatement noncooperatively, taking the matching rate as given. Under the matching agreement, an additional abatement is imposed upon each signatory, which is the amount calculated by multiplying the matching rate fixed in the second stage by the total flat abatement determined by all other countries including non-signatories.

In section ‘‘Solution of the Matching Agreement Game,’’ we examine the solution of the matching agreement game formulated in section ‘‘Model of Matching Agreements.’’ First, we show that the full agreement concluded by all countries achieves an efficient outcome through equilibrium in the second and third stages (Lemma 1). Next, this agreement turns out to be self-enforcing, that is, it has internal stability (Lemma 2). From these lemmas, we show the existence of a self-enforcing agreement that achieves an efficient outcome (Proposition 2). This outcome is also equitable, because each country's total abatement is equal to the sum of the flat abatements of all countries, and thus each country obtains the same payoff. Since countries are symmetric, the equitable outcome is considered to be desirable. Each country enters into the agreement without being forced and the outcome achieved is efficient and equitable. Hence, we have shown the effectiveness of the matching agreement.

In a situation when it is difficult to prevent free riding in international environmental agreements, the concept of matching could provide a major clue. Unlike an agreement on the amount of pollution abatement, even if an individual country refuses to enter into the matching agreement, its payoff will not increase if other countries remain in the agreement. Therefore, there is no incentive to stay outside of the agreement. This is because the decisions of the matching rate and flat abatement in the second and third stages function as a punishment for the non-signatory. Moreover, as shown in Buchholz *et al.* (2009), such matching schemes are equivalent to the tax-subsidy schemes in Andreoni and Bergstrom (1996). Therefore, our results can be extended to the case of agreements on international tax-subsidy policies.

Future issues to be studied include the investigation of broader scenarios such as the cases when the benefit functions of the countries are more general, when players are asymmetric, and when a coalition of multiple countries become free riders (i.e., the problem of coalitional rationality).

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Appendix A Proofs

Proof That (2) Does Not Hold

Based on the discussions in the proof of Proposition 1, we have

$$\begin{aligned}\pi^F(n-1) - \pi^S(n) = &\ C(\Omega(n)) - C(\Omega(1)) - (n-1)\{\Omega(n) - \Omega(n-1)\} \\ &- \{\Omega(n) - \Omega(1)\}.\end{aligned} \tag{A1}$$

The convexity of the cost function $C(\cdot)$ and property $\Omega' > 0$ yield

$$\begin{aligned}C(\Omega(n)) &> C(\Omega(n-1)) + C'(\Omega(n-1))\{\Omega(n) - \Omega(n-1)\} \\ &= C(\Omega(n-1)) + (n-1)\{\Omega(n) - \Omega(n-1)\}.\end{aligned} \tag{A2}$$

Similarly we have

$$\begin{cases} C(\Omega(n-1)) > C(\Omega(n-2)) + (n-2)\{\Omega(n-1) - \Omega(n-2)\}, \\ \qquad\qquad \vdots \\ \quad C(\Omega(2)) > C(\Omega(1)) + \Omega(2) - \Omega(1). \end{cases} \tag{A3}$$

Substituting (A3) into (A2), we obtain

$$\begin{aligned}C(\Omega(n)) - C(\Omega(1)) > &\ (n-1)\{\Omega(n) - \Omega(n-1)\} \\ &+ (n-2)\{\Omega(n-1) - \Omega(n-2)\} \\ &+ \cdots + \Omega(2) - \Omega(1).\end{aligned} \tag{A4}$$

From (A1) and (A4), we have when $n \geq 4$

$$\begin{aligned}\pi^F(n-1) - \pi^S(n) &> (n-2)\{\Omega(n-1) - \Omega(n-2)\} \\ &\quad + \cdots + \Omega(2) - \Omega(1) - \{\Omega(n) - \Omega(1)\} \\ &= (n-2)\{\Omega(n-1) - \Omega(n-2)\} \\ &\quad + \cdots + \Omega(2) - \Omega(1) - \{\Omega(n) - \Omega(n-1)\} \\ &\quad - \{(\Omega(n-1) - \Omega(n-2)\} - \cdots - \{\Omega(2) - \Omega(1)\} \\ &> \{\Omega(3) - \Omega(2)\} - \{\Omega(n) - \Omega(n-1)\}.\end{aligned} \tag{A5}$$

Using (A5) and $\Omega'' \leq 0$, we can show $\pi^F(n-1) - \pi^S(n) > 0$.

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***Proof of* (10)**

The definition of $\bar{b}$ leads to $\Omega(1+\bar{b}(n-2)) = \bar{b}\Omega(1+\bar{b}(n-1))$. Therefore,

$$\begin{aligned}\left.\frac{d\pi_j^S(b)}{db}\right|_{b=\bar{b}} &= (n-1)\Omega(1+\bar{b}(n-1)) \\ &\quad + \{1+\bar{b}(n-1)\}(n-1)\Omega'(1+\bar{b}(n-1)) \\ &\quad - \{\Omega(1+\bar{b}(n-1)) + \bar{b}(n-1)\Omega'(1+\bar{b}(n-1))\} \\ &\quad \times C'(\bar{b}\Omega(1+\bar{b}(n-1))) \\ &= (n-1)\Omega(1+\bar{b}(n-1)) \\ &\quad + \{1+\bar{b}(n-1)\}(n-1)\Omega'(1+\bar{b}(n-1)) \\ &\quad - \{\Omega(1+\bar{b}(n-1)) + \bar{b}(n-1)\Omega'(1+\bar{b}(n-1))\}\{1+\bar{b}(n-2)\} \\ &= (1-\bar{b})(n-2)\Omega(1+\bar{b}(n-1)) \\ &\quad + (n-1)\{1+\bar{b}(1-\bar{b})(n-2)\}\Omega'(1+\bar{b}(n-1)).\end{aligned}$$

It is clear that (10) holds when $n \geq 2$.

***Proof of* (11)**

Since $b\Omega(1+b(n-1))$ is monotonically increasing with b, we have $b\Omega(1+b(n-1)) \geq \Omega(n)$. The monotonicity of $C'(\cdot)$ leads to $C'(b\Omega(1+b(n-1))) \geq C'(\Omega(n)) = n$. Hence, if $b \geq 1$,

$$\begin{aligned}\frac{d\pi_j^S(b)}{db} &\leq (n-1)\Omega(1+b(n-1)) + \{1+b(n-1)\}(n-1)\Omega'(1+b(n-1)) \\ &\quad - n\{\Omega(1+b(n-1)) + b(n-1)\Omega'(1+b(n-1))\} \\ &= -\Omega(1+b(n-1)) + (1-b)(n-1)\Omega'(1+b(n-1)) < 0,\end{aligned}$$

and (11) holds.

Appendix B Numerical Example

Let us explain how the matching agreement works by using a simple numerical example. We assume $n = 4$ and $C(x_i) = x_i^2/2$. The payoff to country i is

$$\pi_i = \sum_{i=1}^{4} x_i - \frac{x_i^2}{2}.$$

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The first-best abatements are $x_1 = \cdots = x_4 = 4$, and the corresponding payoffs are $\pi_1 = \cdots = \pi_4 = 8$; however, when each country decides its abatement noncooperatively, $x_1 = \cdots = x_4 = 1$ and each country's payoff decreases to 7/2.

Let us suppose that a full matching agreement has been concluded. The matching rate should be determined as unity in the second stage. Equation (5) becomes $a_i^* = 4 - 3a_i^*$ for all $i \in N$, and equilibrium in the third stage is the flat abatement vector $\boldsymbol{a}^* = (1, 1, 1, 1)$. In this case, $x_i^* = 4$ for all i is satisfied and efficiency is achieved. Each country's payoff is 8.

Suppose that only one country (e.g., country 1) does not accede to the agreement. If the matching rate determined by signatories (countries 2, 3, and 4) is b, from (5), the flat abatements become

$$a_1^* = 1 + 3b, \quad a_2^* = a_3^* = a_4^* = \begin{cases} 1 - \dfrac{b(1+3b)}{1+2b} & \left(0 \leq b \leq \dfrac{1+\sqrt{13}}{6}\right), \\ 0 & \left(b > \dfrac{1+\sqrt{13}}{6}\right), \end{cases}$$

and total abatements are

$$x_1^* = 1 + 3b, \quad x_2^* = x_3^* = x_4^* = \begin{cases} 1 + 2b & \left(0 \leq b \leq \dfrac{1+\sqrt{13}}{6}\right), \\ b(1+3b) & \left(b > \dfrac{1+\sqrt{13}}{6}\right). \end{cases}$$

Hence, the payoff to country $j (\neq 1)$ is

$$\pi_j^S = \begin{cases} 1 + 3b + 3(1+2b) - \dfrac{(1+2b)^2}{2} & \left(0 \leq b \leq \dfrac{1+\sqrt{13}}{6}\right), \\ 1 + 3b + 3b(1+3b) - \dfrac{b^2(1+3b)^2}{2} & \left(b > \dfrac{1+\sqrt{13}}{6}\right). \end{cases}$$

Since π_j^S takes a maximum value when $b = \frac{-1+\sqrt{145}}{12} \approx 0.92$, the matching rate determined in the second stage is approximately 0.92. Subsequently, the payoffs to signatories and the non-signatory (country 1) are calculated as

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approximately 8.15 and 7.07, respectively. The payoff to country 1 is smaller than that when it accedes to the full agreement; thus, there is no incentive for country 1 to stay outside of the agreement.

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